December 3, 2008 WOCOMAL Varsity Meet

Round 1: Fractions, Decimals, and Percent

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The x-value satisfying this equation is a fraction. When reduced to lowest terms, what is the positive difference of its numerator and denominator?

$$\frac{\dfrac{\dfrac{\frac{1}{2}+1}{3}+1}{4}}{} = \frac{\frac{1}{2}+x}{6}$$

2. At noon, a flock of geese land on a pond. At 1 PM, 20% of the flock flies away. At 2 PM, $\frac{1}{8}$ of the remaining geese fly away. At 3 PM, 3 times as many geese as had flown away at 1 PM fly away leaving 28 geese on the pond. How many geese landed on the pond at noon?

3. Wanted . . . a simplified improper fraction: $$\frac{\dfrac{\dfrac{\frac{1}{2}+3}{4}-5}{6}+7}{\dfrac{\dfrac{\frac{6}{7}+5}{4}-3}{2}+1}=?$$

ANSWERS

(1 pt.) 1. ______________________________

(2 pts.) 2. ________________________ geese

(3 pts.) 3. ______________________________

Nashoba, Quaboag, St. Peter-Marian

December 3, 2008 WOCOMAL Varsity Meet

Round 2: Algebra I (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Jack bought 10 boxes of tiles and 3 cans of paint for \$575. If a box of tiles costs twice as much as a can of paint, how much did he pay for all the paint?

2. Find the value of $(x+y)$ if $x^3+y^3=117$, $x^2y=-50$, and $xy^2=20$.

3. A boy was sent for \$1.20 worth of eggs. On his way home he broke five eggs and the actual cost was thereby \$0.24 more per dozen than the original purchase price. How many eggs did he buy?

ANSWERS

(1 pt.) 1. \$ ______________________

(2 pts.) 2. ______________________

(3 pts.) 3. ______________________ eggs

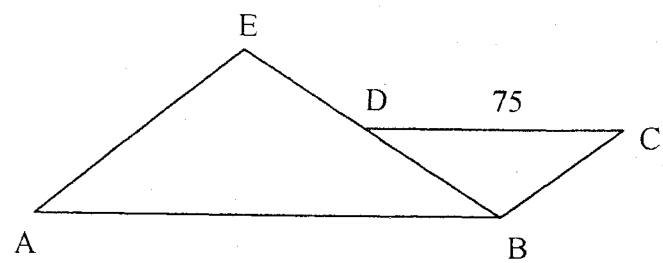
Hudson Catholic, St. John's, Shrewsbury

December 3, 2008 WOCOMAL Varsity Meet

Round 3: Parallel Lines and Polygons (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If ED=DB,
$\overline{BC} \parallel \overline{AE}$,
$\overline{CD} \parallel \overline{AB}$,
and $CD = 75$.
Find AB.



2. Each angle of a regular n-gon is 4° larger than an angle of a regular (n-1) gon. Find n.

3. When two parallel diagonals of a regular hexagon are drawn, a rectangle is formed. Calculate the area of this rectangle for a hexagon whose side is of length S. Leave answer in simple radical form.

ANSWERS

(1 pt.) 1. ______________________

(2 pts.) 2. ______________________

(3 pts.) 3. ______________________

Assabet Valley, Auburn, Doherty

December 3, 2008 WOCOMAL Varsity Meet

Round 4: Sequence and Series (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Starting with 1, what is the maximum number of consecutive integers that can be added together before the sum exceeds 200?

2. Find all values of k such that the sequence: $3k^2+k+1,\ 2k^2+k,\ 4k^2-6k+1$ is an arithmetic progression.

3. Find the n^{th} term of the sequence: 10, 16, 24, 46, 94, 180.

ANSWERS

(1 pt.) 1. ______________________

(2 pts.) 2. ______________________

(3 pts.) 3. ______________________

St. Peter-Marian, South, Tantasqua

December 3, 2008 WOCOMAL Varsity Meet

Round 5: Matrices (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. $\left[\left[\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}^T\right]^{-1}\right]^T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find a+b+c+d, where T stands for transpose.

