Worcester County Mathematics League

WOCOMAL Varsity Meet #2

Coaches' Booklet

December 1, 2004

December 1, 2004 **WOCOMAL Varsity Meet**

Round 1: Fractions, Decimals, and Percents

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What number must be added to both the numerator and the denominator of ¼ to make the value of the resulting fraction ¾ ?

2. In the expression xy^2, the values of x and y are each decreased by 25%. To the nearest whole percent, compare the decreased value to original value.

3. A chemist has a solution consisting of 5 ounces of propanol and 17 ounces of water. She would like to change the solution into a 40% propanol solution by adding z ounces of propanol. How many ounces of propanol should she add?

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Mass Academy, Quaboag

December 1, 2004 **WOCOMAL Varsity Meet**

Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If the reciprocal of $(x + 1)$ is $(x - 1)$, find all possible values of x.

2. If $(3x + 2)(2x - 1)(3x - 2) = ax^3 + bx^2 + cx + d$, then determine the value of $(a + b + c + d)$.

3. Let x and y be two real numbers satisfying $x + y = 6$ and $xy = 7$. Determine the value of $x^3 + y^3$.

ANSWERS

(1 pt.) 1. ________________

(2 pts.) 2. ________________

(3 pts.) 3. ________________

Burncoat, Bartlett, and Tantasqua

December 1, 2004 **WOCOMAL Varsity Meet**

Round 3: Parallel Lines and Polygons

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. An exterior angle of a regular polygon is 1/35 th of the interior angle. How many sides does the polygon have?

2. If the number of sides of an n-sided regular polygon is increased by 4, each interior angle is increased by 32°. How many sides are in the newly formed regular polygon?

3. In triangle ABC, Z is a point on side AB and Y is a point on side AC such that ZY is parallel to side BC. In addition, $AZ = x - 1$, $AY = x + 2$, $YC = x - 4$, $ZY = x + 5$, and $BC = 2x + 2$. Determine the value of $(x + ZB)$.

ANSWERS

(1 pt.) 1. ________________

(2 pts.) 2. ________________

(3 pts.) 3. ________________

Hudson, Assabet Valley

December 1, 2004 **WOCOMAL Varsity Meet**

Round 4: Sequences and Series (NO CALCULATORS)

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1. In an arithmetic sequence, the first term is 4 and the common difference is –2. Determine the value of the 99^{th} term.

2. If x, $2x + 2$, $3x + 3$ are a geometric progression, what is the 4^{th} term ?

3. For what values of x does the infinite geometric series converge?

$$x^2/3 - x^4/6 + x^6/12 - \ldots$$

ANSWERS

(1 pt.) 1. ________________

(2 pts.) 2. ________________

(3 pts.) 3. ________________

Bromfield, Burncoat, Clinton

December 1, 2004 **WOCOMAL Varsity Meet**

Round 5: Matrices and Systems of Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $\begin{matrix} 2 & 0 \\ -2 & 1 \end{matrix} \quad \begin{matrix} x \\ y \end{matrix} \quad \begin{matrix} 6 \\ -7 \end{matrix}$, determine the value of (x + y).

2. Determine the value of x if the following determinant has a value of zero.

$$\begin{matrix} x & x & 1 \\ 2 & 0 & 5 \\ 6 & 7 & 1 \end{matrix}$$

3. Given $A = \begin{matrix} 3 & 1 & 0 \\ 6 & 4 & 0 \\ 2 & 3 & 1 \end{matrix} \qquad B = \begin{matrix} 2 & 1 & 0 \\ 3 & 3 & 9 \\ 6 & 4 & 6 \end{matrix}$

Determine $(B - 2A)^2$

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Notre Dame Academy, Tantasqua, and Westborough

December 1, 2004 **WOCOMAL Varsity Meet**

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. **(2 points each)**

1. The sum of 40% of 1/4 and 50% of 1/5 is a/b where a and b are relatively prime. What is the value of (a + b)?

2. Write an equation expressing a in terms of just b and c if $1/x + 1/y = 1/a$, $x + y = b$, and $x^2 + y^2 = c^2$

3. A circle passes through two adjacent vertices of a square and is tangent to the opposite side of the square. If the side length of the square is 2, what is the radius of the circle?

4. If $a_0 = 1$, $a_1 = 3$, $a_2 = 7$, and $a_{n+3} = 3a_{n+2} - 3a_{n+1} + a_n$ for $n \geq 0$, then what is a general formula for a_n? Write your answer in terms of n.

5. Determine the value of (x + y + z) if,

$$\begin{aligned} 3x - 2y - 3z &= -1 \\ 6x + y + 2z &= 7 \\ 9x + 3y + 4z &= 9 \end{aligned}$$

6. What is the largest integer k such that $n^3 - n$ is divisible by k for all positive integers n? .

7. In racing over a given distance at uniform speed Bryan can beat Sean by 20 meters, Sean can beat Stephen by 10 meters, and Bryan can beat Stephen by 28 meters. How many meters is the distance over which they are racing?

8 Chris and Dick take turns, with Chris going first. Chris tosses a fair coin at each of her turns, while Dick rolls a fair die at each of his turns. The game ends when either Chris tosses a HEAD, in which case Chris wins, or Dick rolls a 3, in which case Dick wins. What is the probability that Dick wins?

9. Each side of triangle ABC has length 2. A circle with center at A and radius 1 cuts AB at M. A tangent to the circle from B and lying outside the triangle meets the circle at P. What is the area of the region bounded by BP, BM, and the minor arc MP?

Mass Academy, Shepherd Hill, Bromfield, Worcester Academy, St. John's

December 1, 2004 **WOCOMAL Varsity Meet ANSWERS**

Round 1: Fractions, Decimals, and Percents

1.	**(1 pt.)**	**8**
2.	**(2 pts.)**	**42**
3.	**(3 pts.)**	**$6\frac{1}{3}$**

Round 2: Algebra 1-open

1.	**(1 pt.)**	**$\pm\sqrt{2}$**
2.	**(2 pts.)**	**5**
3.	**(3 pts.)**	**90**

Round 3: Parallel Lines & Polygons

1.	**(1 pt.)**	**72**
2.	**(2 pts.)**	**9**
3.	**(3 pts.)**	**9**

Round 4: Sequences and Series

1.	**(1 pt.)**	**-192**
2.	**(2 pts.)**	**-13.5**
3.	**(3 pts.)**	**$-\sqrt{2} < x < \sqrt{2}$**

Round 5: Matrices

1.	**(1 pt.)**	**2**
2.	**(2 pts.)**	**2**
3.	**(3 pts.)**	$\begin{matrix} 25 & 9 & -9 \\ 99 & 16 & -9 \\ 18 & 0 & -2 \end{matrix}$

<u>TEAM ROUND (2 pts. Each)</u>

1. 6

2. $a = (b^2 - c^2)/(2b)$

3. 5/4

4. $n^2 + n + 1$

5. 2/3

6. 6

7. 100

8. 1/7

9. $(\sqrt{3})/2 - \pi/6$

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TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. **(2 points each)**

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________

9. ______________________________

December 1, 2004 **WOCOMAL Varsity Meet**

TEAM ROUND **School: ________________________**

Team #:________________________

Team Members:

1. ______________________________________
2. ______________________________________
3. ______________________________________
4. ______________________________________
5. ______________________________________