Worcester County Mathematics League

1

Freshman Meet 1 – November 7, 2007

Round 1: Evaluation of Algebraic Expressions and Order of Operations

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Following the proper order of operations, evaluate:

$$1+2\left(3-4\left[5+6(7-8)\right]+9\right)-10$$

2. If $x=-\frac{1}{2}$, $y=\frac{2}{3}$, find the simplified value of $18x^2y^2-12xy-54xy^3+36$.

3. If $a*b$ represents the larger of *a* and *b*, $a\#b$ represents the smaller of *a* and *b*, and $a\$b$ represents the average of *a* and *b*, evaluate

$$\left[(8\$(-2))*(3\$10)\right]\$\left[(5\#(-7))*(-3)\right].$$

ANSWERS

(1 pt.) 1._________________________

(2 pts.) 2._________________________

(3 pts.) 3._________________________

Assabet Valley, Auburn, Hudson

Worcester County Mathematics League

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Round 2: Solving Linear Equations

2

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Write an equation in the form $ax+b=0$, where a and b are integers having no common factors and $a > 0$, whose solution is $x=\frac{6}{7}$.

2. Three times the sum of a number and -5 equals -6 times the sum of the number and -2. Find the number.

3. Solve for x: $$\frac{x+2}{2}-\frac{x+3}{3}+\frac{x+4}{4}-\frac{x+5}{5}=\frac{x+6}{6}$$

<u>ANSWERS</u>

(1 pt.) 1._________________________

(2 pts.) 2._________________________

(3 pts.) 3._________________________

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Round 3: Logic Problems

3

All answers must be in simplest exact form

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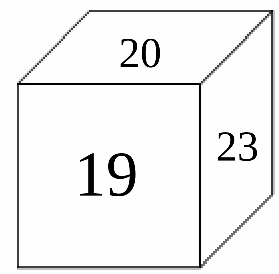
1. A group of 37 students is to be divided into smaller groups. If each group must contain exactly 3 or 4 students, what is the smallest number of groups possible?

2. The faces of a cube are numbered with consecutive integers. Three of the numbers are shown in the diagram to the right. List <u>all</u> of the possible <u>sums</u> of the numbers on all of the faces of the cube.



3. A fortress, holding 700 men, contains enough full rations for those men to eat for two weeks. After two days pass another 500 men enter the fortress, and the king decides that all of the men are to be put on rations that are two-thirds the full amount. How many days will the food last for these 1,200 men, starting with the day the 500 arrive?

<u>ANSWERS</u>

(1 pt.) 1._________________________ groups

(2 pts.) 2._________________________

(3 pts.) 3._________________________ days

Auburn, Algonquin, Hudson

Worcester County Mathematics League

Freshman Meet 1 – November 7, 2007

Round 4: Ratio, Proportion and Variation

4

All answers must be in simplest exact form

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1. On hot summer nights, crickets chirp at a rate which is directly proportional to the number of degrees above $20°$ Celsius. If crickets make 180 chirps per minute at $28°$ Celsius, how many chirps per minute do they make at $32°$ Celsius?

2. A basketball team currently has a record of 28 wins and 19 losses. There are 28 games remaining in the season. What is the least number of games the team must win to finish the season having won more than 51% of its games?

3. Two marks are made on an ordinary 12-inch ruler, one on each side of the ruler's midpoint. If one mark divides the ruler into two parts in the ratio 3:5, and the other mark divides the ruler into two parts in the ratio 5:11, how many inches separate these two marks?

<u>ANSWERS</u>

(1 pt.) 1._________________________ chirps per minute

(2 pts.) 2._________________________

(3 pts.) 3._________________________ inches

Worcester Academy, Quaboag, Hudson

Worcester County Mathematics League

Freshman Meet 1 – November 7, 2007

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

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1. A dinner check was to be divided equally among 8 people. When one of the people refused to pay, each of the others had to pay an additional \$2.00. What was the total amount of the check (in dollars)?

2. Solve the following equation for x:
$$3[x-(3x+4)+(x-1)]+x=2(x+2)-3(x-2)-5x+7$$

3. At a family reunion, every niece is a cousin, half of all cousins are nieces, half of all aunts are cousins, and no aunt is a niece. There are 50 aunts and 30 nieces in attendance. How many cousins at the reunion are neither nieces nor aunts?

4. Two classes took the same test. One class of 20 students averaged 90%, while the other class, consisting of 30 students, averaged 70%. If the two classes were to be combined, what would be the average grade on the test?

