WOCOMAL

Varsity Meet #1

October 9, 2002

WoCoMaL

October 9, 2002 Varsity Meet#1

ROUND#1: **Arithmetic** - **No Calculators Allowed**

1. The formula $W = 5.5(d+8)$ gives the normal weight W in pounds of a person who is d inches more than 4 feet tall. What then should be the weight, in pounds, of a person who is 5 feet, 6 inches tall ? [one foot = 12 inches]

2. In Reverse Polish Notation (RPN), the operator follows the two terms it affects. For example: 2, 4, + equals 6 . If one of two preceding terms of an operator is also an operator, you must do that operation first. Ex: 2, 6, 4, +, x equals 2, 10, x equals 20 .

Evaluate: 1, 11, 13, x, 2, 2, 5, +, x, x, + .

3. Suppose we define $\}a,b,c\{ = a + 100(3ab - c) + 10\left(2a^2 + (b-c)^2\right)$.
Now evaluate: $\}2,3,-2\{$.

Answer here: 1. (1 pt.) _________ pounds

2. (2 pts.) _________

3. (3 pts.) _________

Tantasqua, Northbridge, Shepherd Hill

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ROUND#2: **Algebra 1**

1. Factor into primes: $6xz - 9yz - 4xy + 6y^2$

2. The rectangular serving area of a restaurant is 4000 square feet, while its perimeter is 260 feet. Find in feet the length and width of the room.

3. Find the ordered pair (p,q) given that $x^2 + 2x + 5$ is a factor of $x^4 + px^2 + q$.

Answer here: 1. (1 pt.) ____________________

2. (2 pts.) _____ft. by _____ft.

3. (3 pts.) (p,q) = _________

Bartlett, Holy Name, Worcester Acad

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ROUND#3: **Set Theory**

1. Let the universal set be the set of whole numbers $\{0,1,2,3,....\}$. Let $A = \{\text{multiples of } 4\}$ and $B = \{\text{integral factors of 25 or 36}\}$. Find the intersection of A and B.

2. Let $P = \{1,\ 2,\ 3\}$ and $Q = \{1,\ 3,\ 5,\ 7\}$. Let N be the number of subsets that P and Q have in common and let X be the numbers of subsets of P or Q that they do not have in common. Compute the ratio $\dfrac{N}{X}$.

3. In Lalaland, only three TV stations are in service: one for sports, one for news, one for sitcoms. During one week, a rating service in a survey determined these facts: Ten people watched all three stations. Of those who watched only two, 20 did not watch sports, 15 did not watch sitcoms, and 21 did not watch news. There were 14 who watched no TV at all. If 131 people watched sports, 79 watched news, and 95 watched sitcoms, how many people were surveyed ?

Answer here: 1. (1 pt.) _______________

2. (2 pts.) ______________

3. (3 pts.) _______ people

Bancroft, Quaboag, Assabet

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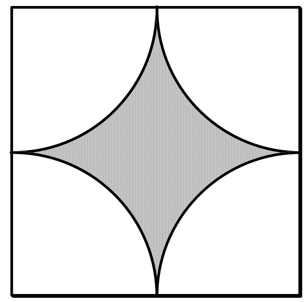
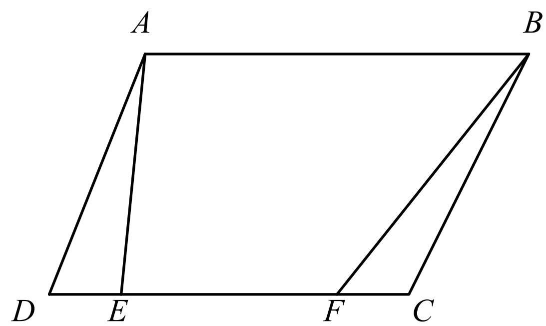
ROUND#4: **Measurement**

1. As shown, four congruent quarter circles are cut out of a square of side 2. Determine the exact area of the shaded star that remains.