2. Let $T=\begin{pmatrix} 3 & 0 \\ -4 & 5 \end{pmatrix}$, $U=\begin{pmatrix} -1 & 7 \\ 6 & -2 \end{pmatrix}$, and $V=\begin{pmatrix} 9 & -11 \\ -8 & -3 \end{pmatrix}$. Find $U+T-V^2$.

3. IF $M=\begin{pmatrix} -1 & 2 \\ 1 & y \end{pmatrix}$, find all values of y such that the determinant of $M^2+2M=-3$

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. $\begin{pmatrix} & \\ & \end{pmatrix}$

(3 pts.) 3. ____________________

Assabet Valley, Hudson, Southbridge

December 3, 2000 TEAM ROUND ANSWER SHEET WOCOMAL Varsity Meet

(2 points each)

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________

9. ______________________________

School ______________________________

Team # ______________________________

Student Names:

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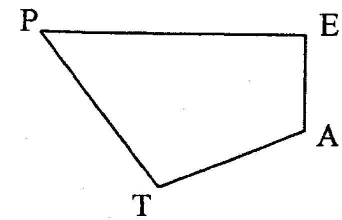
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND
ON THE SEPARATE TEAM ANSWER SHEET (2 points each)

1, If M is 30 % of Q, Q is 20 % of P, and N is 50 % of P, find the value of M/N in fraction form reduced to lowest terms.

2. Simplify: $\dfrac{\dfrac{x}{ym}-\dfrac{y}{xm}+\dfrac{m}{xy}}{\dfrac{1}{y^2m^2}-\dfrac{1}{x^2m^2}+\dfrac{1}{x^2y^2}}$



3. In the diagram quadrilateral PEAT contains right angles at vertices E and T.
If m$\measuredangle$ EAT=120°, AE=12, and AT=60, find PT. Write answer in simple radical form.

4. A pattern of integers is formed as follows:

$$3^2+4^2=5^2,\ 5^2+12^2=13^2,\ 13^2+n^2=(n+1)^2,\ (n+1)^2+y^2=(y+1)^2.$$

Find the sum of n and y.

5. If $\frac{1}{13}A=\frac{1}{7}B+\frac{1}{11}C$ and $\frac{1}{125}A=\frac{1}{5}B+\frac{1}{25}C$ then $C=\frac{P}{Q}A$. Find $\frac{\text{P}}{\text{Q}}$ as an improper fraction reduced to lowest terms.

6. For what values of k will the system: $x-12y+kz=26,\ 2x-3y+z=1$, and $-x-2y+5z=8$ have no solution?

7. Let A=$\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$, B=$\begin{pmatrix}6 & -1\\ 1 & 2\end{pmatrix}$, and C=$\begin{pmatrix}x\\ y\end{pmatrix}$. Find C^T(A+B)C. C^T is the transpose of C.

8. Apples cost 40 cents each. Pears cost 70 cents each. Joe has \$50 to spend on apples and pears for the office personnel. How many ordered pairs of the form (number of apples, number of pears) are possible if Jack is to spend all of the money and purchase at least one of each type of fruit?

9. Let the positive integers be arranged in the "spiral" pattern indicated in the diagram at the left. The sequence can be described as 1, 2, turn right, 3, turn right, 4, 5, turn right, 6, 7, turn right, 8, 9, 10, ... If this pattern continues, how many right turns will you make before you reach the integer 2008?

21	22	23	24	25	26
20	7	8	9	10	27
19	6	1	2	11	28
18	5	4	3	12	29
17	16	15	14	13	30
36	35	34	33	32	31

Auburn, Burncoat, Clinton, Doherty, Hopedale, Hudson, Shepherd Hill

December 3, 2008

WOCOMAL Varsity Meet ANSWERS

Round 1: Fractions, Decimals, and Percent

(1 pt.) 3

(2 pts.) 280

(3 pts.) $\dfrac{707}{26}$

Round 2: Algebra I

(1 pt.) 75

(2 pts.) 3

(3 pts.) 20

Round 3: Parallel Lines and Polygons

(1 pt.) 150

(2 pts.) 10

(3 pts.) $S^2\sqrt{3}$

Round 4: Sequence and Series

(1 pt.) 19

(2 pts.) 2, $\frac{1}{3}$ or $0.\overline{3}$ (need both answers)

