All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. If $a \uparrow b = a^b$ and $a \downarrow b = \sqrt[b]{a}$, find the value of $\left[(2 \uparrow 6) \downarrow 3 \right] \uparrow 2$.

2. Following the proper order of operations, evaluate

$$25 \div 5 \cdot \left(\frac{16+8}{-2^2+8} \right) - 5$$

3. Define $a*b = ab - a^2b$ and $c \wedge d = c + 2cd$. Evaluate:

$$\left[-2 \wedge \left(2 * \frac{3}{2} \right) \right] * \frac{1}{3}$$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

All answers must be in simplest exact form
NO CALCULATOR ALLOWED

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1. Solve for x: $4(3-x)=3[10-2(5-x)]$

2. Solve for x: $\dfrac{3}{8}x + 7 = \dfrac{1}{8}x - 19$

3. If $a \neq 2$, solve for x: $ax + ab = (4-a)x + 2b$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

Auburn, Bromfield, Worc. Academy

All answers must be in simplest exact form

1. Suppose a snail is climbing up a slippery 30-inch wall. Each minute the snail climbs 5 inches, then it slides back 4 inches. How many minutes will it take the snail to reach the top of the wall?

2. Using a balance scale, 1 ball and 1 cup balance 12 pennies. Also, 4 balls and 2 pennies balance 1 cup. How many pennies balance the cup?

3. Assume that for a certain town, it is true that:
   - I.   Some students are not honest
   - II.   All fast food workers are honest.

   Based upon the above, which of the following are necessary conclusions?
   - A. Some students are fast food workers.
   - B. Some fast food workers are not students.
   - C. Some students are not fast food workers.
   - D. No fast food worker is a student.
   - E. No student is a fast food worker.

ANSWERS

(1 pt.)   1._____ minutes

(2 pts.)   2._____ pennies

(3 pts.)   3._____

Tahanto, Shrewsbury, Burncoat

All answers must be in simplest exact form

1. Assuming a constant rate of fuel consumption, how many gallons of gasoline will be needed for a 200-mile trip if the car used 7 gallons to go 87.5 miles?

2. The Worcester County Wrestling League has a girl to boy ratio of 7 to 2. Next season, their 63 wrestlers will unite with the 42 wrestlers in the all-boy Bartlett Wrestling League. Find the ratio of girls to boys in this newly formed league. Write your answer as a simplified fraction.

3. Assume that K varies directly as P and inversely as the square of R. If $K = 21$ when $P = 5$ and $R = 16$, find P when $K = 15$ and $R = 2$. Express your answer as a simplified fraction.

ANSWERS

(1 pt.)   1._____ gallons

(2 pts.)   2._____

(3 pts.)   3._____

St. John's, Bartlett, Assabet Valley

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded correctly to at least three decimal places</u>   (3 pts. each)

1. A jar contains 2 white, 3 red, 1 green and 4 blue marbles. If a marble is drawn from the jar at random, what is the probability that the marble is *not* blue? Express your answer as a simplified fraction.

2. If 90% of P is 30% of Q, then Q is what percent of P?

3. A clock is set correctly at 1 p.m. It loses 3 minutes per hour. What will the clock read when the correct time is 10 a.m. the next day? In your answer, indicate a.m. or p.m.

4. Following the proper order of operations, simplify $\left(\dfrac{2\frac{1}{2} \cdot \dfrac{3}{5}}{\dfrac{5}{8} \cdot 3\frac{1}{5}} \right) \cdot \left(2\frac{5}{6} - \left[\dfrac{2}{9} + \dfrac{23}{18} \right] \right)$.

5. An isosceles triangle and a square have the same perimeter. The legs of the triangle are 3 meters longer than twice the triangle's base. The side of the square is 3 meters more than the base of the triangle. Find the perimeter of the triangle in meters.

6. If $y = \dfrac{k}{x^2}$ and x changes from 2 to 5, find the resulting change in y (in terms of k).

7. Solve for x: $\dfrac{x+6}{6} - \dfrac{2x-1}{2} = x - \dfrac{1}{4}(5x + 8)$

8. For a class of 10 students, a teacher calculated the average on a test. Because he entered the digits for one test in the reverse order, the average was 5.4 less than it should have been. If the erroneous score originally was greater than 60, what were that student's possible scores?

Doherty, Algonquin, Hudson, Tahanto, QSC, Auburn, Worc. Academy (6,8)

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2005

ANSWER SHEET – TEAM ROUND

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded correctly to at least three decimal places</u>   (3 pts. each)

1._____

2._____ %

3._____

4. _____

5. _____ meters

6._____

7._____

8._____

Worcester County Mathematics League
Freshman Meet 1 – November 2, 2005
ANSWERS

Round 1

1. 16

2. 25

3. −30

Round 2

1. $\dfrac{6}{5} = 1\frac{1}{5} = 1.2$

2. −104

3. $-\dfrac{b}{2} = -\frac{1}{2}b = -0.5b$

Round 3

1. 26

2. 10

3. C

Round 4

1. 16

2. $\dfrac{7}{8}$

3. $\dfrac{25}{448}$

Team Round

1. $\dfrac{3}{5}$

2. 300

3. 8:57 a.m.

4. 1

5. 36

6. $-\dfrac{21}{100}k = -0.21k$

7. 6

8. 71, 82, 93 (in any order)

Round 1

1. $\left[(2\uparrow 6)\downarrow 3\right]\uparrow 2 = (64\downarrow 3)\uparrow 2 = 4\uparrow 2 = 16$.

