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1. Successive discounts of $33\frac{1}{3}\%$, 20%, and 25% are equivalent to a discount of $37\frac{1}{2}\%$ followed successively by a discount of N%. Compute N.

2. Ginny withdrew some money from her bank account. She spent 5% of this amount on gas for her car, and then she spent one-third of the original amount on groceries. Finally, she spent \$96 on a present for her mom. The amount she had remaining was 1.25 times the amount she spent on groceries. How much did Ginny withdraw from the bank (in dollars)?

3. The king took a cup filled with water and drank one-fifth of its contents. When the king looked away, the court jester refilled the cup by adding alcohol to the remaining water and stirred. The king drank one-fourth of this mixture. When the king looked away again, the jester refilled the cup with more alcohol and stirred. The king drank one-third of this mixture. When the king looked away again, the jester refilled the cup with more alcohol. What percentage of the final mixture was alcohol?

ANSWERS

(1 pt.)   1._____

(2 pts.)   2. $ _____

(3 pts.)   3._____ %

St. John's, Assabet Valley, Algonquin

WORCESTER COUNTY MATHEMATICS LEAGUE
Varsity Meet 2 – December 2, 2009
Round 2: Algebra 1 - Open

$\triangle 2$

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. Find the value of k in the linear equation $3x + ky = 8$ if the line's x-intercept is twice its y-intercept.

2. Three whole numbers, when added two at a time, have sums of 763, 1003, and 1064. Compute the largest of the original three numbers.

3. Solve the following equation for x:

$$\left(\dfrac{1}{\dfrac{1}{x} - 1} + 1 \right) \cdot \left(\dfrac{2}{\dfrac{1}{x} + 1} - x \right) = \dfrac{3}{x+1}$$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

All answers must be placed in the answer section at the bottom

NOTE: The diagrams are not drawn to scale.

NO CALCULATOR ALLOWED

1. In the diagram below, $l \parallel m \parallel n$, triangle ABC is equilateral and $\angle BDF = 110°$. Find the measure of $\angle BCE$ in degrees.
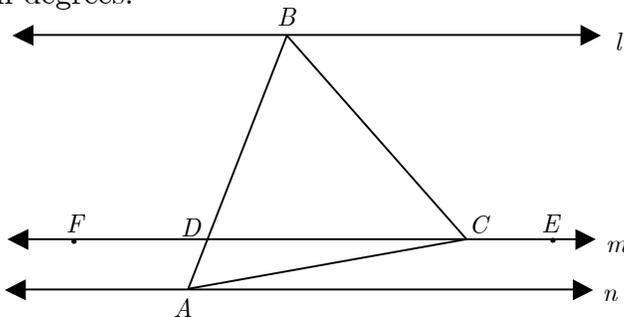
2. Using the diagram below, compute the largest angle (in degrees) in triangle ABC given that $\angle AXB = 42°$, $XB = DB$, $XA = EA$, and $AX = BX$.
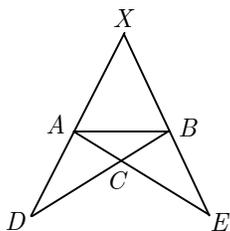
3. In the diagram below, $a \parallel b$, $\angle 3$ is acute, $\angle 1 = x^2 + 4y$, $\angle 2 = 5y + 15x$, and $\angle 3 = x + 4y + 6$. Compute the measure of $\angle 1$ in degrees.
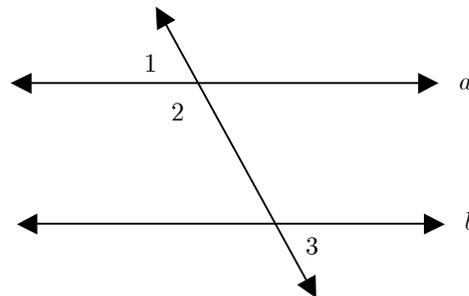
ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Worcester Acad., Hudson, Shrewsbury

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

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1. Consider the arithmetic sequence 5, 14, 23, 32,…. If 5 is the first term, and 239 is the nth term, compute n.

