ROUND I: Arithmetic

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If x#y = (x+y) + (xy) + y, evaluate (5#7) # 2.

2. If a = 1, b = 10, c = 100, and d = 1000, evaluate (a+b+c-d) + (a-b+c+d) + (a+b-c+d)+ (-a+b+c+d).

3. If $\Gamma(x) = 2x + \Omega(x)$, $\Omega(x) = \Psi(x) + 4$, and $\Psi(x) = 3x$, evaluate $\Gamma(\Omega(\Psi(1)))$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bartlett, Bromfield, Quaboag

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. On a number line, a cetain positive integer is 24 units away from twice its opposite. What is the integer?

2. The average of a number greater than one and its reciprocal is $\frac{13}{12}$. What is the number?

3. If $p^2 - 1 = 17$, evaluate $2p - 3(p+5)(p-1) + p^2 + (p)(p) + 10p + 3$.

ANSWERS

(1 pt) 1. _____

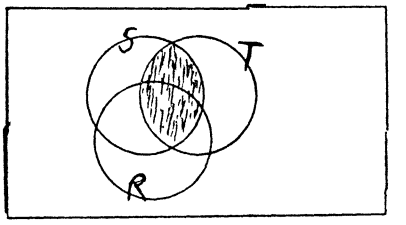
(2 pts) 2. _____

(3 pts) 3. _____

ROUND III: Set theory $\overline{A}$ denotes the complement of set $A$.
$A \subset B$ means that A is a subset of B, not necessarily proper.

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Express the shaded region set in terms of as many as are needed of sets R, S, and T.




2. If A = {1, 3, 5} and B = {0, 3, 6, 9}, find $[\{A \cap (B \cup A)\} \cap B] \cup A$.

3. Given: $A \subset B$, $B \subset C$, and $C \subset A$, which of the following statements are always true? There may be more than one. Answer by number(s).

   1. $A \cup B = A \cap B$
   2. $\overline{A} \cap \overline{B} = \overline{A \cup B}$
   3. If $D \subset A$, then $B \subset D$.
   4. $A$ and $\overline{C}$ are disjoint sets.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

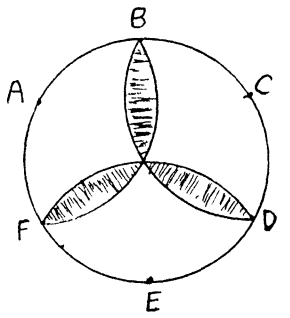
ROUND IV: Measurement

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM

1. Determine the minimum perimeter of a plane figure made up of 12 unit squares with area 12.

2. A wierd shaped rock is dropped into a right circular cylinder containing a wierd blue liquid. The liquid rises 13.5 cm and covers the rock. If the inner radius of the cylinder is 2.75 cm, what is the volume of the rock? Do approximate $\pi$ and answer to the nearest whole cubic centimeter.

3. Trillium BDF is made from three "third of a circle" arcs centered at A, C, and E, and having radius 6 cm. Find in sq cm the exact area of the trillium. (This means simplified radical form and keep $\pi$ as $\pi$.)




ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____ cu cm

(3 pts) 3. _____ sq cm

Bromfield, Worcester Academy

October 11, 2000 WOCOMAL Varsity Meet

ROUND V: Polynomial equations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for m in terms of a and b: $2ab + 2am + b^2 = m^2$.

2. If $kx^2 - 2x + k - 1 = 0$ has x= -3 as a root, find a non-negative root of $x^2 + x + k - 1 = 0$ in exact rational or simplified radical form.

3. The sum of two of the three roots of $x^3 + ax^2 + bx + c = 0$ is 0. What equation expresses c explicitly in terms of $a$ and $b$?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. $c =$ _____

Burncoat, Quaboag, Southbridge

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1. Arrange the digits 5, 6, 7, 8 and 9 on the lines to maximize the product of the resulting 3 and 2 digit numbers. Also write that maximum product.

2. If one stick of lipstick costs $\frac{3}{5}$ the price of one bottle of nail polish, how many sticks of lipstick can be purchased for the same price as 15 bottles of nail polish?

3. In a group of 124 rabbits, 75 have some black on them and 84 have some white. There could be rabbits with neither black nor white.
   a. At least how many rabbits must have both black and white on them?
   b. At most how many rabbits could have both black and white on them?

4. The dimensions of a rectangular box are measured as 5, 7 and 9. The volume is computed. If each measurement is, in fact, x units too small, express the difference between the real volume and the calculated volume as a polynomial in x.

