# WORCESTER COUNTY MATHEMATICS LEAGUE

## Freshman Meet 3 – March 1, 2006

## Round 1: Graphing on a Number Line

1

**NO CALCULATOR ALLOWED**

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Draw the graph of each of the following problems on the corresponding number line provided below. Please specify all endpoints on your graph.

1. $2(x-6) \geq 3(1-x)$

2. $\left|\frac{x}{3}+1\right| \leq 1$

3. $x^2 < 3x+10$

ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

# WORCESTER COUNTY MATHEMATICS LEAGUE

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## Round 2: Operations on Polynomials

2

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. Simplify the following expression as a single polynomial. DO NOT FACTOR YOUR ANSWER.

$$(x+3)^2 + 13(x+3) + 36$$

2. In terms of $K$ and $L$, what must be subtracted from $2K - 3L$ in order to yield the same result as obtaining the sum of $8K + 9L$ and $3K - 8L$?

3. Factor $x^4 - xy^3 - x^3y + y^4$ completely over the integers.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

# WORCESTER COUNTY MATHEMATICS LEAGUE

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## Round 3: Techniques of Counting and Probability

3

All answers must be in simplest exact form

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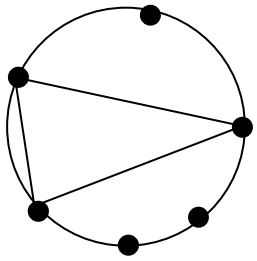
1. Six points lie on a circle. How many inscribed triangles can be formed by choosing any three of these points as vertices? An example of such a triangle is illustrated in the figure to the right.



2. You have a set of 9 books. How many ways are there to select four books from the set of 9 and arrange those 4 on a shelf? Assume that you arrange the books upright and left to right.

3. Let all 8 of the kings and queens be removed from a standard deck of playing cards. If 2 cards are drawn (without replacement) from these 8 cards, find the probability that the cards are either both red or both queens. Write your answer as a simplified fraction.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

# WORCESTER COUNTY MATHEMATICS LEAGUE

## Freshman Meet 3 – March 1, 2006

## Round 4: Perimeter, Area and Volume

4

All answers must be in simplest exact form

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1. Two rectangular solids have the dimensions 4, 6, $h$, and 8, 2, $2h-1$. Find the value of $h$ which will make their volumes equal.

2. When the radius of a circle is increased by 5, the area is increased by $32\pi$. Find the radius of the original circle.

3. The perimeter of a square is 12 cm greater than that of a second square. Also, the area of the first square exceeds the area of the second by 39 sq. cm. Find the perimeter of each square in centimeters.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____cm, and _____ cm

Hudson, Doherty, Westboro

# WORCESTER COUNTY MATHEMATICS LEAGUE

## Freshman Meet 3 – March 1, 2006

## TEAM ROUND

All answers must be in simplest exact form
(3 pts. each)

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1. In a group of 20 girls, 8 are on the soccer team, 9 are on the softball team and 5 are on both teams. What *percent* of the girls are on neither team?

2. Expand and simplify the following expression as a single polynomial in terms of *a* and *b*:

$$(a^2 - ab + b^2)(a^2 + ab + b^2) - a^2b^2$$

3. Find the volume of a cube whose total surface area is 72. Express your answer in simple radical form.

4. A bag contains 4 red marbles and 3 white marbles. One marble is selected at random, returned to the bag, and then a second marble is selected. What is the probability of selecting a red marble followed by a white marble? Express your answer as a fraction reduced to lowest terms.

5. Find the value of *k* for which the graph of the equation $3x + ky = 2$ is parallel to the graph of $x - 3y = 0$.

6. On the space provided on the answer sheet, graph the solution set of:

$$|x - 4| < x$$

7. If $-2 \le x \le 5$, $-3 \le y \le 7$, $4 \le z \le 8$ and $w = xy - z$, find the smallest possible value for *w*.

8. A triangular region is enclosed by the graphs of $y \ge |x| + 8$ and $y \le 12$. Find the area of the region.

# WORCESTER COUNTY MATHEMATICS LEAGUE

All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must be in simplest exact form (3 pts. each)

1. _____ %

2. _____

3. _____

4. _____

5. _____

6. 

7. _____

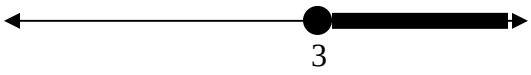
8. _____

# WORCESTER COUNTY MATHEMATICS LEAGUE

## Freshman Meet 3 – March 1, 2006

## ANSWERS

### Round 1

1. 


2. 


3. 


### Round 2

1. $x^2 + 19x + 84$

2. $-9K - 4L$ (or equivalent)

3. $(x-y)^2(x^2+xy+y^2)$
$= (x-y)(x-y)(x^2+xy+y^2)$

### Round 3

1. 20

2. 3,024

3. $\frac{11}{28}$

### Round 4

1. 2

2. $\frac{7}{10} = 0.7$

3. 20 and 32 (any order)

### Team Round

1. 40

2. $a^4 + b^4$ (or equivalent)

3. $24\sqrt{3}$ (only)

4. $\frac{12}{49}$ (only)

5. –9

6. 