2. The sum of three numbers in an arithmetic sequence is 24 and the product of these three numbers is 440. Find the largest of the three numbers.

3. Let a, b, c represent three terms in a geometric sequence. The sum of these three terms is 13. When the third term is subtracted from the first term the result is 8. Find all possible ordered triples (a, b, c).

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

Tantasqua, Tahanto, Doherty

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. Let x and y be integers. Find the ordered pair (x, y) solution to the following system of equations:

$$\frac{5}{x} + \frac{6}{y} = 4 \text{ and } y = x^2$$

2. Find the values of a and b which make the following matrix equation true:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$$

3. The determinant $\begin{vmatrix} 4 & 1 & -1 \\ a & 3 & 3 \\ -2 & 0 & b \end{vmatrix}$ has the value -42. However, if a and b are

switched, the value of the new determinant is 18. Find all possible ordered pairs (a, b) for which this is true.

ANSWERS

(1 pt.)   1._____

(2 pts.)   2. $a = $_____ $b = $ _____

(3 pts.)   3._____

WORCESTER COUNTY MATHEMATICS LEAGUE
Varsity Meet 2 – December 2, 2009
TEAM ROUND

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded</u> <u>correctly to at least three decimal places,</u> unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. Consider a standard digital watch that displays only hours and minutes. What percentage of the day has the watch showing at least one "2"?

2. The quotient, difference and sum of two numbers are in the ratio 1:6:10, respectively. Find the two numbers.

3. A counting number N has four digits. The leading digit is removed and placed after the other three, forming a new four-digit number. When N is subtracted from the new number the result is 1,125. Find the smallest such N for which this is possible.

4. If $3 \leq y \leq 22$, for how many ordered pairs of positive integers (x, y) does y exceed x by at least 2?

5. The first two terms of an arithmetic progression are $\log_2 3$ and $\log_2 9$. If the sixth term is x, compute the numerical value of 2^x.

6. Three people take turns throwing two dice and the first player to throw a sum of seven wins. If no sum of seven is thrown in the first round, the process is repeated until someone wins. Find the probability that the first person to throw the dice eventually wins. Please express your answer as a fraction reduced to lowest terms.

7. The square of the sum of the digits of a two-digit number minus the sum of the squares of the digits is 36. Find the smallest possible two-digit number for which this is true.

8. Suppose that a, b, c, d, and e are numbers that satisfy the system of equations:
$$13a + 26b + 2c + 13d + 3e = 18$$
$$6a + 12b + c + 6d + e = 7$$
$$5a + 10b + c + 5d + e = 6$$
Find the value of e.

9. In the diagram the rows and columns of circles are perpendicular and equally spaced. How many squares can be formed by joining the centers of the circles?
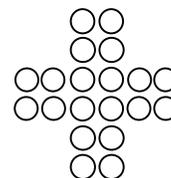
WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 2 – December 2, 2009

ANSWER SHEET – TEAM ROUND

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded</u> <u>correctly to at least three decimal places</u> unless stated otherwise (2 pts. each)

1._____ %

2._____

3._____

4. _____

5. _____

6._____

7._____

8._____

9._____

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 2 – December 2, 2009

ANSWERS

Round 1
1. 36

2. 480

3. 60

Round 2
1. 6

2. 652

3. 3

Round 3
1. 130   (or 130°)

2. 126   (or 126°)

3. 65   (or 65°)

Round 4
1. 27

2. 11

3. (9, 3, 1) and $\left(\dfrac{49}{3}, -\dfrac{35}{3}, \dfrac{25}{3} \right)$

   (need both sets of triples, and any exact equivalent of the second triple is OK)

Round 5
1. (2, 4)

2. $a = -4$ and $b = \dfrac{7}{2} = 3\tfrac{1}{2} = 3.5$ (need both)

3. (15, 10) and (2, −3)   (need both pairs)

Team Round

1. $37\tfrac{1}{2} = 37.5 = \dfrac{75}{2}$

2. 8 and 32   (need both, either order)

3. 1236

4. 210

5. 729

6. $\dfrac{36}{91}$ (only)

7. 29

8. 3

9. 21