2. $25\div 5\cdot\left(\dfrac{16+8}{-2^2+8}\right)-5 = 25\div 5\cdot(\frac{24}{4})-5 = 25\div 5\cdot 6-5 = 5\cdot 6-5 = 25$.

3. $\left[-2\wedge\left(2*\dfrac{3}{2}\right)\right]*\dfrac{1}{3} = [-2\wedge(3-6)]*\frac{1}{3} = [-2+2(-2)(-3)]*\frac{1}{3} = 10*\frac{1}{3} = \dfrac{10}{3}-\dfrac{100}{3} = -30$.

Round 2

1. $4(3-x) = 3\left[10-2(5-x)\right]\Rightarrow 12-4x = 3(10-10+2x)\Rightarrow 12-4x = 6x\Rightarrow$
   $x = \dfrac{6}{5} = 1.2 = 1\frac{1}{5}$

2. $\dfrac{3}{8}x+7 = \dfrac{1}{8}x-19\Rightarrow 3x+56 = x-152\Rightarrow 2x = -208\Rightarrow x = -48$

3. $ax+ab = (4-a)x+2b\Rightarrow ax+xb = 4x-ax+2b\Rightarrow 2ax-4x = 2b-ab\Rightarrow$
   $2x(a-2) = b(2-a)\Rightarrow 2x = -b\Rightarrow x = -\dfrac{b}{2}$

Round 3

1. The snail takes 25 minutes to go 25 inches. The last 5 inches will be completed in the 26th minute.

2. There are many ways to sort the problem out logically, otherwise, use algebra. Set up and solve simultaneous equations: $B+C = 12P$ and $4B+2P = 1C\Rightarrow$ $5C = 50P\Rightarrow C = 10P\Rightarrow$ 1 cup balances 10 pennies.

3. The statements in I and II can be translated into three possible situations. As illustrated with Venn diagrams we have:
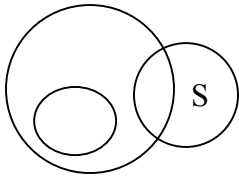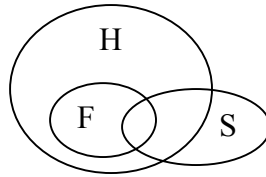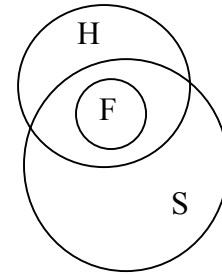

Fig. 1

Fig. 2

Fig. 3

Now, of the possible conclusions, A is false by fig. 1, B false by figure 3, and both D and E are false by figures 2 and 3. C is the only statement that must be true.

Round 4

1. Set-up and solve a proportion: $\dfrac{87.5}{7} = \dfrac{200}{x} \Rightarrow x = 16$.

2. With 63 wrestlers the WCWL has a girl to boy ratio of $\dfrac{49}{14}$. Then, the newly formed league will have a girl to boy ratio of $\dfrac{49}{14+42} = \dfrac{49}{56} = \dfrac{7}{8}$.

3. Let m be the constant of proportionality. So, $K = \dfrac{mP}{R^2}$ and $21 = \dfrac{5m}{256} \Rightarrow m = \dfrac{5376}{5}$.

Using the second pieces of information, we can substitute and solve for P:

$$15 = \dfrac{\dfrac{5376}{5}P}{4} \Rightarrow P = \dfrac{25}{448}.$$

Team Round

1. There are 10 marbles in the jar and 6 of them are not blue. So, the correct probability is $\dfrac{6}{10} = \dfrac{3}{5}$.

2. If $0.9P = 0.3Q$, then $Q = 3P \Rightarrow Q$ is 300% of P.

3. From 1 p.m. to 10 a.m. the next day, 21 hours pass. So, the clock loses a total of 63 minutes during this time. Hence, the clock reads 8:57 a.m.

4. $\left(\dfrac{2\frac{1}{2}\cdot\frac{3}{5}}{\frac{5}{8}\cdot 3\frac{1}{5}}\right)\cdot\left(2\frac{5}{6}-\left[\dfrac{2}{9}+\dfrac{23}{18}\right]\right)=\left(\dfrac{\frac{5}{2}\cdot\frac{3}{5}}{\frac{5}{8}\cdot\frac{16}{5}}\right)\cdot\left(\dfrac{17}{6}-\dfrac{9}{6}\right)=\dfrac{3}{4}\cdot\dfrac{8}{6}=1$

5. Let the base of the triangle be b. Then, its legs are each $2b+3$ and the sides of the square are each $b+3$. Hence, the perimeter of the triangle is $5b+6$ and the perimeter of the square is $4b+12$. Setting these expressions equal and solving for b yields $b=6\Rightarrow$ the perimeter of the triangle $6+15+15=36$.

6. $\Delta y = y_2 - y_1 = \dfrac{k}{25}-\dfrac{k}{4}=\dfrac{-21k}{100}$.

7. $\dfrac{x+6}{6}-\dfrac{2x-1}{2}=x-\dfrac{1}{4}(5x+8)\Rightarrow 2x+12-(12x-6)=12x-3(5x+8)\Rightarrow$
$-10x+18=12x-15x-24\Rightarrow -7x=-42\Rightarrow x=6$

8. Let the correct two-digit score be AB. Then the miss-entered score was BA, and $10A+B-54=10B+A\Rightarrow 9A-9B=54\Rightarrow A-B=6$. If the correct score was greater than a 60, the possibilities are 71, 82 or 93.