2. $ABCD$ is a parallelogram with $\overline{AE}$ and $\overline{BF}$ drawn to side $\overline{DC}$. If $DE = FC = \frac{1}{5}DC$, what is the ratio of the area of $ABFE$ to the area of $ABCD$?



3. Tennis balls are often supplied in cylindrical cans containing three balls, fitting snugly and aligned vertically. Calculate the exact fraction of the can's volume that is occupied by the balls.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Westborough, St. John's, Hudson

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ROUND#5: **Polynomial Equations - No Calculators Allowed**

1. Find the sum of the roots of $2x^3 - 3x + 10 = 0$.

2. Definition: The number c is called a **fixed point** of the function f if and only if $f(c) = c$.
Find the product of all fixed points of $f(x) = x^3 + 3x^2 - 3x - 12$.

3. Write the simplest polynomial equation whose roots are $\{2 \ , \ 1 + i\sqrt{3} \ , \ 1 - i\sqrt{3}\}$.

Answer here: 1. (1 pt.) ________________

2. (2 pts.) ________________

3. (3 pts.) ________________

QSC, St. John's , Doherty

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Team Round: Answers must be **exact** or rounded to **three decimal** places, except where stated otherwise.

1. In a square of side N, counting numbers are arranged in an inward spiral starting in the upper left corner. [For example, the diagram shows the result when N=4.] If N=10, compute the sum of the elements in the main diagonal from top left to bottom right.

1	2	3	4
12	13	14	5
11	16	15	6
10	9	8	7

2. For what value of k will the remainder be the same when $4x^2 + kx + 18$ is divided by $x - 1$ and $x - 3$?

3. Suppose set $A = \{a, b, c, d, e\}$ and $B = \{e, f, g\}$. Count the number of subsets X of set A such that $X \cap B = \varnothing$. Write the count as your answer.

4. A circular pool is surrounded by a brick walkway, 3 meters wide. Find, in meters, the exact diameter of the pool if the area of the walkway is 60π square meters.

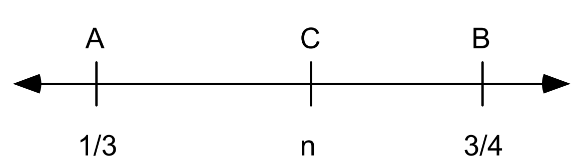
5. One of the roots of a fourth degree polynomial equation with real coefficients is $r_1 = 2 + 3i$. r_2 is the conjugate of r_1, and r_3 is the reciprocal of r_1. Find r_4.

6. A and B as used in the following ratio are the digits of two different 3-digit numbers. Find the 2-digit number AB if $\dfrac{4AB}{AB7} = \dfrac{4}{7}$.

7. The zeros of $f(x) = x^4 - 4x^2 + x - 6$ are $\{a, b, c, d\}$. Find the sum of the coefficients of $g(x) = x^4 + \dots$ having zeros $\{a-1, b-1, c-1, d-1\}$.

8. A cylindrical pipe is 8 feet long. Its inside diameter is 2 inches and the thickness of its wall is $\frac{3}{16}$ of an inch. It is made from a metal that weighs 240 kilograms per cubic foot. Calculate, in kilograms, the weight of the pipe. [1 foot = 12 inches]

9. Find n if $\dfrac{AC}{CB} = \dfrac{5}{4}$.



Auburn, Quaboag, 2 Notre Dame, Quaboag, Northbridge, QSC, Shepherd Hill, Westboro

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October 9, 2002 Team Round Varsity Meet#1

2 Points Each

Answers must be **exact** or
rounded to **three decimal** places,
except where stated otherwise.