(3 pts.) $2n^3 - 11n^2 + 25n - 6$

Round 5: Matrices

(1 pt.) 0.4 or $\frac{2}{5}$

(2 pts.) $\begin{pmatrix} -167 & 73 \\ 50 & -94 \end{pmatrix}$

(3 pts.) -3, 1 (need both answers)

December 3, 2008 TEAM ROUND ANSWERS WOCOMAL Varsity Meet

(2 points each)

1. $\frac{3}{25}$

2. xym (any order)

3. $28\sqrt{3}$ (accept only simple radical form)

4. 3696

5. $\frac{297}{260}$

6. 17

7. $\left(10x^2 + 3xy + 5y^2\right)$

8. 17

9. 88

------------------- Fractions, Decimals, and Percent --------------------

1. Multiply both sides of the equation by 24 to get 1+2+6=2+4x $\Rightarrow$ x=$\frac{7}{4}$. So the difference is $\boxed{3}$.

2. $x-.20x-\frac{1}{8}(.80x)-3(.20x)=28 \Rightarrow x-.90x=28 \Rightarrow .10x=28 \Rightarrow x=\boxed{280}$.

3. $$\dfrac{\dfrac{\dfrac{\frac{1}{2}+3}{4}-5}{6}+7}{\dfrac{\dfrac{\frac{6}{7}+5}{4}-3}{2}+1}=\dfrac{\dfrac{\dfrac{\frac{1}{2}+\frac{6}{2}}{4}-5}{6}+7}{\dfrac{\dfrac{\frac{6}{7}+\frac{35}{7}}{4}-3}{2}+1}=\dfrac{\dfrac{\frac{7}{8}-\frac{40}{8}}{6}+7}{\dfrac{\frac{41}{28}-\frac{84}{28}}{2}+1}=\dfrac{\frac{-33}{48}+\frac{336}{48}}{\frac{-43}{56}+\frac{56}{56}}=\dfrac{\frac{303}{48}}{\frac{13}{46}}=\frac{303}{48}\frac{56}{13}=\frac{303}{6}\frac{7}{13}=\frac{101}{2}\frac{7}{13}=\boxed{\frac{707}{26}}$$

------------------- Algebra I --------------------

1. 10T+3P=575, T=2P $\Rightarrow$ 20P+3P=575 $\Rightarrow$ P=25. Total cost of paint=$\boxed{\$75}$.

2. $(x+y)^3=(x^3+y^3)+3x^2y+3xy^2=117+3(-50)+3(20)=27 \Rightarrow (x+y)=\boxed{3}$.

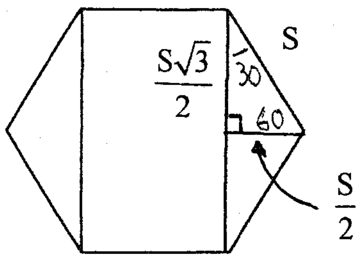
3. $\frac{(120)(12)}{x}$ = price/doz., $\frac{(120)(12)}{x-5}$ = newprice/doz. $\therefore \frac{(120)(12)}{x-5}-\frac{(120)(12)}{x}=24$. Multiplying both sides by $x(x-5)$ and solving the resulting quadratic equation $24x^2-120x-7200=0 \Rightarrow$ $x=\boxed{20}$ eggs.

-------------------- Parallel Lines and Polygons --------------------

1. $\triangle AEB \sim \triangle CBD$ because of the parallel sides. Since ED=DB, $\frac{\triangle AEB}{\triangle CBD}=\frac{2}{1}$. CD=75 $\Rightarrow$ AB=$\boxed{150}$.

2. Each interior angle of a regular polygon of n sides $=\frac{180(n-2)}{n}$. Therefore $\frac{(n-2)180}{n}-\frac{(n-3)180}{n-1}=4$. Multiply both sides of the equation by n(n−1) and solving the resulting quadratic equation $4n^2-4n+360=0 \Rightarrow n=\boxed{10}$.

3. The two smaller triangles on the right side of the regular hexagon of side S are 30-60-90 triangles. Using the 30-60-90 relationships for the sides of these triangles ⇒ that the dimensions for the rectangle are S and $S\sqrt{3}$.



Thus the area of the rectangle is $\boxed{S^2\sqrt{3}}$.

