Worcester County Mathematics League

Freshman Meet 2 – January 9, 2008

Round 1: Algebraic Word Problems

1

All answers must be in simplest exact form

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1. How many hours will it take a person to travel 50 miles if that individual rides half of the time at 9 miles per hour and walks the rest of the time at 3½ miles per hour?

2. Joe spent 10 pennies and 16 dimes. Dawn spent as much as Joe, but her 26 coins consisted of only quarters and nickels. How many nickels did Dawn spend?

3. A full radiator with a 5 gallon capacity contains a 60% antifreeze solution. How many gallons of the solution must be removed and replaced with pure antifreeze in order to have a radiator full of a 75% antifreeze solution?

ANSWERS

(1 pt.) 1._____ hours

(2 pts.) 2._____ nickels

(3 pts.) 3._____ gallons

Doherty, Algonquin, Assabet Valley

Worcester County Mathematics League

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Round 2: Number Theory

2

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Convert the base ten number 321_{10} to a base 5 number.

2. Find the smallest integer greater than 1,000 that is divisible by 5 and 13, but not divisible by 4.

3. From the sum of the 3 greatest prime numbers less than 50 subtract the greatest common factor of 140 and 462.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

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Round 3: Operations on Fractions, Decimals, Percents and Percentage Word Problems

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. My math class has 12 boys and 15 girls in it when everyone is present. Today ¼ of the boys and one-third of the girls are absent. How many students are present in my class today?

2. Jill is paid a 15% commission on her sales plus she earns $6.50 per hour. Last week her sales were $1,240. If she earned $459 last week, how many hours did she work?

3. Simplify the following expression to a fraction reduced to lowest terms:

$$\frac{\dfrac{3+\dfrac{2}{3}}{8-\dfrac{2}{3}}}{\dfrac{(4.5)\cdot\left(6\frac{2}{3}\right)}{(2.25)\cdot\left(\dfrac{10}{3}\right)}}$$

ANSWERS

(1 pt.) 1._____ students

(2 pts.) 2._____ hours

(3 pts.) 3._____

Tahanto, Westborough, Auburn

Worcester County Mathematics League

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Round 4: Set Theory

4

All answers must be in simplest exact form

NOTE: S' indicates the complement of the set S

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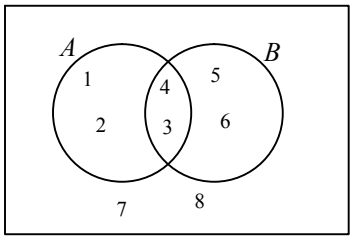
1. Using the diagram to the right, what numbers are in the region defined by $A' \cap B$?



2. Let the universal set $U = \{0,1,2,3,4,5,6,7,8,9\}$. Also, let $A = \{0,1,2,3,4,5,6\}$ and $B = \{0,2,4,6,8\}$. List the elements contained in the set $(A \cup B) \cap (A \cap B)'$.

3. Suppose that out of a group of 150 mathletes it is found that 90 take vitamin A, 88 take vitamin B, 97 take vitamin C, 53 take vitamins A and B, 55 take vitamins A and C, 57 take vitamins B and C, and, finally, 32 take vitamins A, B and C. How many of the mathletes take no vitamins?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. _____

(3 pts.) 3._____ mathletes

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

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1. Find the least prime number greater than 720.

2. You have two triangles which altogether have 6 angles. Five of the angles are $50°$, $60°$, $70°$, $80°$, and $90°$. How large is the sixth angle (in degrees)?

3. In this year's local election Ms. Bonderman received 35% of all of the votes cast, and Mr. Colon received 15% of all the votes cast. Also, in this year's election Mr. Colon received 25% more than the 13,500 votes that he received in the previous election. How many votes did Ms. Bonderman receive in this year's election?

4. Set A contains $x+2$ elements and set B contains $2x-1$ elements. If $A \cap B$ contains 3 elements and $A \cup B$ contains 10 elements, how many elements are in set B?

5. Amy and Francene are next to each other in line with other students waiting to see a movie. There are at most 30 students in the line. Amy says, "There are 3 times as many students after me in this line as in front of me." Francene says, "There are 4 times as many students after me in this line as in front of me." How many students are in line?