7. –23

8. 16

# WORCESTER COUNTY MATHEMATICS LEAGUE
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## SOLUTIONS

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Round 1

1. $2(x-6) \geq 3(1-x) \Rightarrow 2x-12 \geq 3-3x \Rightarrow 5x \geq 15 \Rightarrow x \geq 3$
2. $\left|\frac{x}{3}+1\right| \leq 1 \Rightarrow -1 \leq \frac{x}{3}+1 \leq 1 \Rightarrow -2 \leq \frac{x}{3} \leq 0 \Rightarrow -6 \leq x \leq 0$
3. $x^2 < 3x+10 \Rightarrow x^2-3x-10 < 0 \Rightarrow (x-5)(x+2) < 0 \Rightarrow -2 < x < 5$

Round 2

1. $(x+3)^2+13(x+3)+36 = x^2+6x+9+13x+39+36 = x^2+19x+84$
2. First, $8K+9L+3K-8L = 11K+L$. Next, let $X$ be the desired quantity, then, $2K-3L-X = 11K+L \Rightarrow X = -9K-4L$.
3. $x^4-xy^3-x^3y+y^4 = x^3(x-y)-y^3(x-y) = (x-y)(x^3-y^3)$
   $= (x-y)(x-y)(x^2+xy+y^2)$ (using the factorization for "the difference of two cubes."

Round 3

1. There are ${}_6C_3 = 20$ ways to choose three of the six points.
2. There are ${}_9C_4 = 126$ ways to choose the four books. Then, there are $4! = 24$ arrangements of these 4 books. Hence, there are $126 \cdot 24 = 3,024$ total ways to select and arrange.

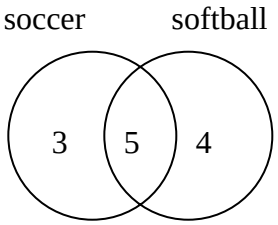
3. Using the inclusion-exclusion principle: $P(\text{both red}) = \frac{4}{8}\cdot\frac{3}{7} = \frac{12}{56}$,
$P(\text{both queens}) = \frac{4}{8}\cdot\frac{3}{7} = \frac{12}{56}$, $P(\text{both red and queens}) = \frac{2}{8}\cdot\frac{1}{7} = \frac{2}{56}$, and hence,
$P(\text{both red or both queens}) = \frac{12}{56} + \frac{12}{56} - \frac{2}{56} = \frac{22}{56} = \frac{11}{28}$.

Round 4

1. Set the volumes equal: $24h = 16(2h-1) \Rightarrow 8h = 16 \Rightarrow h = 2$.
2. $\pi(r+5)^2 = \pi r^2 + 32\pi \Rightarrow r^2 + 10r + 25 = r^2 + 32 \Rightarrow 10r = 7 \Rightarrow r = \frac{7}{10}$
3. If the second square has sides $s$, then the first square has sides $s+3$. And, $s^2 + 39 = (s+3)^2 \Rightarrow \cancel{s^2} + 39 = \cancel{s^2} + 6s + 9 \Rightarrow s = 5$ and the perimeters are 20 and 32.

Team Round

1. Use a Venn diagram:




Therefore, 12 are on teams, and so 8 or 40% are not.

2. Expand and simplify:
$a^4 + a^3b + a^2b^2 - a^3b - a^2b^2 - a^3b + a^2b^2 + a^3b + b^4 - a^2b^2 = a^4 + b^4$
3. Each face has area 12. So, each edge has length $\sqrt{12}$ and the volume of the cube is $12\sqrt{12} = 24\sqrt{3}$.
4. The probability of drawing a red marble is $\frac{4}{7}$ and the probability of pulling a white marble is $\frac{3}{7}$. Hence, the probability of drawing a red then white marble is $\frac{4}{7}\cdot\frac{3}{7} = \frac{12}{49}$.

5. $x-3=0 \Rightarrow y=\frac{1}{3}x$ and the slope is $\frac{1}{3}$. Also, $3x+ky=2 \Rightarrow y=-\frac{3}{k}x+\frac{2}{k}$ and we want $-\dfrac{3}{k}=\dfrac{1}{3} \Rightarrow k=9$.
6. $\left|x-4\right|-x<0 \Rightarrow x-4-x<0$ or $-x+4-x<2 \Rightarrow x>2$
7. We want the smallest product $xy$. So choose $x=5$ and $y=$ -3. Then, we want the largest $z$, $z=8$. Hence the smallest $w$ is $-15-8=-23$.
8. Draw a graph. The area is $\frac{1}{2}\cdot 8\cdot 4=16$