5. Let $a \updownarrow b$ represent the sum of the prime numbers lying between a and b. For example, $4 \updownarrow 9 = 5 + 7 = 12$. Find the value of $(12 \updownarrow 20) \updownarrow (24 \updownarrow 34)$.

6. Two cubes have edges whose lengths are a positive integer number of centimeters. The total volume for the two cubes is 189 cubic centimeters. Find the total surface area of the smaller cube (in square centimeters).

7. You have an urn containing red, black and blue marbles. The ratio of red marbles to black marbles is 5:2 and the ratio of black marbles to blue marbles is 4:3. If there are a total of 68 marbles in the urn, how many more red marbles than blue marbles are there?

8. Determine the value of k so that the line containing the points (-2, 3) and (6, k) has a y-intercept of 4.

Algonquin, Hudson, Westborough (3, 8), St, John's, Worcester Academy, QSC, Quaboag

Worcester County Mathematics League

Freshman Meet 1 – November 7, 2007

ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

1.________________________________

2.________________________________

3.________________________________

4. _______________________________

5. _______________________________

6.________________________________ square centimeters

7.________________________________

8.________________________________

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Freshman Meet 1 – November 7, 2007

ANSWERS

Round 1

1. 23

2. 50

3. $\frac{7}{4} = 1\frac{3}{4} = 1.75$

Round 2

1. $7x - 6 = 0$ OR $7x + (-6) = 0$

2. 3

3. 20

Round 3

1. 10

2. 123 and 129 (need both, in any order)

3. 10.5 ($10\frac{1}{2} = \frac{21}{2}$ are acceptable)

Round 4

1. 270

2. 11

3. $\frac{15}{4} = 3\frac{3}{4} = 3.75$

Team Round

1. 112 (or \$112.00)

2. 8

3. 5

4. 78 (or 78%)

5. 112

6. 96

7. 28

8. 7

Worcester County Mathematics League

Freshman Meet 1 – November 7, 2007

BRIEF SOLUTIONS

Round 1

1. $1+2\left(3-4\left[5+6(7-8)\right]+9\right)-10=1+2(2-4(-1)+9)-10=1+2(3+4+9)-10=1+2(16)-10=33-12=23$

2. Using substitution we have, $18\left(\frac{1}{4}\right)\left(\frac{4}{9}\right)+12\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+54\left(\frac{1}{2}\right)\left(\frac{8}{27}\right)+36=2+4+8+36=50$.

3. $[(8\$(-2))*(3\$10)]\$[(5\#(-7))*(-3)]=[2*6.5]\$[-7*(-3)]=6.5\$(-3)=1.75$.

Round 2

1. $x=\frac{6}{7}\Rightarrow 7x=6\Rightarrow 7x-6=0$.

2. Let the number be x. We have $3(x-5)=-6(x-2)\Rightarrow 3x-15=-6x+12\Rightarrow 9x=27\Rightarrow x=3$.

3. Multiply by 60 to clear the fractions:
$$60\cdot\left[\frac{x+2}{2}-\frac{x+3}{3}+\frac{x+4}{4}-\frac{x+5}{5}=\frac{x+6}{6}\right]\Rightarrow 30(x+2)-20(x+3)+15(x+4)-12(x+5)=10(x+6)$$
$$\Rightarrow 13x=10x+60\Rightarrow x=20.$$

Round 3

1. We want to maximize the number of 4-person groups, while having the remainder of the students placed in 3-person groups. There cannot be 9 or 8 four-person groups since those groups would leave a remainder of 1 or 5 students, respectively. There can be 7 four-person groups, since 37 – 28 = 9 $\Rightarrow 3$ three-person groups. So, the smallest number of groups is 7 + 3 = 10.

2. If 23 is the largest of the consecutive integers, the sum is 23 + 22 + 21 + 20 + 19 + 18 = 123. This also shows that 18 is the smallest possible number showing on a face. Also, if 19 is the smallest number, then we have 19 + 20 + 21 + 22 + 23 + 24 = 129. This exhausts the possibilities.

3. Consider a full ration to be what a man could eat in one day. There are $700\cdot 14=9{,}800$ full rations to start. After 2 days pass, there are 9,800 – 1,400 = 8,400 full rations. But, putting the men on 2/3 rations increases the amount of rations available per man 1.5 times. So, 1,200 men must share $8{,}400\cdot 1.5=12{,}600$ rations. This will last $12{,}600\div 1{,}200=10.5$ days.

