Round 1:   Fractions, Decimals, and Percents

1.   Container A has a volume of $\dfrac{4}{5}\pi^2 x^8$. Container B has a volume of $\dfrac{5}{4}\pi^2 x^8$.
       What percent of B is the volume of container A?

2.   My favorite store was having a 15% off everything sale. The jeans I wanted
       were originally $\$49.99$, but they were already marked down to $\$35.99$
       before the 15% discount. When I bought the jeans, what percent discount off
       the original price did I pay? (Round to the nearest tenth of a percent.)

3.   Jan's income is $\dfrac{5}{8}$ that of Fran's. Jan's expenses are $\dfrac{1}{2}$ of Fran's expenses,
        and Jan saves 40% of her income. What percent of her income does Fran save?
       (Assume that what is not spent is saved.)

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Leicester, Bromfield, Athol

Round 2:   Algebra 1 (open)

1.   The arithmetic mean of two numbers is A, and one of the numbers is N.
      Write a simplified expression for the other number.

2.   Solve: $|x-3|+|3-x|=12$

3.   A farmer had a herd of 24 animals, some sheep and some goats. He sold the sheep
      for $140 each and the rest of the herd, the goats for $100 each. If he had sold the
      whole herd at the uniform price of $120 per animal, he would have gained 360
      more dollars. How many goats did he sell?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Tantasqua, Shrewsbury, Hudson

Round 3:   Parallel Lines and Polygons

1.   Lines *a* and *b* are parallel; lines *p* and *q* are perpendicular and they both intersect both *a* and *b*. In degrees, if the measure of the acute angle formed by *a* and *p* is *x*, what is the measure of the obtuse angle formed by *b* and *q*?

2.   In this badly drawn figure, $\overline{IJ} \parallel \overline{KL}$, $\overrightarrow{IM}$ bisects $\angle JIO$, and $\overrightarrow{NM}$ bisects $\angle LNO$. The angle measures $m\angle JIO$, $m\angle ION$, and $m\angle KNO$ are in the ratio 1 to 2 to 4. Find the degree measure of $\angle M$.
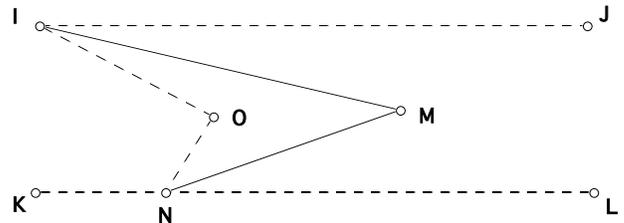
3.
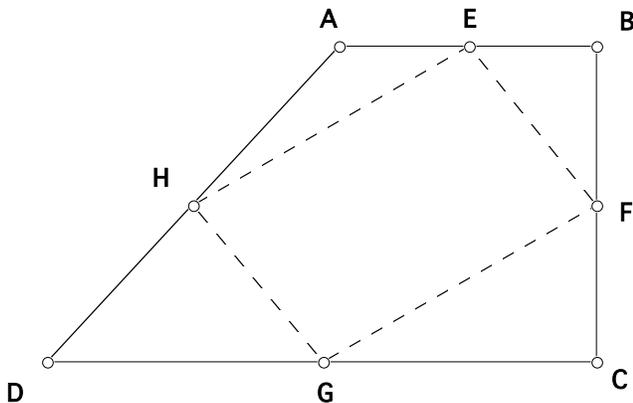
Angles B and C of quadrilateral ABCD are right angles. E, F, G, and H are the midpoints of its sides. AB = 6, BC = 8, CD = 15, and AD = $\sqrt{145}$. Compute the perimeter of EFGH.

ANSWERS

(1 pt.)   1. _____degrees

(2 pts.)   2. _____degrees

(3 pts.)   3. _____

Hudson, Auburn, Doherty

Round 4:   Sequences and Series   (**NO CALCULATORS**)

1.   The 4th term of a sequence is 4 and the 6th term is 6. Every term of the sequence
        is the sum of the two preceding terms. Find the 2nd term.

2.   Find the sum of all counting numbers less than 100 which are <u>not</u> multiples of 4.

3.   The sum of the first two terms of an infinite geometric series is 30, while the sum
        of the whole series is 32. What is the first term?

