Worcester County Mathematics League

WOCOMAL Varsity Meet #1

Coaches' Booklet

October 8, 2003

Round 1:   Arithmetic (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Simplify completely:

$$(1997-1904)(1997-1905)(1997-1906)......(1997-2003).$$

2.   If $a * b = (a^2 - b^2) / (a - b)$, determine the value of $(a * b) * c$ where $a = 3$, $b = 4$, and $c = 5$.

3.   If $y_n = 1 + 1/(x_n - 1)$, and $x_1, x_2, \ldots, x_9$ equals $-2,-3,-4,\ldots,-10$, respectively, then determine the product of $y_1 * y_2 * y_3 * \ldots * y_9$.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bartlett, Bancroft, Worcester Academy

Round 2:   Algebra (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   The average of 10 numbers a,b,c,d,…,i,j is 190. If "a" is replaced by "2a", the
         new average is 500. What is the value of "a"?

2.   A boy buys oranges at 3 for $1 and will sell them at 5 for $2. If he wishes to
         make a profit of $10, how many oranges must he sell?

3.   The number 200 is increased by a certain percent. The result equals the number
         obtained when 800 is decreased by the same percent. What is the percent?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Hudson, Bromfield, Doherty

Round 3:   Set Theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Set A contains 15 elements, set B contains 16 elements, and A∩B contains 8 elements.. How many elements are contained in A U B where U means "union" and ∩ means "intersection"?

2.   A set containing k elements has 16 more subsets than a set containing k – 1 elements. Determine the value of k.

3.   A survey was taken by 99 people on places they liked to vacation. 57 people liked Yosemite National Park, 46 people liked Disneyworld, and 45 people liked Cape Cod. 7 people liked only Disneyworld, 8 people liked only Disneyworld and Cape Cod, and 6 people liked all three destinations. How many people liked only Yosemite National Park?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Algonquin, Bartlett, West Boylston

Round 4:   Measurement

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM. RADICALS MUST BE WRITTEN IN SIMPLEST RADICAL FORM.

1.   Determine the area of an isosceles trapezoid with sides of lengths 5,8,5,14 units.

2.   If the area of a regular hexagon inscribed in a circle whose circumference is 12π is written in the form of $a\sqrt{b}$, then what is the numerical value of (a+b) ?

3.   If a 30° - 60° - 90° triangle has a perimeter of 12 units and its area is in the form of $(a\sqrt{b} - c)$, then determine the numerical value of $(a + b + c)$.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bromfield, Quaboag, Notre Dame Academy

Round 5:   Polynomial Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Determine the absolute value of the difference of the roots of: $x^2 - 7x - 9 = 0$.

2.   If the simplified cubic equation whose roots are double the roots of

       $x^3 + x^2 + x + 1 = 0$ is written in the form of $x^3 + ax^2 + bx + c = 0$, then what is the

       numerical value of $(a + b + c)$?

3.   Determine all solutions of the equation: $2x^4 - 5x^3 - 12x^2 - x + 4 = 0$.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bromfield, Worcester Academy, Shepherd Hill

TEAM ROUND

**ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE
SEPARATE TEAM ANSWER SHEET.**   **(2 points each)**

1.   The cube of a two-digit positive number has four digits, the last of which is a 3.
        What is the number?

2.   If the graphs of the equations $2y + x + 3 = 0$ and $ax + 3y + 2 = 0$ intersect at
        right angles in the plane, what is the numerical value of "a"?

3.   If the sum of the numerical values of the area and perimeter of a triangle whose
        sides have lengths of 7,8,9, is $a + b\sqrt{c}$, then what is the numerical value of
        $(a + b + c)$? **RADICALS MUST BE WRITTEN IN SIMPLEST RADICAL
        FORM.**

4.   Determine the number of solutions in: $\{1, 2\} \subseteq X \subseteq \{1,2,3,4,5\}$ where X is a set.

5.   The solutions of $x^3 - 2x^2 + 4x - 8 = 0$ are a,b,c. Determine the only possible
        values of c^2.

6.   The lateral area of a cone is 3/5 of the total area. Determine the ratio of the radius
        of the cone to the slant height of the cone.

7.   If the roots of the equation $2x^2 - 7x + k = 0$ are complex, determine the smallest
        integral value of k ?

8.   A plane flies from Worcester to Chicago at an average rate of 380 mph and
        returns along the same route at an average rate of 420 mph. What was the
        average rate in miles per hour, for the round trip?

9.   In the equation: $x^2 + ax + b = 0$, one solution is twice as large as the other.
        Express the value of "b" in terms of "a".

St. John's, Burncoat, Leicester, Bromfield, Westborough, Quaboag, Hudson

Round 1: Arithmetic

1.   **(1 pt.)**   **0**

2.   **(2 pts.)**   **12**

3.   **(3 pts.)**   **2/11**

Round 2: Algebra-open

1.   **(1 pt.)**   **3100**

2.   **(2 pts.)**   **150**

3.   **(3 pts.)**   **60**

Round 3: Set Theory

1.   **(1 pt.)**   **23**

2.   **(2 pts.)**   **5**

3.   **(3 pts.)**   **22**

Round 4: Measurement

1.   **(1 pt.)**   **44**

2.   **(2 pts.)**   **57**

3.   **(3 pts.)**   **63**

Round 5: Polynomial Equations

1.   **(1 pt.)** $\sqrt{85}$

2.   **(2 pts.)**   **14**

3.   **(3 pts.)**   **x = -1, 4, ½**

TEAM ROUND (2 pts. Each)

1.   17

2.   -6

3.   41

4.   8

5.   4, -4

6.   2/3 or .6

7.   7

8.   399

9. $2a^2 / 9$ or $.2a^2$

October 8, 2003   **WOCOMAL Varsity Meet**

TEAM ROUND   **School:** _____

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.   (2 points each)

  1.   _____

  2.   _____

  3.   _____

  4.   _____

  5.   _____

  6.   _____

  7.   _____

  8.   _____

  9.   _____

_____

Total Points for Team Round: _____