--------------------- Sequence and Series ---------------------

1. Using the formula for the sum of an arithmetic sequence, where the first term is 1 and the last term is n, solve $\frac{n}{2}(1+n)<200$ for the largest n possible. This implies that $n(n+1)<400$. Looking at factors of 400, 20 times 20 is too big. But 19 times 20 works. So $n = \boxed{19}$.

2. Given any three consecutive terms of an arithmetic progression: (1) The average of the first and third = the second term. Or (2) the second term minus the first term = the third term minus the 2nd term. Using either (1) or (2) leads to the quadratic equation $3k^2-7k+2=0$. Solving $k=\boxed{\frac{1}{3}, 2}$.

3. If we look at the finite differences of a sequence of terms, there is a polynomial function or pattern for generating the terms! It looks something like:

$f(n) = f(1) + (\text{first term of the first set of common differences})(n-1)$

$+$ (first term of the second set of common differences)$\left(\frac{(n-1)(n-2)}{2!}\right)$

$+$ (first term of the third set of common differences)$\left(\frac{(n-1)(n-2)(n-3)}{3!}\right)$

$+$ ...... until the common difference becomes constant.

Looking at the common differences of the terms:

n	1	2	3	4	5	6
$f(n)$	<u>10</u>	16	24	46	94	180
first common difference	<u>6</u>	8	22	48	86	
second common difference	<u>2</u>	14	26	38		
third common difference	<u>12</u>	12	12			

$$\text{Thus } f(n) = 10 + 6(n-1) + \frac{2(n-1)(n-2)}{2!} + \frac{12(n-1)(n-2)(n-3)}{3!}$$
$$= 10 + (6n-6) + (n^2 - 3n + 2) + (2n^3 - 12n^2 + 22n - 12)$$
$$= \boxed{2n^3 - 11n^2 + 25n - 6}.$$

------------------ Matrices and Systems of Equations ------------------

1. $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}^T = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$, $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}^{-1} = \dfrac{\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}}{\det\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}} = \dfrac{\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}}{10} = \begin{pmatrix} 0.3 & -0.1 \\ -0.2 & 0.4 \end{pmatrix}$,

$\begin{pmatrix} 0.3 & -0.1 \\ -0.2 & 0.4 \end{pmatrix}^T = \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix}$. *Therefore* $0.3 + (-0.2) + (-0.1) + 0.4 = \boxed{0.4}$

2. $\begin{pmatrix} -1 & 7 \\ 6 & -2 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ -4 & 5 \end{pmatrix} - \begin{pmatrix} 9 & -11 \\ -8 & -3 \end{pmatrix}\begin{pmatrix} 9 & -11 \\ -8 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 169 & -66 \\ -48 & 97 \end{pmatrix} = \boxed{\begin{pmatrix} -167 & 73 \\ 50 & -94 \end{pmatrix}}$

3. $\begin{pmatrix} -1 & 2 \\ 1 & y \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 1 & y \end{pmatrix} + 2\begin{pmatrix} -1 & 2 \\ 1 & y \end{pmatrix} = \begin{pmatrix} 3 & -2+2y \\ -1+y & 2+y^2 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 2 & 2y \end{pmatrix} = \begin{pmatrix} 1 & 2+2y \\ 1+y & 2+2y+y^2 \end{pmatrix}$

Now determinant of $\begin{pmatrix} 1 & 2+2y \\ 1+y & 2+2y+y^2 \end{pmatrix} = -3 \Rightarrow (2+2y+y^2) - (2+4y+2y^2) = -3$

$\Rightarrow y^2 + 2y - 3 = 0 \Rightarrow y = \boxed{-3,\ 1}$.

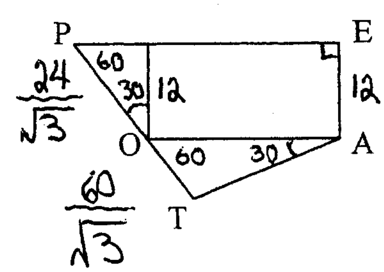
-------------------- Team --------------------

1. $M = 0.3Q$, Q=0.2P, N=0.5P $\Rightarrow$ M=0.3(.0.2P)=0.06P. Therefore $\dfrac{M}{N} = \dfrac{0.06P}{0.5P} = \dfrac{6}{50} = \boxed{\dfrac{3}{25}}$.