Answers here ↓ :

1.__________________________

2.__________________________

3.__________________________

4.__________________________

5.__________________________

6.__________________________

7.__________________________

8.__________________________

9.__________________________

School:______________________________

Team#: ______

Players' Names ↓ :

#1:______________________

#2:______________________

#3:______________________

#4:______________________

#5:______________________

WOCOMAL Answers Varsity Meet #1 October 9, 2002

R#1:
1. 143 lbs.
2. 2003
3. 2332

R#2:
1. $(3z-2y)(2x-3y)$ & vars.
2. 80 ft. by 50 ft.
3. $(p,q)=(6,25)$

R#3:
1. $\{4,\ 12,\ 36\}$
2. $\frac{1}{4}$
3. 243 people

R#4:
1. $4-\pi$
2. $\frac{4}{5}$
3. $\frac{2}{3}$

R#5:
1. zero
2. 12
3. $x^3+8=0$ [or $x^3=-8$]

Team:
1. 620
2. $k=$ -16
3. 16
4. 17 meters
5. $\frac{2+3i}{13}=\frac{2}{13}+\frac{3}{13}i$
6. 84
7. - 4
8. $\frac{175\pi}{32}$ kilograms $\approx 17.181\ kg$
9. n = $\frac{61}{108}=0.56\overline{481}\approx 0.565$

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V1 - Solutions Oct. 9, 2002

Round#1 1. One who is $5' \ 6''$ is 18 inches over 4 feet. $\therefore W = 5.5(18+8) = 143 \ lbs$.

2. 1, 11, 13, x, 2, 2, 5, +, x, x, + 1, 143, 2, 2, 5, +, x, x, +
1, 143, 2, 7, x, x, + 1, 143, 14, x, + 1, 143, 14, x, +
1, 2002, + 2003 .

3. $\}2,3,-2\{ = 2+100\left(3\cdot 2\cdot 3-(-2)\right)+10\left(2\cdot 2^2+(3-(-2))^2\right)$

$= 2+100(20)+10(8+25) = 2002+330 = 2332$

Round#2 1. This is a bi-bi: $3z(2x-3y)-2y(2x-3y) = (2x-3y)(3z-2y)$.

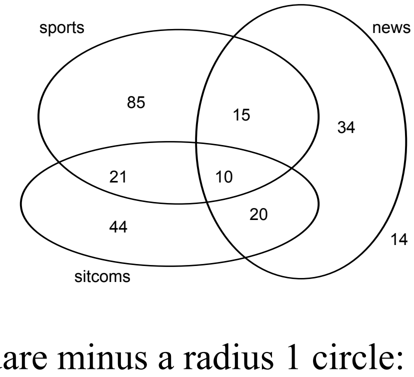
2. Assume $\ell \times w$ and solve the system $\ell w = 4000$ *and* $2\ell + 2w = 260$. The 2nd eqn. reduces to $\ell + w = 130$. Now try this: Remove a factor of 10.
ie. $LW = 40$ *and* $L + W = 13$. 8 and 5 are pretty obvious. [Orange concentrate method.]
So, ans. is 80 feet by 50 feet.

3. Every good algebraic observer should know that the only way $(x^2+2x+5)(factor) = x^4 + px^2 + q$ can be true is if $factor = x^2 - 2x + 5$.
[It's like the theory of conjugates. Or, in reverse, a hidden difference of two squares.]
Multiplying, we get: $(x^2+5)^2 - (2x)^2 = x^4 + 10x^2 + 25 - 4x^2 = x^4 + 6x^2 + 25$, and indeed, the linear and cubic terms are missing.

Round#3 1. $A = \{0, \ 4, \ 8, \ 12, \ ...\}$ and $B = \{1, \ 5, \ 25, \ 1, \ 2, \ 3, \ 4, \ 6, \ 9, \ 12, \ 18, \ 36\}$.
$A \cap B = \{4, \ 12, \ 36\}$.

2. The subsets they have in common are the subsets of $\{1, \ 3\}$. So, $N = 2^2 = 4$.
P has 8 subsets; Q has 16. So, $X = (8-4)+(16-4) = 16$ and $\frac{N}{X} = \frac{1}{4}$.

