Worcester County Mathematics League

Freshman Meet 2 – January 5, 2005

Round 1: Algebraic Word Problems

All answers must be in simplest exact form

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1. There are two numbers such that one is twice the other and the sum of their reciprocals is $\frac{3}{10}$. Find the smaller of the two numbers.

2. A baseball team has won 50 games out of the 75 they have played. There are 45 games remaining in the team's schedule. How many of the 45 remaining games must the team win to finish with exactly 60% of all of their games won?

3. Alex had averaged 156 mph during the first 300 miles of the Daytona 500. By how many miles per hour must she increase her average speed on the final 200 miles so that she averages 160 mph for the entire race?

<u>ANSWERS</u>

(1 pt.) 1.________________________

(2 pts.) 2.________________________

(3 pts.) 3.________________________ mph

Hudson, Auburn, Worc. Academy

Worcester County Mathematics League

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Round 2: Number Theory

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Find the greatest common factor of 24, 42 and 102.

2. Convert 211_3 to base 9. (Note: the subscript indicates the number's base.)

3. Find the largest prime factor of 3003.

ANSWERS

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Bartlett, South, Shrewsbury

Worcester County Mathematics League

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Round 3: Operations on Fractions, Decimals, Percents and Percentage Word Problems

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Write 11.5% as a fraction reduced to lowest terms.

2. Simplify the following complex fraction: $$\frac{\frac{5}{3}}{3+\frac{\frac{1}{2}}{5+\frac{1}{3+1}}}$$

3. Express $\sqrt{0.69\overline{4}}$ as a fraction reduced to lowest terms.

<u>ANSWERS</u>

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Worc. Academy, W. Boylston, Hudson

Worcester County Mathematics League

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Round 4: Set Theory

All answers must be in simplest exact form

NOTE: S' indicates the complement of the set S

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1. Participants in a two-day sports camp could register for only one of the days or for both of the days. There were 231 participants on Friday and 252 on Saturday. The total number of registered participants was 350. How many students attended the camp for both days?

2. Using the Venn diagram in the answer section below, shade the region represented by the set $(A \cup B)' \cap C$.

3. Set *A* is a universal set containing 30 elements. Sets *B*, *C* and *D* are subsets of set *A*. How many elements does set *C* have if the following is true:

$(B \cup C)'$ has 8 elements

$B \cap D$ has 4 elements

$\left[(B \cup C \cup D) \cap A\right]'$ has 3 members

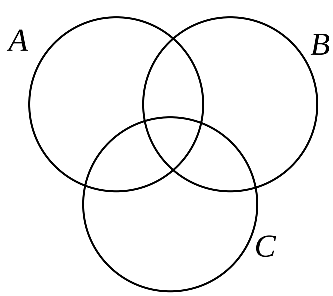
$B \cap C \cap D = \varnothing$

$(C \cup D)'$ has 9 elements

<u>ANSWERS</u>

(1 pt.) 1.__________________________

(2 pts.) 2.



(3 pts.) 3.__________________________

Worcester County Mathematics League

Freshman Meet 2 – January 5,2005

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

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1. Find the smallest positive perfect cube that is divisible by both 5 and 16.

2. The average of 40 numbers is X. The average of 20 of these numbers is Y. In reduced form, express the average of the remaining 20 numbers in terms of X and Y.

3. The value of 92 coins consisting of nickels and quarters is \$15.80. Find the number quarters in the collection.

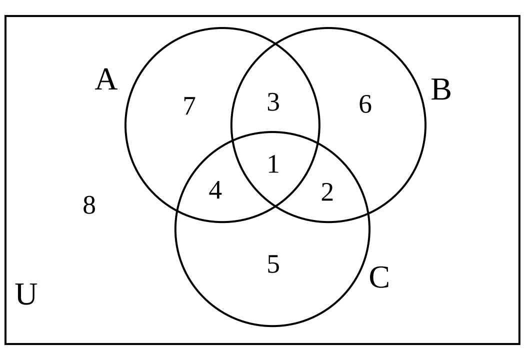
4. The area of a rectangle is 192 square centimeters. If the perimeter of the rectangle is 56 cm, find the dimensions of the rectangle (in centimeters).

5. If A is 80% of B and B is 140% of C, then A is *X*% of C. Find *X*.

6. The square of a positive two-digit number is decreased by 24 and then divided by 10. If half of this result is 15, find the original number.

7. A *palindrome* is a number that reads the same left to right as it does from right to left (858 and 2442 are palindromes, but 674 and 3753 are not). Find the sum of all positive three-digit prime palindromes less than 200.

8. Using the Venn diagram below, list the elements contained in the set $A \cap \left[(B' \cap C) \cup (B \cap C')\right]$, where S' indicates the complement of set *S*. (Notice that the universal set U contains the number 8.)



St. John's, Worc. Academy, Auburn, Holy Name, Bartlett, Hudson

Worcester County Mathematics League

All answers must be in simplest exact form!

Freshman Meet 2 – January 5, 2004

ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1.________________________________

2.________________________________

3.________________________________

4. ____________ cm by _____________ cm

5. _______________________________

6.________________________________

7.________________________________

8.________________________________

Worcester County Mathematics League

Freshman Meet 2 – January 5, 2005

ANSWERS

Round 1

1. 5

2. 22

3. $10.4 = 10\frac{2}{5} = \frac{52}{5}$

Round 2

1. 6

2. 24 or 24_9

3. 13

Round 3

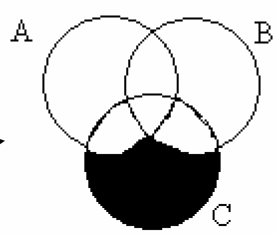
1. $\frac{23}{200}$

2. $\frac{7}{13}$

3. $\frac{5}{6}$

Round 4

1. 133

2. 

