Worcester County Mathematics League

Freshman Meet 3 – March 30, 2011

Round 1: Graphing on a Number Line

1

NO CALCULATOR ALLOWED

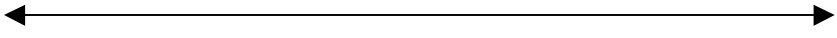
Draw the graph of each of the following inequalities on the corresponding number line provided below. Please specify all endpoints on your graph.

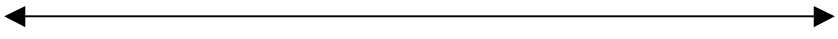
1. $-30 < 30(x+4) \leq 60$

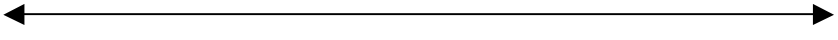
2. $1 < |x-1| < 3$

3. $2 \leq |x+3|$ and $5x - 6 < 2x + 3$

ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

Worcester County Mathematics League

Freshman Meet 3 – March 30, 2011

Round 2: Operations on Polynomials

2

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

---

1. Simplify the following expression to a single polynomial. Do NOT factor the polynomial.

$$\left[(x^3 - 2x^2 + 3x) + (4x^2 - 5x + 1)\right] - \left[(3x^3 + 8) + (2x^2 - 4x + 3)\right]$$

2. Factor the following polynomial as the product of three binomials:

$$8b^3 - 4ab^2 - 18b + 9a$$

3. When the polynomial $6x^4 - 5x^3 - 37x^2 + 46x + c$ is divided by $2x - 5$, the remainder is 5. Find the numerical value of c.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3. $c =$_____

Worcester County Mathematics League

Freshman Meet 3 – March 30, 2011

Round 3: Techniques of Counting and Probability

3

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. I have forgotten my sister's seven-digit phone number. However, I recall that it contains four 3's and three 7's. How many such numbers are there?

2. Each of four students hands in a homework paper. Later the teacher hands back the papers randomly, one to each student. In how many ways can every student receive someone else's paper?

3. Sarah's free throw average is 60%. What is the probability that she will successfully make exactly three of her next 5 free throw attempts?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Worcester County Mathematics League

Freshman Meet 3 – March 30 2011

Round 4: Perimeter, Area and Volume

4

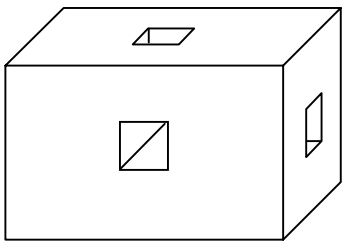
All answers must be in simplest exact form

NO CALCULATOR ALLOWED & THE DIAGRAM IS NOT DRAWN TO SCALE

1. Tape that is 2 inches wide is used to cover a cube 14 inches on each edge. Find the length of tape needed to accomplish this, in inches. Assume that no overlap or waste of the tape occurs.

2. Find the maximum area, in square inches, for a rectangle with integer side lengths and a perimeter of 74 inches.

3. A concrete block whose dimensions are 3 feet by 2 feet by 5 feet has square holes drilled completely through the centers all six faces, as suggested by the diagram. Each hole is 6 inches by 6 inches and is perpendicular to the faces that it has been drilled through. What is the volume of the block, in cubic feet?



ANSWERS

(1 pt.) 1._____ inches

(2 pts.) 2._____ square inches

(3 pts.) 3._____ cubic feet

Worcester County Mathematics League

Freshman Meet 3 – March 30, 2011

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. How many square tiles, 9 inches on a side, are needed to cover a square floor measuring 21 feet by 21 feet?

2. On weekdays (Monday through Friday) Harriet usually washes and dries the dinner dishes, but her daughter washes them every sixth day and her son dries them every fourth day. If the children first did the dishes together on Wednesday, September 1st, 2010, when (month, day and year) did they celebrate their 10th time working together? Assume that they only wash & dry dishes on weekdays, including holidays.

3. What is the only positive integral solution to the equation
$$n^2 + (n+1)^2 + (n+2)^2 = (n+3)^2 + (n+4)^2$$

4. A **palindrome** is a number that can be read the same way in either direction. For example, 27972 is a palindrome. Given that a number is a three-digit palindrome, what is the probability that it is divisible by 11?

5. In a recent election for the student council president at Tahanto Regional High School, Annette received 324 votes, Susan received 201 votes, and Jonathan received 258 votes. If 80% of the student population voted (rounded to the nearest whole number), how many students did not vote?

6. On the space provided on the answer sheet, graph the solution set of:
$$0 < |x+1| - |x-2|$$

7. In a particular game you get 7 points for a "ring" and 5 points for a "lean." In how many ways can 199 points be scored? For this problem assume that the order that "rings" and "leans" occur does not matter.

8. Dilcue computes the "average" of 72, 81 and 107 by averaging 72 and 81 first, and then he takes that result and averages it with 107. What is the absolute value of the difference between Dilcue's method and the correct average?

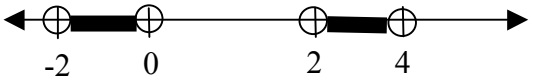
Worcester County Mathematics League

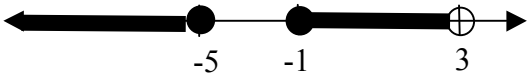
Freshman Meet 3 – March 30, 2011

ANSWERS

Round 1

1. 

2. 

3. 

Round 2

1. $-2x^3 + 2x - 10$ (only)

2. $(2b - a)(2b + 3)(2b - 3)$
(or equivalent, allowing for the commutativity of multiplication & addition)

3. -35

Round 3

1. 35

2. 9

3. $\dfrac{216}{625} = 0.3456 = 34.56\%$

Round 4

1. 588

2. 342

3. $27.75 = 27\frac{3}{4} = \dfrac{111}{4}$

Team Round

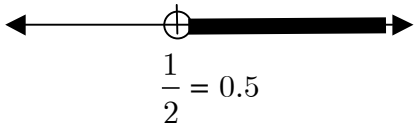
1. 784

2. January 31, 2011
or 1/31/11 or equivalent

3. 10

4. $\dfrac{4}{45} = 0.0\overline{8} \approx 0.089$
(any decimal answer needs to be rounded correctly to at least three decimal places)

5. 196

6. 

7. 6

8. $\dfrac{61}{12} = 5\frac{1}{12} = 5.08\overline{3} \approx 5.083$