5. The polynomial p(x) has minimal degree, real coefficients, and two of its zeros are -2i and 3+i. Evaluate p(1). ✓ $P(-1) = 17$.

6. How many hours does it take a train traveling at an average speed of 50 mph between stops to travel d miles if it makes n stops of m minutes each?

7. Solve for x: $\frac{9^{x^2-10x}}{9^{3x}} = 81^{-21}$

8. If $a > 0$, graph the solution set for $|x| + |x - a| = a$ on a number line where you supply relevant coordinates.

9. Evaluate $x^3 + y^3$ if $x+y = 30$ and $xy = 223$.

Assabet Valley, Bromfield, Burncoat, St.John's, Shepherd Hill, Shrewsbury, Tahanto, Worcester Academy

October 11, 2000 WOCOMAL Varsity Meet ANSWERS

ROUND I arith
1. 1 pt 166
2. 2 pts 2222
3. 3 pts 69

ROUND II alg 1
1. 1 pt 8
2. 2 pts $\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5
3. 3 pts 0

ROUND III sets
1. 1 pt $S \cap T$ or $T \cap S$
2. 2 pts $\{1,3,5\}$ or A
3. 3 pts 1,2,4

ROUNDIV meas
1. 1 pt 14
2. 2 pts 321 cu cm
3. 3 pts $36\pi - 54\sqrt{3}$ sq cm

ROUND V poly eq
1. 1 pt $-b$ or $2a+b$ need both
2. 2 pts $\frac{-1+\sqrt{7}}{2}$
3. 3 pts $c = ab$

TEAM ROUND 2 pts each

1. $875 \cdot 96 = 84{,}000$

2. 25

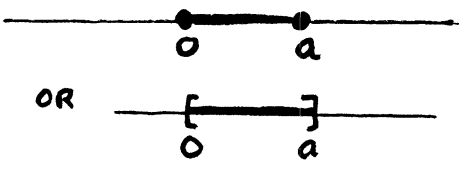
3a. 35 3b. 75 no part credit

4. $x^3 + 21x^2 + 143x$

5. ~~25~~ 5

6. $\frac{6d+5mn}{300}$ OR $\frac{d}{50} + \frac{mn}{60}$

7. $x = 6$ or 7 need both and or , CK

8. 


9. 6930

October 11, 2000 WOCOMAL VARSITY MEET BRIEF SOLUTIONS

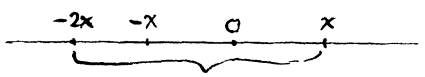
## ROUND I

1 $5\#7 = 12+35+7 = 54$ and
$54\#2 = 56+108+2 = 166$

2. The expressions total $2(a+b+c+d)$
$= 2(1111) = 2222$

3 $\Psi(1) = 3$
$\Omega(3) = 9+4 = 13$
$\Gamma(13) = 26+39+4 = 69$

## ROUND II

1. 

$3x = 24 \Rightarrow x = 8$

2. $\frac{x+\frac{1}{x}}{2} = \frac{13}{12}$. Mult by 12 to get
$6x + \frac{6}{x} = 13$
$6x^2 - 13x + 6 = 0$
$(3x-2)(2x-3) = 0$
$x > 1$ so $x = \frac{3}{2}$

3. $p^2 = 18$ and expression to evaluate =
$2p - 3p^2 - 12p + 15 + 2p^2 + 10p + 3$
$= 18 - p^2 = 0$

## ROUND III

1. Just SNT. R is irrelevant

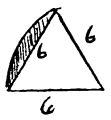
2 $A \cap (B \cup A) = A$
Then $[A \cap B] \cup A = A$ also

3 The Given makes $A = B = C$
1 T, both sides are A
2 T, both sides are $\bar{A}$
3 F, would need $\emptyset = A$
4 T, $\bar{C}$ is $\bar{A}$


## ROUND IV

1. A 3 by 4 rectangle gets minimum perimeter, 14

2. Volume of rock = vol of cylinder with $r = 2.75$ cm and $h = 13.5$ cm
$V = \pi r^2 h \approx 321$ cu.cm

3 

Shaded area
= 60° sector − equilat. Δ
$= \frac{1}{6}\pi 6^2 - \frac{6^2}{4}\sqrt{3}$
$= 6\pi - 9\sqrt{3}$
Trillium area = 6 times this
$= 36\pi - 54\sqrt{3}$ cm$^2$

