WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

Round 1: Arithmetic

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. If $a \# b = a^b - a$, evaluate: $(3 \# 5) - (5 \# 3)$

2. If $a \oplus b$ represents $a\%$ of $(a+b)$, $p \otimes q$ represents $q\%$ of $(p-q)$, and $x \odot y$ represents $\frac{x}{y}\%$ of $(x+y)$, find the value of $(250 \oplus 50) \otimes (2400 \odot 3600)$.

3. Following the proper order of operations, evaluate:

$$16\tfrac{4}{5} \cdot \left[\frac{\frac{1}{3} \cdot \frac{5}{4} + \frac{7}{18}}{9\frac{2}{3}} \right] + \frac{6}{7} - \frac{9}{35}$$

ANSWERS

(1 pt.) 1.________________________

(2 pts.) 2.________________________

(3 pts.) 3.________________________

St. John's, Tahanto, Bromfield

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

Round 2: Algebra 1 - Open

2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. In woodshop, a student is asked to cut a board into three pieces such that the longest piece is three times the length of the middle-sized piece and the shortest piece is 10 inches shorter than the middle-sized piece. If the board is 50 inches long, how long should the student cut the longest of the three pieces (in inches)?

2. When a two-digit number is divided by the sum of its digits the quotient is 6 and the remainder is 8. If the units digit of the number is 3 less than the tens digit, find the number.

3. Find all of the values of x that satisfy the equation $|x+1|^2 - |6x+6| = 7$.

ANSWERS

(1 pt.) 1.__________________________ inches

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Westborough, Doherty, Worcester Acad.

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

Round 3: Set Theory

3

All answers must be placed in the answer section at the bottom

NOTE: $\overline{S}$ indicates the complement of the set S

NO CALCULATOR ALLOWED

1. Let $A = \{1,2,3,4,5,6\}$, $B = \{2,4\}$, $C = \{2,4,8,9\}$, $D = \{4,5\}$, $E = \{4,5,6,7,8,9\}$ and $F = \{2\}$. Which of the sets listed above equals the set X if $X \not\subset A$ and $X \not\subset C$? Give your answer as the letter name of the set.

2. Set A has 50 elements. How many subsets of A have exactly 48 elements?

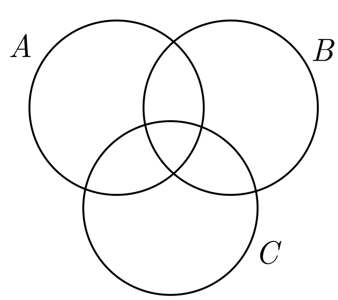
3. Using the Venn diagram in the answer section below, clearly shade the region represented by the set $\left(A \cap (B \cup C)\right) \cap \left(\overline{A} \cup \overline{B} \cup \overline{C}\right)$.

ANSWERS

(1 pt.) 1. ________________________

(2 pts.) 2. ________________________

(3 pts.) 3.



Doherty, South, Westborough

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

Round 4: Measurement

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. A square is inscribed in a circle whose area is 36π square millimeters. Find the area of the square (in square millimeters).

2. How many cubes, each 2 inches on an edge, are needed to make a volume equal to that of a rectangular solid whose dimensions are 1 foot by 2 feet by 4 feet.

3. A right circular cone whose height is 9 cm is inscribed in a sphere of radius 6 cm. In terms of π, find the volume of the region inside of the sphere but outside of the cone (in cubic cm).

ANSWERS

(1 pt.) 1.________________________ mm^2

(2 pts.) 2.________________________ cubes

(3 pts.) 3.________________________ cm^3

South, West Boylston, Hudson

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009
Round 5: Polynomial Equations

5

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. Find the real value(s) of x that satisfy the equation $6x^3 - 9x^2 + 2x - 3 = 0$.

2. The quadratic polynomial $\frac{1}{9}x^2 + \frac{4}{5}x + k$ is the perfect square of a binomial. Find the value of k.

3. If a, b, and c are the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, find the numerical value of $a^2 + b^2 + c^2$

ANSWERS

(1 pt.) 1.________________________

(2 pts.) 2.________________________

(3 pts.) 3.________________________

Quaboag, Hudson, Worcester Acad.

