

**Worcester County Mathematics
League**

WOCOMAL Varsity Meet #2

Coaches' Booklet

December 6, 2006

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WOCOMAL Varsity Meet

Round 1: Fractions, Decimals, and Percents

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. An item for sale is marked down 20%. By what percent must it then be marked up in order to return to the original selling price?

2. Both solution X and solution Y contain alcohol and water. In solution X the ratio of alcohol to water is 3:2. When equal amounts of solution X and solution Y are mixed, the ratio of alcohol to water is 3:4. What is the ratio of alcohol to water in solution Y? *Write your answer in the form A:W.*

3. The decimal expansion of $\left(\frac{1}{2}\right)^{100}$ has what digit for its third nonzero digit from the right?

ANSWERS

- (1 pt.) 1. _____%
- (2 pts.) 2. _____:
- (3 pts.) 3. _____

Bancroft, Quaboag, and Shepherd Hill

December 6, 2006

WOCOMAL Varsity Meet

Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $xy = 2$ and $x^2 + y^2 = 5$, then $\frac{x}{y} + \frac{y}{x} = ?$

2. If Bryan wants to mail a package, which requires \$1.53 in postage, and has only 5-cent and 8-cent stamps, what is the smallest number of stamps he could use to total \$1.53?

3. If Beth gets a 97 on her next math test, her average will be 90. If she gets 73, her average will be 87. How many tests has Beth already taken?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Bromfield, Leicester, and Notre Dame

December 6, 2006

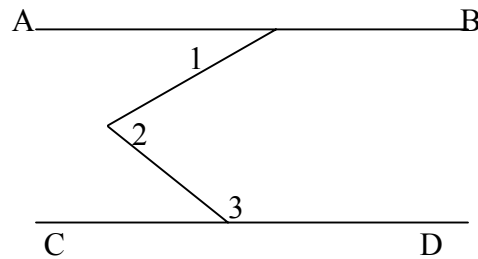
WOCOMAL Varsity Meet

Round 3: Parallel Lines and Polygons

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. A regular polygon has an interior angle that measures 144 degrees, and a side of which is 12 units long. What is the perimeter of the regular polygon?

2. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel.
 $\angle 1 = 90 - 2x$; $\angle 2 = 50 + x$;
 $\angle 3 = 3x + 10$.
What is the value of x *in degrees*?



3. If a regular octagon has a side-length 1, find the distance between opposite sides.

ANSWERS

- (1 pt.) 1. _____
(2 pts.) 2. _____ degrees
(3 pts.) 3. _____

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WOCOMAL Varsity Meet

Round 4: Sequences and Series (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Evaluate the sum: $1-2+3-4+5-6+7-8+9-10+\dots-998+999-1000$

2. Five numbers are in geometric progression. If the third number is 13, what is the product of the first number and the fifth number?

3. Evaluate: What is the sum of all the *one's digits* in the following finite sequence?

$$3^0, 3^1, 3^2, 3^3, \dots, 3^{2006}$$

ANSWERS

- (1 pt.) 1. _____
- (2 pts.) 2. _____
- (3 pts.) 3. _____

Tahanto, Hudson, and West Boylston

December 6, 2006

WOCOMAL Varsity Meet

Round 5: Matrices and Systems of Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $\mathbf{A} = \begin{bmatrix} 3 & -7 \\ -4 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & -4 \\ 1 & -9 \end{bmatrix}$, find the sum of the elements of $\mathbf{AB}$.

2. Find the difference between the largest and smallest values of x for which

$$\begin{vmatrix} x & -1 & 3 \\ 2 & x & 1 \\ -1 & 0 & 2 \end{vmatrix} = 10$$

3. Consider the system of equations:

$$\begin{aligned} x + y + \alpha z &= 14 \\ \alpha x + 2y + 4z &= 7 \\ 2x + \alpha y + 3z &= 0 \end{aligned}$$

For what value(s) of α does the system not have a unique solution for x , y , and z ?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

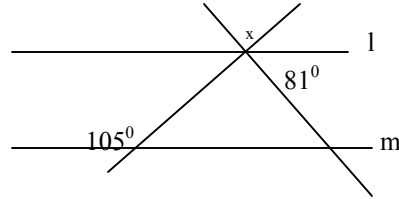
West Boylston, Bromfield, and Worcester Academy

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET.

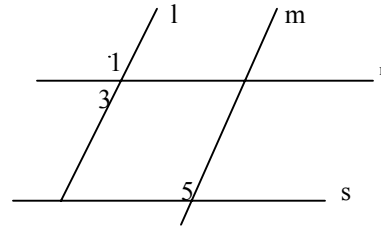
(2 points each)

1. In the adjacent diagram $l \parallel m$. Find x *in degrees*.



2. Find the sum $\sum_{n=1}^{\infty} 4 \left(\frac{1}{3^{n-1}} \right) \left(\frac{1}{x^{-n}} \right)$.

3. In the figure, $l \parallel m$ and $r \parallel s$. If $m\angle 1 = 2x + 3y$, $m\angle 3 = x + y$, and $m\angle 5 = 2x + y + 30^\circ$, find x *in degrees*.



4. Determine the value of the product: $(1 - \frac{1}{2^2})(1 - \frac{1}{3^2})(1 - \frac{1}{4^2}) \dots (1 - \frac{1}{99^2})(1 - \frac{1}{100^2})$.

5. $\sum_{n=0}^{\infty} n \left(\frac{1}{3} \right)^n = ?$

6. How many whole numbers between 1 and 100 inclusive can be written as a sum of integers constructed by using each of the digits 0 through 9 exactly once. (For example, $0 + 1 + 52 + 3 + 4 + 6 + 7 + 8 + 9 = 90$ is one.)

7. Factor completely over the set of **REAL** numbers: $x^4 - x^3 - 5x^2 + 3x + 6$.

8. A clockmaker wants to design a clock such that the area swept by each hand (second, minute, and hour) in one minute is the same (with all hands moving continuously). What is the length of the hour hand divided by the length of the second hand?

9. A 2×2 matrix A is such that $A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find the sum of the elements of A .

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WOCOMAL Varsity Meet ANSWERS

Round 1: Fractions, Decimals, and Percents

1. (1 pt.) 25
 2. (2 pts.) 9:26
 3. (3 pts.) 6
-

Round 2: Algebra 1-open

1. (1 pt.) $\frac{5}{2}, \text{ or } 2\frac{1}{2}, \text{ or } 2.5$
 2. (2 pts.) 21
 3. (3 pts.) 7
-

Round 3: Parallel Lines & Polygons

1. (1 pt.) 120
 2. (2 pts.) 35
 3. (3 pts.) $1 + \sqrt{2}$
-

Round 4: Sequences and Series

1. (1 pt.) -500
 2. (2 pts.) 169
 3. (3 pts.) 10033
-

Round 5: Matrices

1. (1 pt.) 52
 2. (2 pts.) $\frac{7}{2}$ or $3\frac{1}{2}$ or 3.5
 3. (3 pts.) $-1 \pm 2\sqrt{2}, 2$
-

TEAM ROUND (2 pts. Each)

1. 24

2. $\frac{12x}{3-x}$

3. 40

4. 0.505 or $\frac{101}{200}$

5. $\frac{3}{4}$ or 0.75

6. 7

7. $(x-2)(x+1)(x+\sqrt{3})(x-\sqrt{3})$

8. $12\sqrt{5}$

9. 5

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TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. _____degrees

2. _____

3. _____degrees

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

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WOCOMAL Varsity Meet

TEAM ROUND

School: _____

Team #: _____

Team Members:

1. _____

2. _____

3. _____

4. _____

5. _____