2. $$\frac{\dfrac{x}{ym} - \dfrac{y}{xm} + \dfrac{m}{xy}}{\dfrac{1}{y^2m^2} - \dfrac{1}{x^2m^2} + \dfrac{1}{x^2y^2}} \cdot \frac{(x^2y^2m^2)}{(x^2y^2m^2)} = \frac{x^3ym - xy^3m + xym^3}{x^2 - y^2 + m^2} = \frac{xym(x^2 - y^2 + m^2)}{x^2 - y^2 + m^2} = \boxed{xym}.$$

3. Make a rectangle as shown. The triangles that result are 30-60-90. Thus P0=$\dfrac{24}{\sqrt{3}}$ and TO=$\dfrac{60}{\sqrt{3}}$.

Hence PT=$\dfrac{84}{\sqrt{3}}$ or in simple radical form $\boxed{28\sqrt{3}}$.



4. Solve $13^2 + n^2 = (n+1)^2$ for $n \Rightarrow n = 84$. Replace n in the second equation to get $85^2 = (y+1)^2 - y^2$. Solve for y $\Rightarrow y = 3612$. Hence $n + y = \boxed{3696}$.

5. (One method of many) Multiply the first equation by $\frac{1}{5}$ and the second equation by $-\frac{1}{7}$ *to* get $\frac{1}{65}A = \frac{1}{35}B + \frac{1}{55}C$ and $-\frac{1}{875}A = -\frac{1}{35}B - \frac{1}{175}C$. Add the two equations to get $\left(\frac{1}{65} - \frac{1}{875}\right)A = \left(\frac{1}{55} - \frac{1}{175}\right)C$. Solve for C and work on combining and reducing the complex fractions. $C = \frac{\left(\frac{1}{65} - \frac{1}{875}\right)}{\left(\frac{1}{55} - \frac{1}{175}\right)}A$ or $C = \frac{\frac{810}{56875}}{\frac{120}{9625}}A \Rightarrow \frac{P}{Q} = \left(\frac{810}{56875}\right)\left(\frac{9625}{120}\right) = \boxed{\frac{297}{260}}$.

6. The system will have no solution when the determinant of $\begin{pmatrix} 1 & -12 & k \\ 2 & -3 & 1 \\ -1 & -2 & 5 \end{pmatrix} = 0$.

Thus $-15 + 12 - 4k - 3k + 2 + 120 = 0 \Rightarrow -7k + 119 = 0 \Rightarrow k = \boxed{17}$.

7. $(x \ \ y)\left(\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -1 \\ 1 & 2 \end{pmatrix}\right)\begin{pmatrix} x \\ y \end{pmatrix} = (x \ \ y)\left(\begin{pmatrix} 10 & 1 \\ 2 & 5 \end{pmatrix}\right)\begin{pmatrix} x \\ y \end{pmatrix} = (10x + 2y \ \ x + 5y)\begin{pmatrix} x \\ y \end{pmatrix}$

$(10x^2 + 2xy + xy + 5y^2) = \boxed{(10x^2 + 3xy + 5y^2)}$.

8. Let x= number of apples and y = number of pears. Then $.4x + .7y = 50$. Multiply by 10 and solve for x $\Rightarrow$ that $x = \frac{500 - 7y}{4}$. In order to keep x and y as whole numbers y has to be a multiple of 4 and $7y \leq 500$ or $y \leq 71$. Counting the multiples of 4 $\leq$ 71 gives $\boxed{17}$ possibilities.

9. Let n keep track of each group of 2 right turns. Then $f(n)$ indicates the number at which 2 more right turns have been taken. Note: $2n$ will be the number of right turns.

n	1	2	3	4	5	$f(n) = 3 + 4(n-1) + 2(n-1)(n-2)/2! = n^2 + n + 1$
$f(n)$	$\underline{3}$	7	13	21	31	Must find an n such that $f(n) \leq 2008$.
1st difference	$\underline{4}$	6	8	10		Picking n's finds $f(44) = 1981$ and $f(45) = 2071$.
2nd difference	$\underline{2}$	2	2			Hence $2(44) = \boxed{88}$.