3. The facts lead to



Round#4 1. A 2 by 2 square minus a radius 1 circle: $2^2 - \pi = 4 - \pi$.

2. Area(ΔADE) = area(ΔBFC) = $\frac{1}{10}$ of area($ABCD$). Reason: Same height, triangles, and one-fifth base. So, $\frac{area(ABFE)}{area(ABCD)} = \frac{1-\frac{2}{10}}{1} = \frac{4}{5}$.

3. Suppose the radius of each ball is r. The volume of the balls is $3 \times \frac{4}{3}\pi r^3 = 4\pi r^3$.
The can is a cylinder of radius r and height $6r$. Its volume is $\pi r^2 \times 6r = 6\pi r^3$.
Even Archimedes knew the ratio was 2 to 3.

Round#5 1. The sum of the roots of $ax^3+bx^2+cx+d=0$ is $-\frac{b}{a}$.
So, the sum of the roots of $2x^3-3x+10=0$ is zero.

2. The fixed points of the equation are the solutions of $c^3+3c^2-3c-12=c$, and the product of roots is $-\frac{-12}{1}=12$.

3. To accomodate $1\pm i\sqrt{3}$, $sum=2$ and $prod=1^2-(i\sqrt{3})^2=1+3=4$. So, factor must be x^2-2x+4. Full equation is $(x^2-2x+4)(x+2)=0$ or $x^3+8=0$.

Team 1. There is a pattern, but by the time you discover it, it is easier to make the grid and add them up: $1+19+37+51+65+75+85+91+97+99=620$.

2. If $f(x)=4x^2+kx+18$, the respective remainders are $f(1)=f(3)$.
So, solving $22+k=54+3k$, we get $k=-16$.

3. All this says is that X cannot contain element $\boldsymbol{e}$. So, X must be a subset of $\{a,\ b,\ c, d\}$. There are 16 such sets.

4. Let r be the radius of the pool. Then $\pi(r+3)^2-\pi r^2=60\pi$ or $r=8.5$ and $d=17$ meters.

5. If $r_1=2+3i$, then $r_2=2-3i$. Since r_3 is the reciprocal of r_1, for the coeffs. to be real, r_4 must be the reciprocal of r_2. Thus, $r_4=\frac{1}{2-3i}=\frac{2+3i}{13}$.

6. It is allowed to keep AB together as the number which names the two digits.
So, $\dfrac{4AB}{AB7}=\dfrac{4}{7}$ translates into $\dfrac{400+AB}{10AB+7}=\dfrac{4}{7}$ or $2800+7AB=40AB+28$.
$2772=33AB$ and $AB=84$.

7. The zeros of f are decreased by 1 (translated 1 to the left) by replacing x with $x+1$.
ie. $g(x)=f(x+1)$. Now, the sum of the coeffs. of any polynomial $P(x)$ is $P(1)$.
$\therefore$ We want $g(1)=f(2)=2^4-4\times 2^2+2-6=-4$

8. The inner radius is $r_i=1\ \ in.=\frac{1}{12}\ \ ft.$ The outer radius is $r_o=\frac{1}{12}+\frac{3}{16}\times\frac{1}{12}=\frac{19}{192}\ \ ft.$
The volume is $\pi\left[\left(\frac{19}{192}\right)^2-\left(\frac{1}{12}\right)^2\right]\times 8\ \ cuft.$ Thus, the weight is $240\times vol.=\frac{175\pi}{32}\ \ kg.$

9. This can be done by using a weighted average: $c=\frac{4a+5b}{9}$ or
$n=\dfrac{4\left(\frac{1}{3}\right)+5\left(\frac{3}{4}\right)}{9}=\dfrac{\frac{4}{3}+\frac{15}{4}}{9}=\frac{16+45}{108}=\frac{61}{108}$.