Round 4

1. Let the equation $C=kT$ model the situation, where C is the number of chirps per minute, T is the temperature above 20° C, and k is the constant of proportionality. From the given we have, $180=k\cdot 8\Rightarrow k=22.5$. So, $C=(22.5)(12)=270$ chirps per minute.

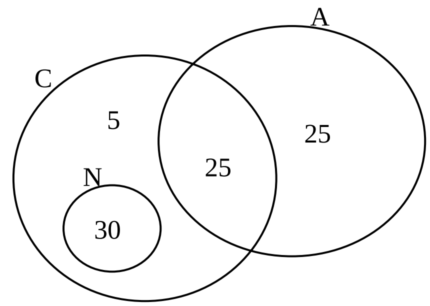
2. The team has already played 28 + 19 = 47 games, and with 28 games remaining, they will have played 75 games at the conclusion of the season. Let x be the number of the remaining games they must win to have a record above 51%. Hence, we want $\frac{x+28}{75}>\frac{51}{100}\Rightarrow x+28>38.25\Rightarrow x>10.25$. Therefore, they must win 11 games.

3. Here is one way: without loss of generality, let x be the location for the mark to the left of the ruler's midpoint and dividing the ruler in the ratio 3:5, and let y be the location for the mark to the right of the ruler's midpoint and dividing the ruler in the ratio 11:5. Using the ratios we have: $\frac{x}{12-x}=\frac{3}{5}\Rightarrow 5x=36-3x\Rightarrow x=4.5$ and $\frac{y}{12-y}=\frac{11}{5}\Rightarrow 5y=132-11y\Rightarrow y=8.25$. Therefore, the distance between the marks is $y-x=8.25-4.5=3.75$ inches.

Team Round

1. Let x equal the amount that each person pays. Then, $8x=7(x+2)\Rightarrow x=14$. And so the total bill was $8\cdot\$14=\$112$.

2. $3[x-(3x+4)+(x-1)]+x=2(x+2)-3(x-2)-5x+7\Rightarrow 3(-x-5)+x=2x+4-3x+6-5x+7\Rightarrow$ $-2x-15=-6x+17\Rightarrow 4x=32\Rightarrow x=8$



3. A Venn diagram is one way to organize the information that is given. Let N stand for the set of nieces, C represent the set of cousins and A stand for the set of aunts. There are 50 aunts and since half of all aunts are cousins, the intersection of A and C must be 25. Since there are 30 nieces and half of all cousins are nieces implies that there must be 60 cousins in total and, as a result, 5 that are neither nieces nor aunts.

4. In the class of 20 students, the sum of the scores is $20(0.90)=18$. In the class of 30 students, the sum of the scores is $30(.7)=21$. Therefore, the sum of the scores of all 50 students is 18 + 21 = 39. Hence, the average of the two classes combined is $\frac{39}{50}=78\%$.

5. The sum of the primes between 12 and 20 is 12 + 17 + 19 = 49, and the sum of the primes between 24 and 34 is 29 + 31 = 60. Therefore, $(12\updownarrow 20)\updownarrow(24\updownarrow 34)=49\updownarrow 60$. Now, the sum of the primes between 49 and 60 is 53 + 59 = 112.

6. Knowing that the edges of the cubes are whole numbers, we need to find two perfect cubes whose sum is 189. A little searching uncovers 64 and 125. Therefore, the volume of the smaller cube is 64 cubic centimeters, and hence its edge length is 4 centimeters. The surface area of this cube is $6\cdot 4^2=96$ square centimeters.

7. If the ratio of red marbles to black marbles is 5:2 and the ratio of black marbles to blue marble is 4:3, then the ratio of red marbles to black marble to blue marbles is 5:2:1.5 or 10:4:3. Let x be the constant multiple between the amount of marbles in the urn. We have, $10x+4x+3x=68\Rightarrow x=4$. Therefore, there are $10\cdot 4=40$ red marbles and $3\cdot 4=12$ blue marbles, resulting in a difference of 28 marbles.

8. The slope of the line determined by (-2, 3) and (6, k) is $m=\frac{k-3}{6-(-2)}=\frac{k-3}{8}$. Using point-slope form gives us the equation for the line: $y-3=\frac{k-3}{8}(x+2)$. Changing this equation into slope-intercept form, the equation becomes $y-3=\frac{k-3}{8}x+\frac{2k-6}{8}\Rightarrow y=\frac{k-3}{8}x+\frac{2k-6}{8}+3$. Therefore, we want the y-intercept to equal 4, or $\frac{2k-6}{8}+3=4$. Solving for k yields $2k-6=8\Rightarrow k=7$.