

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - April 7, 2010

Round 1: Graphing on a Number Line

1

NO CALCULATOR ALLOWED

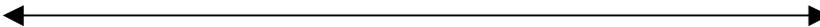
Draw the graph of each of the following inequalities on the corresponding number line provided below. Please specify all endpoints on your graph.

1. $5x - 3 \geq 12$ or $4x + 13 < 9$

2. $-3 \cdot |2x - 4| + 5 \geq -7$

3. $x + \frac{1}{x} \leq -2$

ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

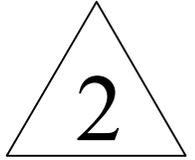
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Round 2: Operations on Polynomials

All answers must be in simplest exact form

NO CALCULATOR ALLOWED



1. After $P(x) = -5(x-1)^3 + 4(x-1)^2 + 6(x-1) + 7$ is expanded and simplified as a single polynomial, find the numerical coefficient of the quadratic term.

2. Determine the remainder when $3x^2 - x + 8$ is divided by $3x + 2$.

3. Factor the following polynomial as the product of one monomial and three binomials:

$$2x^3y + 6x^2y - 8xy^3 - 24y^3$$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Techniques of Counting and Probability

All answers must be in simplest exact form

NO CALCULATOR ALLOWED



3

1. A jar contains 1 red marble, 2 white marbles and 3 blue marbles. If 3 marbles are drawn from the jar at random (and without replacement), determine the probability that all three marbles are blue.
2. Three couples, Mr. and Mrs. Brown, Mr. and Mrs. Green, and Mr. and Mrs. White, are about to line up for a photograph. In how many ways can the six people line up if every man must stand next to his wife?
3. The Combinatorics Club at Neil's school has 6 members (including Neil). The club needs to choose a 3-person Counting Committee and a 4-person Probability Committee. Students can serve on either committee or on both committees. Neil, however, refuses to serve on both committees; he will only serve on one committee or the other, or neither committee. Find the total number of ways that both committees can be chosen.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Freshman Meet 3 - April 7, 2010

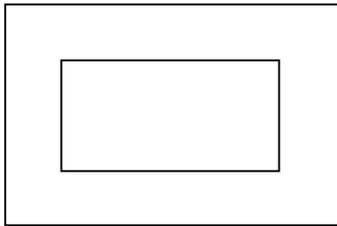
Round 4: Perimeter, Area and Volume

4

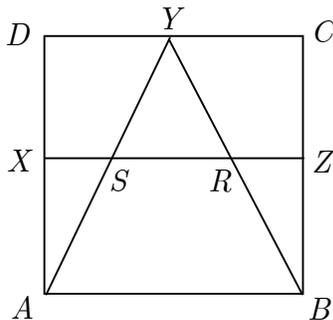
All answers must be in simplest exact form

NO CALCULATOR ALLOWED & THE DIAGRAMS ARE NOT DRAWN TO SCALE

1. The diagram below shows a brick walkway that is 2 meters wide surrounding a rectangular pool that is 3 meters wide by 5 meters long. Find the area of the walkway, in square meters.



2. The diagram below shows square $ABCD$ with side length 20. If X , Y and Z are the midpoints of three sides of the square, compute the area of quadrilateral $ABRS$.



3. A rectangular box measuring 6 inches by 12 inches by 15 inches is partially filled with sand. When the box is sitting on its largest face, the sand reaches 4 inches from the bottom of the box. If the box is placed on its smallest face, *how far from the top* will the sand reach (in inches)?

ANSWERS

(1 pt.) 1. _____ square meters

(2 pts.) 2. _____

(3 pts.) 3. _____ inches

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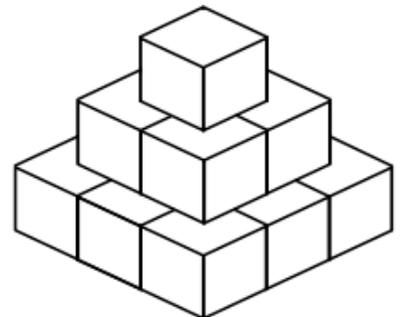
TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. Compute the largest integer n such that 3^n is a factor of $30!$ (Note that $30!$ is 30 factorial or $30 \cdot 29 \cdot 28 \cdot \dots \cdot 2 \cdot 1$)
2. On a certain test the average score for the girls in the class is 83, whereas the average score for the boys in the class is 71. If the average score for all of the students in the class is 80, what percentage of the total number of students in the class are girls?
3. I am thinking of a number from 1 to 100 (inclusive). What is the probability that the number that I am thinking of is divisible by 6 or 10?
4. Compute the sum of all of the digits in all of the positive integers less than 100.
5. Determine the value of k so that the line containing the points $(5, -3)$ and $(-1, k)$ has a y -intercept of 7.
6. On the space provided on the answer sheet, graph the solution set of:
$$3|x - 5| + 2|x - 1| \leq 13$$
7. The average of a set of numbers is $x^2 - 2x - 3$ and the sum of all of the numbers in the set is $2x^4 - 5x^3 + x^2 - 7x - 15$. In terms of x , how many numbers are in the set? Please express your answer as a single polynomial. Do not factor your answer.

8. An artist has 14 cubes, each with an edge of one meter. She stands them on the ground to form a sculpture as shown. She then paints the exposed surface of the sculpture (she does not paint the bottom). How many square meters does she paint?



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ANSWER SHEET - TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

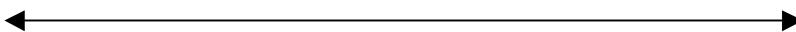
1. _____

2. _____ %

3. _____

4. _____

5. _____

6. 

7. _____

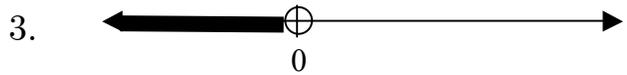
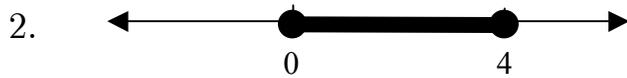
8. _____ square meters

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ANSWERS

Round 1



Round 2

1. 19

2. 10 (OK to accept $\frac{10}{3x+2}$)

3. $2y(x+3)(x-2y)(x+2y)$
(or equivalent, accounting for the commutativity of multiplication and addition)

Round 3

1. $\frac{1}{20} = 0.05 = 5\%$

2. 48

3. 200

Round 4

1. 48

2. 150

3. 5

Team Round

1. 14

2. 75

3. $\frac{23}{100} = 0.23 = 23\%$

4. 900

5. 9

6.



7. $2x^2 - x + 5$ (only)

8. 33