WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 4 – March 3, 2010

Round 1: Number Theory

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

Note: a subscript indicates a number's base

1. Express the sum $354_6 + 432_6$ as a base-six number.

2. The three-digit base-ten number $3A3$ is added to the base-ten number 424 to give the three-digit base-ten number $7B7$. If $7B7$ is divisible by 9, find the sum $A + B$.

3. The greatest common factor of A and B is 14, while the least common multiple of A and B is 168. Find the sum of all of the possible values of A.

ANSWERS

(1 pt.) 1._____ (base-6)

(2 pts.) 2. _____

(3 pts.) 3._____

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Round 2: Algebra 1 – Open

2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. A boy has nickels and quarters worth a total of \$6.00. If there are three times as many nickels as quarters, how many nickels does he have?

2. If $f(x) = 3x - 4$, $g(x) = ax + b$, and $f(g(x)) = g(f(x))$, find an expression for b in terms of a. Please express your answer as a single polynomial (that is, do not factor your expression).

3. A farmer has four cows that he bought for a total of \$800. The first cow cost as much as the total of the second cow and half of the third cow. The second cow cost as much as the fourth cow minus the cost of the third cow. Finally, the third cow cost one-third of the first cow. Compute the price of the first cow (in dollars).

ANSWERS

(1 pt.) 1._____ nickels

(2 pts.) 2. $b =$ _____

(3 pts.) 3. \$_____

Oxford, St. John's, Tahanto

WORCESTER COUNTY MATHEMATICS LEAGUE

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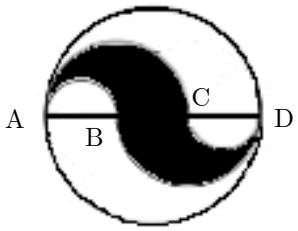
Round 3: Geometry – Open

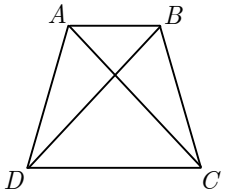
3

All answers must be placed in the answer section at the bottom
NO CALCULATOR ALLOWED The diagrams are NOT drawn to scale

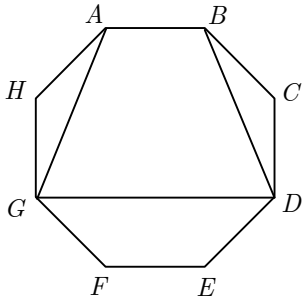
1. The diagram below shows a circle and four smaller semicircles with A, B, C, and D all collinear. If AB = BC = CD = 2, find the area of the shaded region in terms of π.



2. In isosceles trapezoid ABCD, with bases $\overline{AB}$ and $\overline{CD}$, the diagonals, $\overline{AC}$ and $\overline{BD}$ are perpendicular. If $AB = 4$ and $AD = 7$, compute CD in simplest radical form.



3. In the diagram below, $ABCDEFGH$ is a regular octagon with side length 12. When expressed in simplest radical form, the area of trapezoid $ABDG$ can be written as $a + b\sqrt{2}$, where a and b are positive integers. Find the sum $a + b$.



ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Nashoba (1 and 2), Westborough

WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 4: Logarithms, Exponents and Radicals

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. The expression $\left(\sqrt[3]{\sqrt[6]{x^9}}\right)^4 \cdot \left(\left[\left(x^9\right)^{\frac{1}{3}}\right]^{\frac{1}{6}}\right)^2$ can be written as x^n, where n is an integer. Find the value of n.

2. Find both real values of x that satisfy the equation

$$3x^2 - 4x - 6 + \sqrt{3x^2 - 4x - 6} = 12$$

3. If $\log_x 2 = \frac{2}{3}$ and $\log_x y = \frac{3}{2}$, find the numerical value of $\log_2\left(\sqrt{x} \cdot y^3\right)$.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____ and _____

(3 pts.) 3._____

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Round 5: Trigonometry – Open

5

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. If $0 < \theta < 2\pi$ and $\sec\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right) = \cos\left(\theta - \pi\right)$, compute the radian measure of angle θ (in terms of π).

2. Let x be a first quadrant angle such that $\sin x = 3\cos x$. Find the numerical value of the product $\sin x \cdot \cos x$.

3. Evaluate $\cos 15° - \sin 15°$ in simplest radical form.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. _____

(3 pts.) 3. _____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. When expanded, how many digits are in the base-ten number 2010^{2010}?

2. Compute the remainder when $17+177+1777+\ldots+17777777777777777777$ is divided by 8. Note that the last summand is 21 digits long; it is a 1 followed by twenty 7's.

3. Paul has twice as many brothers as sisters. His sister Mary has five times as many brothers as sisters. How many sons do their parents have?

4. Find the largest real value of b such that the solutions to the following equation are integers where $x \neq 1$: $\left(\log_{1024} x^{2b}\right)^2 = \log_{1024} x^4$. Please express your answer in simplest radical form.

5. Triangle A has sides of length 3, 5 and 7. Triangle B has sides of length 7, 8 and 13. Triangle A and triangle B have one angle measure in common. Find the measure of that angle in degrees.

6. Let x be an integer such that $-10 \le x \le 10$. If x is chosen at random, determine the probability that it will be a solution to the system of inequalities $|x-3| \ge 4$ and $x^2 \le 64$.

7. Find the sum of the infinite series $\frac{1}{8}+\frac{3}{64}+\frac{1}{512}+\frac{3}{4096}+\ldots$, where the denominators are consecutive powers of 8 and the numerators alternate between 1 and 3.

8. A *binary day* has the property that its representation in the form mm/dd/yy contains only 0's and 1's. For example, January 10, 2010 is a binary day because its representation is 01/10/10. How many binary days occur in any consecutive span of 100 years?

9. In quadrilateral $ABCD$, $\angle B = \angle C = 120°$, $AB = 2$, $BC = 4$, and $CD = 6$. Compute the area of $ABCD$ in simplest radical form.

Algonquin, Westborough (2 and 4), Nashoba (3 and 9), Doherty, Hudson, (6 and 7), QSC

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ANSWERS

Round 1

1. 1230 (or 1230_6)

2. 6

3. 280

Round 2

1. 45

2. $2-2a=-2a+2$ (only these possibilities)

3. 240

Round 3

1. 3π

2. $\sqrt{82}$

3. 360

Round 4

1. 3

2. 3 and $-\frac{5}{3}=-1\frac{2}{3}=-1.\overline{6}$

3. $\frac{15}{2}=7\frac{1}{2}=7.5$

Round 5

1. π

2. $\frac{3}{10}=0.3$

3. $\frac{\sqrt{2}}{2}$ (or any simplified equivalent)

Team Round

1. 6640

2. 4

3. 5

4. $\sqrt{10}$ (only)

5. 120° (or 120)

6. $\frac{10}{21}=0.\overline{476190}$
(any decimal approximation must be correct to at least three decimal places)

7. $\frac{11}{63}=0.\overline{174603}$
(any decimal approximation must be correct to at least three decimal places)

8. 36

9. $11\sqrt{3}$ (only)