6. If $x^2 + y^2 = 7$ and $x^2 - y^2 = 3$, compute all of the possible values of xy.

7. In a basketball game, a team can score 1 point, 2 points or 3 points each time the ball is shot through the hoop. Our team shot the ball through the hoop 50 times and scored 80 points. What is the largest possible number of 3-point shots our team made?

8. A certain chapter in a math textbook has a big list of problems. Alex solved half of the problems plus two more. Brenda solved ¼ of the problems plus four more. Chandra solved one-sixth of the problems plus 6 more. Every problem in the chapter was done by exactly one of these three students. How many problems are in the chapter?

Doherty, Worc. Academy, Hopedale, Shepherd Hill, Hudson, QSC (6, 7), Bromfield

Worcester County Mathematics League

All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. _____

2. _____

3. _____ votes

4. _____

5. _____ students

6. _____

7. _____

8. _____

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ANSWERS

Round 1

1. 8

2. 24

3. $1.875 = 1\frac{7}{8} = \frac{15}{8}$

Round 2

1. 2241 or 2241_5

2. 1,105

3. 117

Round 3

1. 19

2. 42

3. $\frac{1}{8}$

Round 4

1. 5 and 6 (need both in any order)

2. 1, 3, 5, and 8 (need all four in any order)

3. 8

Team Round

1. 727

2. 10 (or $10°$)

3. 39,375

4. 7

5. 21

6. $\sqrt{10}$ and $-\sqrt{10}$ (need both)
   (also acceptable $\pm\sqrt{10}$)
   (also acceptable ± 3.162)

7. 15

8. 144

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BRIEF SOLUTIONS

Round 1

1. Let the total travel time be t. Then since the total distance traveled is $r_1\left(\frac{1}{2}t\right)+r_2\left(\frac{1}{2}t\right)$, we have $50 = 9\left(\frac{1}{2}t\right)+(3.5)\left(\frac{1}{2}t\right) \Rightarrow 50 = 4.5t+1.75t \Rightarrow 50 = 6.25t \Rightarrow t = 8$.

2. Dawn spent \$1.70 using 26 coins (nickels and quarters). Let n = the number of nickels that she spent. So, $0.05n+0.25(26-n) = 1.70 \Rightarrow 0.05n+6.50-0.25n = 1.70 \Rightarrow 0.2n = 4.80 \Rightarrow n = 24$.

3. A chart makes a nice method of organizing the information, with x representing the amount of solution to be added and then replaced with antifreeze:

Start	- Remove	+ Add	= Result
60%	60%	100%	75%
5 gal.	x	x	5

So, we have $0.6(5)-0.6x+x = 0.75(5) \Rightarrow 3-0.6x+x = 3.75 \Rightarrow 0.4x = 0.75 \Rightarrow x = 1.875$.

Round 2

1. $321_{10} = 2\cdot125+2\cdot25+4\cdot5+1 = 2\cdot5^3+2\cdot5^2+4\cdot5^1+1\cdot5^0 = 2241_5$.

2. The number must be divisible by $5\cdot13 = 65$. Since $65\cdot15 = 975$, the factor must be greater than 15. But, it cannot be 16, since 16 is divisible by 4. Hence the number is $65\cdot17 = 1{,}105$.

3. First, the three greatest prime numbers less than 50 are 41, 43 and 47. Their sum is 131. Next, to find the GCF of 140 and 462, prime factor each $\Rightarrow 140 = 2^2\cdot5\cdot7$ and $462 = 2\cdot3\cdot7\cdot11$. Hence, their GCF is comprised of the prime factors that they have in common, or $2\cdot7 = 14$. The difference between 131 and 14 is 117.

Round 3

1. There are 27 students enrolled in the class. One-fourth of 12 is 3, and one-fifth of 15 is 5. Therefore, there are $27-3-5 = 19$ students present today.

2. The amount of money that Jill made on her sales was $\$1{,}240(0.15) = \$186$. Therefore, she made $\$459-\$186 = \$273$ based on her hourly wage. The number of hours that she worked was $\frac{\$273}{6.5} = 42$.