## ROUND V

1. Change to
$0 = m^2 - b^2 - 2am - 2ab$
$0 = (m+b)(m-b) - 2a(m+b)$
$0 = (m+b)(m-b-2a)$
$\therefore m = -b$ or $m = b+2a$

2. $kx^2 - 2x + k - 1 = 0$ and $x = -3$ gets
$9k + 6 + k - 1 = 0 \Rightarrow k = -\frac{1}{2}$
Other equation becomes
$x^2 + x - \frac{3}{2} = 0$
Apply the quadratic formula to get
$x = \frac{-1 \pm \sqrt{1+6}}{2}$
The nonneg root is $\frac{-1+\sqrt{7}}{2}$

3. Call the roots $r, -r$, and $s$ Then
$x^3 + ax^2 + bx + c = (x-r)(x+r)(x-s)$
$= (x^2 - r^2)(x-s)$
$= x^3 - sx^2 - r^2x + r^2s$
Equating coefficients from $x$ exponents
$c$ $r^2s = (-b)(-a)$ or $c = ab$

TEAM ROUND

1. Calculator aided try possibilities, but use the bigger digits in the hundreds and tens positions.

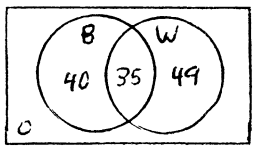
$875 \cdot 96 = 84\,000$

2. $L = \frac{3}{5}N$ or $\frac{5}{3}L = N$

$\therefore 15N = 15 \cdot \frac{5}{3}L = 25L$. Ans: 25

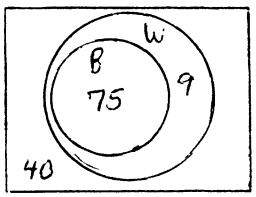
OR use easy numbers.

3a. For at least, use this situation:




All have either black or white. min B∩W

b. For at most, use:




All with black also have white max B∩W

4. $V = \ell \cdot w \cdot h$

Real − Calculated

$(5+x)(7+x)(9+x) - 5 \cdot 7 \cdot 9$

$= (5 \cdot 7 + 12x + x^2)(9+x) - 5 \cdot 7 \cdot 9$

$= 5 \cdot 7 \cdot 9 + 35x + 108x + 12x^2 + 9x^2 + x^3 - 5 \cdot 7 \cdot 9$

$= x^3 + 21x^2 + 143x$

5 Real coefficients makes other two zeros $2i$ and $3-i$. Degree is 4.

Write $p(x)$ as

$a(x+2i)(x-2i)(x-(3+i))(x-(3-i))$

$= a(x^2+4)(x^2-3x-ix-3x+ix+9+1)$

$= a(x^2+4)(x^2-6x+10)$.

Then $p(-1) = a(5)(17) = 17 \Rightarrow a = \frac{1}{5}$

and $p(1) = \frac{1}{5}(5)(5) = 5$

6. While moving, time $= \frac{\text{dist}}{\text{rate}} = \frac{d}{50}$

Time stopped $= \frac{m}{60} \cdot n$ hr

Total $= \frac{d}{50} + \frac{mn}{60} = \frac{6d + 5mn}{300}$

7. Change to

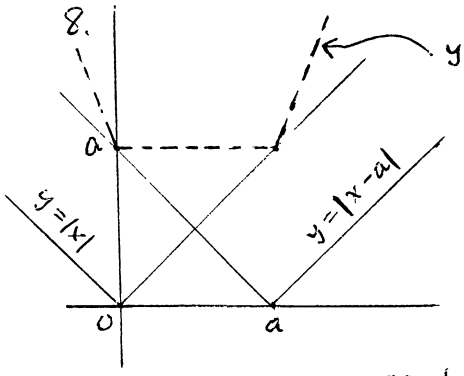
$9^{x^2-13x} = 9^{-42}$

Then $x^2 - 13x + 42 = 0$

$(x-6)(x-7) = 0$

$x = 6$ or $x = 7$

8.




The dashed sum graph shows that $|x| + |x-a| = a$ for $0 \le x \le a$, which you then graph on a number line

9. Use $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

and $(x+y)^2 = x^2 + 2xy + y^2$

$900 = x^2 + 446 + y^2$

$454 = x^2 + y^2$.

Then $x^3 + y^3 = (30)(454 - 223)$

$= 30 \cdot 231$

$= 6930$