

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 1 - November 4, 2009

Round 1: Evaluation of Algebraic Expressions and Order of Operations

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. If $\textcircled{x} = x + \frac{1}{x}$, compute $\textcircled{\textcircled{2}}$.

2. Following the proper order of operations, evaluate:

$$2\left[9 \div 2 \cdot 5 \div 3 + 6(6 \div 2 + 8 \div 3 - 2 \div 6)\right]$$

3. Let $a * b = 2a + 2b$ and $a \Delta b = ab - (a * b)$. Find the simplified value of the following expression: $(3 * -1) \Delta (2 * 1)$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 2: Solving Linear Equations



All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Solve for x : $0.3x - 12 = 18 + 0.8x$

2. Solve for x : $2x - 5[3x - 7(4x - 9)] = 66$

3. Solve for x : $\frac{1}{5}\left(\frac{2}{3}x + 8\right) - \frac{1}{4} = \frac{1}{4}(2x + 3) - \frac{1}{3}x$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Logic Problems



All answers must be placed in the answer section at the bottom

NO CALCULATOR ALLOWED

1. How many times is the numeral 4 required to number the pages of a book from page 1 to page 70?

2. Russ, Bryan and Steve play first, second, and third base on their school's baseball team, but not necessarily in that order. Russ and the third baseman went with Steve to the movies yesterday. Russ does not play first base. Who does play first base?

3. Starting on January 1st, the first dwarf visits Snow White every day. The second dwarf visits Snow White on January 2nd and every second day thereafter. The third dwarf visits Snow White on January 3rd and every third day thereafter. This pattern continues for each of the seven dwarves. Find the total number of dwarf visits up to and including the first day on which all seven dwarfs visit Snow White.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 4: Ratio, Proportion and Variation

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. A recipe for cookies requires $\frac{1}{2}$ cup of chocolate chips to make 18 cookies.

Assuming the same proportion, how many cups are needed to make five-dozen cookies?

2. The ratio to freshman to sophomores in attendance at yesterday's soccer match was 2:3. The ratio of sophomores to juniors was 12:5 and the ratio of juniors to seniors was 15:8. Find the ratio of seniors to freshmen in attendance. Write your answer as a whole number ratio in lowest terms using the colon in the answer space.

3. In the combined variation, P varies directly with T and inversely with V . When $P = 3.2$, $V = 10$ and $T = 400$. Find the value of P when $T = 350$ and $V = 14$.

ANSWERS

(1 pt.) 1. _____ cups

(2 pts.) 2. _____ : _____

(3 pts.) 3. _____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. If $a \# b = \frac{2a}{b} - \frac{b}{2a}$ and $a * b = b \# a$, evaluate $(4 * 5) \# (8 * 10)$.

2. Solve the following equation for x : $\frac{x+3}{2} + \frac{1}{3}(x-4) - \frac{2x-8}{6} + 4 = \frac{1}{2}$

3. Find the four-digit number such that when multiplied by nine yields a result whose digits are exactly the reverse of the original number. That is, find $ABCD$ if

$$\begin{array}{r} A \ B \ C \ D \\ \times \qquad \qquad 9 \\ \hline D \ C \ B \ A \end{array}$$

4. A man walked a certain distance at a constant rate. If he had gone $\frac{1}{2}$ mile per hour faster, he would have walked the distance in four-fifths of the time. If he had gone $\frac{1}{2}$ mile per hour slower, he would have spent 2.5 more hours walking. Find the number of miles that the man walked.

5. A customer received a 20% discount on a suit during a sale. He also received a 4% discount on the sale price for paying cash. If he paid \$80.25 for the suit, find the regular price of the suit *to the nearest cent*. Please express your answer in dollars and cents.

6. How many numbers in the set $\{1, 2, 3, 4, \dots, 2008, 2009\}$ are both perfect squares *and* perfect cubes?

7. The room numbers down one side of a corridor in a motel include all of the odd numbers from 101 to 189. If Eileen is staying in one of these rooms, find the probability that her room number is divisible by 7. Please express your answer as a fraction reduced to lowest terms.

8. Megan and her brother Sean decide to ride their bikes to the candy store each taking the same route. Megan leaves home at 3 pm and rides at a constant rate of 8 miles per hour until she reaches the store. She spends 5 minutes and 30 seconds at the store, then rides home at the constant speed of 5 mph. At 3:25 pm Megan meets Sean, who left the house at 3:12 pm riding at a constant speed of 6 mph. How far is the house from the store (in miles)?

Douglas, Wachusett, Assabet Valley, Westborough, Shrewsbury, QSC (6 & 8), Hudson

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ANSWER SHEET - TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

1. _____

2. _____

3. _____

4. _____ miles

5. \$ _____

6. _____

7. _____

8. _____ miles

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ANSWERS

Round 1

1. $\frac{29}{10} = 2\frac{9}{10} = 2.9$

2. 79

3. 4

Round 2

1. -60

2. 3

3. 18

Round 3

1. 17

2. Steve

3. 1089

Round 4

1. $\frac{5}{3} = 1\frac{2}{3} = 1.\overline{6}$

2. 1 : 3

3. 2

Team Round

1. $\frac{3}{2} = 1\frac{1}{2} = 1.5$

2. -10

3. 1089

4. 15

5. 104.49

6. 3

7. $\frac{7}{45}$ (only)

8. $\frac{9}{5} = 1\frac{4}{5} = 1.8$