 (1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Westborough, Bromfield, Assabet

Round 5:   Matrices and Systems of Equations   (**NO CALCULATORS**)

1.   Let A and B be matrices such that $A^{-1} \cdot B = \begin{bmatrix} 8 & -12 \end{bmatrix}$.

What is the matrix $(2A)^{-1} \cdot B$?

2.   Solve this determinant equation for x.

$$\begin{vmatrix} x & 1 & 3 \\ 0 & 1 & 4 \\ 0 & 1 & x \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & x \\ \frac{1}{2} & 1 & 4 \end{vmatrix}$$

3.   Solve for matrix X: $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} 4 & 43 \\ 2 & 25 \end{bmatrix}$

ANSWERS

(1 pt.)   1. [_____]

(2 pts.)   2. _____

(3 pts.)   3. $X =$ [   ]

St. John's , Quaboag, Leicester

ALL ANSWERS MUST BE IN SIMPLEST FORM AND
ON THE SEPARATE TEAM ANSWER SHEET   (2 points each)

1.   If each foot of water screens out 60 % of the light above, what percent of light remains after passing vertically through 5 feet of water?

2.   If $x^2 + y^2 = 6xy \neq 0$, compute the numerical value(s) of $\dfrac{x+y}{x-y}$.

3.   Three regular polygons are used to tile the floor around point A, as shown. Two of them are n-gons and one is a p-gon. Find the sum of all possible p-values.
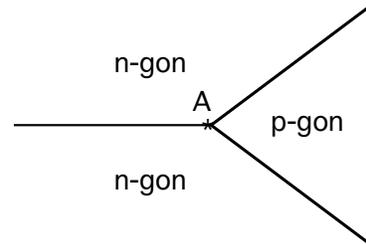
4.   After a period of bad weather, the king decreed that there would be no weather on even numbered days. During one month, three Wednesdays had no weather. On what day of the week did the first day of that month occur?

5.   Evaluate $\displaystyle\sum_{n=1}^{20} \log_2\left(\sqrt{2}\right)^n$.

6.   Solve this system for (x, y): $2^x \cdot 4^y = 16$ and $4^{3x-2} \cdot \left(\frac{1}{2}\right)^y = 128$

7.   If your school had 20 students coming to a WoCoMaL meet, in how many ways could the students be separated into four teams of 5? (Assume no other restrictions.) The correct answer is closest to 10^p. Write p as your answer.

8.   With numerical coefficient in simplified integer form, write the term in the expansion of $(a-b)^{17}$ which contains b^{11}.

9.   Starting with 100, the terms of a sequence are found by adding the previous term to its digits. How many of the terms of this sequence less than 200 are prime?

Mboro, WA, QSC, Wboro, NDA, Doh, North, Hopktn, Burn

_____

Round 1:   Fract, Dec & %

(1 pt.)   64 %

(2 pts.)   38.8 %   (% symbol
                                    is optional
                                    throughout)

(3 pts.)   25 %

_____

Round 2:   Algebra 1

(1 pt.) $2A - N$

(2 pts.)   -3 , 9   (need both)
            or   -3 or 9
            or   -3 and 9

(3 pts.)   21

_____

Round 3:   Par Lins & Polys

(1 pt.) $90 + x$

(2 pts.)   36

(3 pts.)   27

_____

_____

Round 4:   Seqs & Sers

(1 pt.)   6

(2 pts.)   3750

(3 pts.)   24 , 40   (need both)
            or 24 or 40
            or 24 and 40

_____

Round 5:   Mats & Systs

(1 pt,)   4 -6   or   4 , -6

(2 pts.)   -1, 5   (need both)
            or -1 or 5
            or -1 and 5

(3 pts.) $\begin{bmatrix} 0 & 7 \\ 2 & 4 \end{bmatrix}$

_____

December 5, 2007   TEAM ROUND ANSWERS (2 points each)

1. $1.024\,\%$ *or* $\dfrac{128}{125}\,\%$ (% symbol is optional)

2. $\pm\sqrt{2}$ or $\pm\,2^{\frac{1}{2}}$ or ± 1.414

3.   23

4.   Tuesday

5.   105

6. $(2,1)$

7.   10

8. $-12376\,a^{6}\,b^{11}$

9.   6

Total Points for Team Round: _____