

# WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - January 6, 2010

Round 1: Algebraic Word Problems

1

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. Find the largest of three consecutive odd integers such that the product of the second and third minus the product of the first and second is equal to 44.
  
  
  
  
  
  
  
  
  
  
2. A chemist has 160 mL of a solution which is 20% acid. How many milliliters of water must he evaporate in order to make a solution which is 60% water.
  
  
  
  
  
  
  
  
  
  
3. The combined age of a man and his twin sons is 66 years. Two years from now, the father will be  $2\frac{1}{2}$  times the age of his sons. How old are the sons now?

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_ mL

(3 pts.) 3. \_\_\_\_\_ years old



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Round 3: Operations on Fractions, Decimals, Percents  
and Percentage Word Problems



All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. If the length of a rectangle is decreased by 20% and the width is increased by 40%, by what percent is the area of the rectangle increased?
  
2. Let  $A = \frac{11}{15}$ ,  $B = \frac{13}{19}$  and  $C = \frac{16}{23}$ . Using their letter-names, write these fractions in order from smallest to largest.
  
3. All of the 30 students in Mr. Harris's Honors Geometry class took an exam. If the average passing grade was 85%, the average failing grade was 60%, and the overall average was 80%, how many students passed the exam?

## ANSWERS

(1 pt.) 1. \_\_\_\_\_ %

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_ students passed

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Round 4: Set Theory

4

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. Let  $A = \{\text{the odd natural numbers less than 20 but divisible by 3}\}$ ,  
 $B = \{\text{the odd natural numbers less than 20 but divisible by 5}\}$ , and  
 $C = \{\text{the even natural numbers less than 20 but divisible by 5}\}$ . List the  
elements in the set  $(A \cap B) \cup C$ .
2. Let  $A = \{\text{the letters in the word MATHLETE}\}$  and  $B = \{\text{the letters in the  
word DOMINATE}\}$ . If  $2^{x+1}$  is equal to the number of subsets of  $A \cap B$ ,  
compute the value of  $x$ .
3. One hundred and fifty students were asked what activities they did while on  
vacation. The results indicated the following: 66 went snowboarding, 88  
went to the movies, 6 only went skating, 13 did all three activities, 14 did  
none of these activities, 24 went snowboarding and saw a movie, 40 only saw  
a movie, and 58 did more than one activity. How many students went  
skating?

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_ students

# WORCESTER COUNTY MATHEMATICS LEAGUE

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## TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places EXCEPT for #5! (3 pts. each)

### **APPROVED CALCULATORS ALLOWED**

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1. John has a lemonade stand. He sells small lemonades for 50¢ each and large lemonades for \$1 each. A small contains 1 cup of lemonade and a large is 1.5 cups of lemonade. At the end of the day, John has made \$9 and sold 15.5 cups of lemonade. How many of each size of lemonade did he sell?
2. A perfect square is written in base 5. List the possible values for the units digit of the perfect square.
3. How many pieces of wire  $1\frac{5}{16}$  inches long can be cut from a piece of wire measuring 2 yards, 2 feet and 9 inches long?
4. The product of two positive numbers is twice their sum. Find the sum of their reciprocals.
5. Copy machine A makes 48 copies per minute. Copy machine B makes 64 copies per minute. The boss needs 50,000 copies of an announcement. Beginning at 8:00 am, both machines work together for 3 hours. But then machine A breaks down. It is repaired in 2 hours, while machine B continues to copy. However, just as A is ready after its repairs, B breaks down and cannot be fixed. Machine A finishes the job. To the nearest minute, compute the time of the day that the entire job will be completed. Indicate am or pm.
6. A group of workers were cutting two fields of hay. One field had twice the area of the other. All of the workers worked on the larger field for half of a day, then half of the workers started on the smaller field for the other half of the day. At the end of the day, the larger field was cut, but one worker had to work an entire second day in order to finish the smaller field. Assuming that each worker cuts hay at the same rate, how many workers were in the entire group?
7. How many three-digit positive integers  $X$  are there with the property that  $X$  and  $2X$  have only even digits?
8. On an airplane there are 9 boys, 5 American children, 7 foreign boys, 14 Americans, 6 American males, and 7 foreign females. Compute the number of people on the airplane. (Assume that foreign means non-American and children and boys are younger than 18 years of age.)

Nashoba (1, 6), Westborough (2, 4), Leicester, Doherty, QSC, St. John's

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All answers must be in simplest exact form!

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ANSWER SHEET - TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places EXCEPT for problem #5! (3 pts. each)

1. # of small = \_\_\_\_\_ # of large = \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_ pieces

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_ workers

7. \_\_\_\_\_

8. \_\_\_\_\_ people

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## ANSWERS

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### Round 1

1. 13
2. 80
3. 14

### Round 2

1. 222
2. 10001 (or  $10001_2$ )
3. 67

### Round 3

1. 12
2. B, C, A (this order only)
3. 24

### Round 4

1. 10, 15 (need both, either order)  
( $\{10, 15\}$  is OK)
2. 3
3. 53

### Team Round

1. # small = 8 # large = 5
2. 0, 1, 4  
(need all 3, in any order)
3. 80
4.  $\frac{1}{2} = 0.5$
5. 8:42 pm
6. 8
7. 18
8. 28