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. If $a * b = \dfrac{a+b}{b}$ and $a \# b = \dfrac{a-b}{a}$, find the value of $2 * \left(3^{2\#6}\right)$.

2. A number q is 5 less than twice another number p. If q is multiplied by three more than p, the result is 21. Find all possible pairs (p, q) that satisfy these conditions.

3. Let the universal set $U = \{\text{natural numbers between 5 and 20, inclusive}\}$. Also, let $A = \{6, 8, 10, 12, 14, 16, 18\}$, $B = \{7, 9, 11, 13, 15\}$ and $C = \{6, 8, 10, 15, 17, 19\}$. List the elements contained in the set $\overline{\left((A \cap B) \cup C\right)} \cap A$. (Note: $\overline{S}$ indicates the complement of set S.)

4. A jumbo chocolate bar, in the shape of a rectangular solid, measures 12 cm by 7 cm by 3 cm. Due to escalating costs, management decided to reduce the bar's volume by 10%. To accomplish this reduction, they decided that the new bar should still be 3 cm thick, but the other dimensions should be reduced by the same amount. To the ***nearest thousandth***, what is the surface area of the new bar (in square centimeters)?

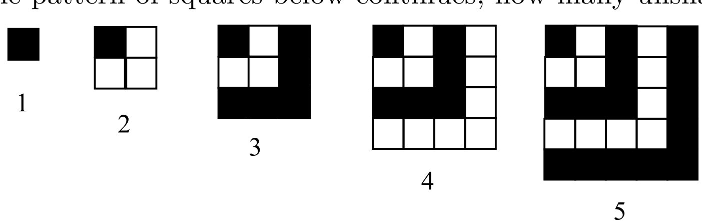
5. The equation $x^2 + x - 2009 = 0$ has two non-zero real roots m and n. Find the quadratic equation with integer coefficients in the form $ax^2 + bx + c = 0$ whose roots are $\dfrac{1}{m}$ and $\dfrac{1}{n}$.

6. If the hundreds digit of a three-digit number is increased by n, and both the tens and units digits are decreased by n, the new number formed is n times the original number. Find the original number.

7. A lattice point in the coordinate plane is a point whose coordinates are integers. How many lattice points lie on the line segment joining the endpoints (1, 1) and (101, 151)? Please count the endpoints as lattice points.

8. Sean, Amy, Dana, Rick and Joe are to be seated in a row. However, Rick and Joe refuse to sit next to each other. How many ways can the five people be seated if it is planned for Rick and Joe not to be seated next to each other?

9. If the pattern of squares below continues, how many unshaded squares will be in the 11^{th} diagram?



St. John's (1, 7), Bartlett, Leicester, Hudson (4, 6), Algonquin, Tantasqua, Tahanto

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 14, 2009

ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (2 pts. each)

1.________________________________

2.________________________________

3.________________________________

4. ________________________________ cm^2

5. ________________________________

6.________________________________

7.________________________________ points

8.________________________________

9.________________________________ squares

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 - October 14, 2009

ANSWERS

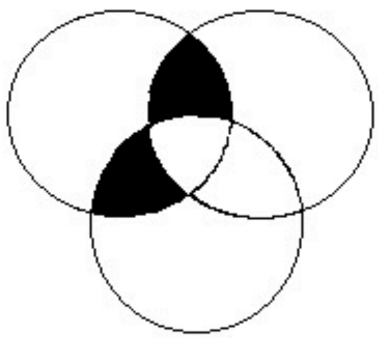
Round 1

1. 120
2. 284
3. 2

Round 2

1. 36
2. 74
3. 6, −8 (need both, in either order)

Round 3

1. E
2. 1,225
3. 

Round 4

1. 72
2. 1,728
3. 207π

Round 5

1. $\frac{3}{2} = 1.5 = 1\frac{1}{2}$
2. $\frac{36}{25} = 1\frac{11}{25} = 1.44$
3. -2

Team Round

1. 19
2. (4, 3) and $\left(-\frac{9}{2}, -14\right)$
   or equivalent, need both
3. {12, 14, 16, 18}
   (without { } OK)
4. 259.765 (only)
5. $2009x^2 - x - 1 = 0$
6. 178
7. 51
8. 72
9. 55