3. 12

Team Round

1. 8,000

2. 2X – Y

3. 56

4. 16 by 12 (or 12 by 16)

5. 112

6. 18

7. 755

8. 3 and 4 (need both in any order)

Worcester County Mathematics League

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SOLUTIONS

Round 1

1. Call the numbers x and $2x$. Then, $\frac{1}{x}+\frac{1}{2x}=\frac{3}{10}\Rightarrow\frac{3}{2x}=\frac{3}{10}\Rightarrow x=5$.

2. Let x be the number of wins needed. Then, $\frac{50+x}{120}=0.6\Rightarrow 50+x=72\Rightarrow x=22$.

3. Using "$\frac{D}{r}=t$" and letting r be the rate that she must attain on the final 200 miles, we have, $\frac{300}{156}+\frac{200}{r}=\frac{500}{160}\Rightarrow r=166.4$. Hence, she must increase her rate by 10.4 miles per hour.

Round 2

1. Find the prime factorization for the three numbers: $24=2^3\cdot 3$, $42=2\cdot 3\cdot 7$, and $102=2\cdot 3\cdot 17$. Hence the GCF is $2\cdot 3=6$.

2. $211_3=2\cdot 3^2+3+1=2\cdot 9+4=24_9$.

3. Find the easiest path through division: $3003=3\cdot 1001=3\cdot 11\cdot 91=3\cdot 7\cdot 11\cdot 13$. Therefore the largest prime factor is 13.

Round 3

1. $11.5\%=\frac{115}{1000}=\frac{23}{200}$.

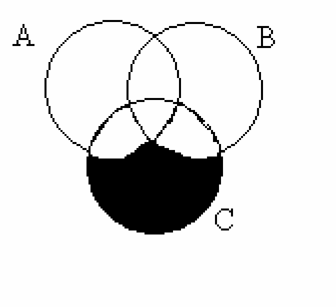
2. Carefully simplify: $$\frac{\frac{5}{3}}{3+\frac{\frac{1}{2}}{5+\frac{1}{3+1}}}=\frac{\frac{5}{3}}{3+\frac{\frac{1}{2}}{\frac{21}{4}}}=\frac{\frac{5}{3}}{3+\frac{2}{21}}=\frac{35}{63+2}=\frac{7}{13}.$$

3. Let $x = 0.69\overline{4}$. Then, $10x - x = 6.9\overline{4} - 0.69\overline{4} \Rightarrow 9x = 6.25 \Rightarrow x = \frac{625}{900} = \frac{25}{36}$. Hence, $\sqrt{0.69\overline{4}} = \sqrt{\frac{25}{36}} = \frac{5}{6}$.

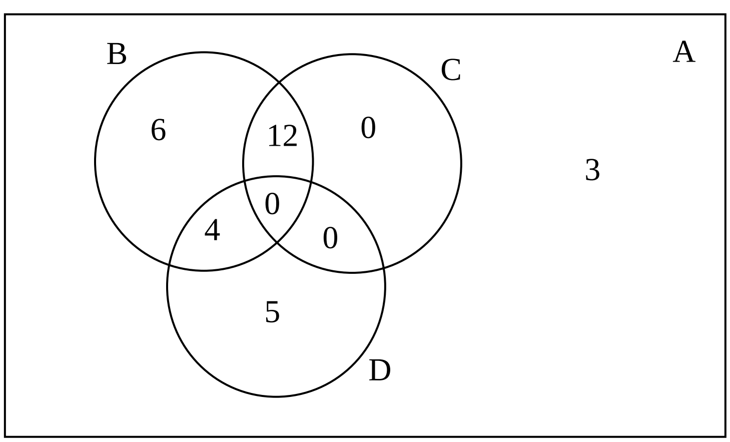
Round 4

1. If we let x be the number of participants for both days, we can use the inclusion-exclusion principle: $231 + 252 - x = 350 \Rightarrow x = 133$.

2. Carefully shade the region by "cross-hatching."



3. We can use a carefully constructed Venn diagram $\Rightarrow C$ contains 12 elements.



Team Round

1. The smallest perfect cube divisible by 5 and 16 is $5^3 \cdot 2^6 = 8000$.

2. Let A be the average of the remaining 20 numbers, then $40X = 20Y + 20A \Rightarrow A = 2X - Y$

3. Letting N be the number of nickels and Q be the number of quarters, we can set up and solve simultaneous equations: $N+Q=92$ and $5N+25Q=1580 \Rightarrow N+5Q=316$. Therefore, $Q=56$.

4. Letting l and w be the length and the width of the rectangle, we can set up and solve a pair of simultaneous equations: $lw=192$ and $2l+2w=56$. Therefore, $l=16$ and $w=12$

5. $A=0.8B$ and $B=1.4C \Rightarrow A=0.8(1.4C)=1.12C$. So A is 112% of C.

6. Set up and solve an equation: $\frac{x^2-24}{20}=15 \Rightarrow x^2=324 \Rightarrow x=18$.

7. The three-digit prime palindromes less than 200 are 101, 131, 151, 181, and 191. The sum of these is 755.

8. $\left[(B'\cap C)\cup(B\cap C')\right]$ contains 3, 4, 5 and 6, so $A\cap\left[(B'\cap C)\cup(B\cap C')\right]$ contains 3 and 4.