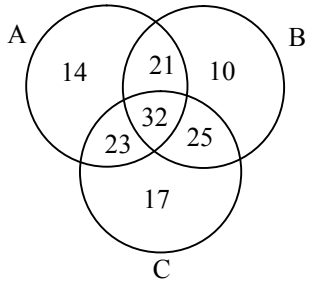
3. Carefully simplify the fractions by multiplying and reducing: $\dfrac{3+\frac{2}{3}}{8-\frac{2}{3}} \Big/ \dfrac{(4.5)\cdot\left(6\frac{2}{3}\right)}{(2.25)\cdot\left(\frac{10}{3}\right)} = \dfrac{\frac{11}{3}}{\frac{22}{3}} \Big/ \dfrac{\frac{9}{2}\cdot\frac{20}{3}}{\frac{9}{4}\cdot\frac{10}{3}} = \dfrac{\frac{1}{2}}{\frac{30}{15}} = \dfrac{\frac{1}{2}}{2} = \dfrac{1}{8}$.

Round 4

1. $A' \cap B$ is the set of elements in B but not in $A \Rightarrow 5, 6$.

2. First, $A \cup B = \{0,1,2,3,4,5,6,8\}$. Next, $A \cap B = \{0,2,4,6,\}$, and as a result $(A \cap B)' = \{1,3,5,7,8,9\}$. Finally, $(A \cup B) \cap (A \cap B)' = \{1,3,5,8\}$.

3. One way: use a Venn diagram to account for the given information, working from the inside-out. In the diagram, 142 mathletes are accounted for leaving 8 who take no vitamins.



Team Round

1. Start checking for factors, skipping the evens, which are divisible by 2. First, 721 is divisible by 7. Then, 723 is divisible by 3, next 725 is divisibly by 5. Finally, 727 has no prime factors other than itself; it is prime (a quick check using the calculator can show that 727 has no prime factors up to 27).

2. We need two sets of three angles whose sum is 180. The angles of one triangle must be $50°$, $60°$, and $70°$ (there is no other possibility to choose 3 angles from the 5 that are given and get a sum of 180). Therefore, the second triangle contains the $80°$ and $90°$ angles; so, the sixth angle must be $180° - 80° - 90° = 10°$.

3. Mr. Colon received $1.25(13,500) = 16,875$ votes this year. As a result, since he received 15% of this year's total votes, there were $\frac{16,875}{0.15} = 112,500$ votes cast in the election. Ms. Bonderman received 35% of these or $112,500(0.55) = 39,375$ votes.

4. Using the "inclusion-exclusion principle" $\left(|A| + |B| - |A \cap B| = |A \cup B|,\ \text{where } |S| \text{ represents the \# of elements in } S\right)$, we have $(x+2) + (2x-1) - 3 = 10 \Rightarrow x = 4$. So, B has $2(4) - 1 = 7$ elements.

5. Using Amy's statement, if there are x students before her, then there are $3x$ after her, and $4x + 1$ students in line. Using Francene's statement, if there are y students before her, then there are $4y$ after her, and $5y + 1$ students in line. We need $4x + 1 = 5y + 1 \Rightarrow 4x = 5y < 30$. Since x and y are whole numbers, the only values of x and y that satisfy this equation are 5 and 4. So there are $4 \cdot 5 + 1 = 21$ students in line.

6. Add the equations to get $2x^2 = 10 \Rightarrow x = \pm\sqrt{5}$. Then, $5 + y^2 = 7 \Rightarrow y = \pm\sqrt{2}$. Therefore, the product is $xy = \pm\sqrt{10}$.

7. Let $N =$ the number of 1-point baskets, $W =$ the number of two point baskets and $H =$ the number of three point baskets. Then, from the given, we have $N + W + H = 50$ and $N + 2W + 3H = 80$. Subtract the first equation from the second to get $W + 2H = 30$. H will be as large as possible when $W = 0 \Rightarrow H = 15$.

8. Half of the problems plus ¼ of the problems plus 1/6 of the problems is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$ of the problems. The extra 2 + 4 + 6 =12 problems must account for the other $\frac{1}{12}$ of the problems. Therefore, there are $12 \cdot 12 = 144$ problems.