Worcester County Mathematics League

Freshman Meet 1 – November 3, 2004

Round 1: Evaluation of Algebraic Expressions and Order of Operations

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Following the proper order of operations, evaluate:

$$2^4 - 3^2 \div 3 + 4 \cdot \sqrt{36} - 2 \cdot 4$$

2. If $x * y = xy - y$ and $x @ y = x^2 - y^3$, evaluate $(2 * 6) @ (4 * -3)$.

3. If $A = \frac{1}{4}$ and $B = \frac{2}{3}$, find the simplified value of $\frac{A + B + AB}{A - B}$.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

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Round 2: Solving Linear Equations

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Let $A=\frac{1}{2}h(b+c)$. If $c=8$, $h=5$, and $A=55$, find the value of b.

2. Solve for x: $\frac{3}{8}(x+2)=\frac{1}{4}x-6$

3. Solve for x: $\frac{2}{3}x-\frac{x+3}{2}+\frac{3}{4}(x+5)=\frac{x+1}{4}+\frac{3x-6}{2}$

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Bartlett, Worc. Academy, Auburn

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Round 3: Logic Problems

All answers must be in simplest exact form

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1. John had his senior privileges taken away from him on January 11, 2004. He lost them for 90 days (including weekends and holidays). On what date (month and day) will his privileges be reinstated?

2. The table below can be filled so that each row and each column contains each of the numbers 1, 2, 3 and 4 exactly once. Find the value of x.

			1
	2		
		x	
1			4

3. Politician A lies on Mondays, Tuesdays and Wednesdays, but tells the truth on all of the other days of the week. Politician B tells the truth on Mondays, Tuesdays, Wednesdays and Sundays, but lies on the other days of the week. On one particular day, they both say, "Yesterday was one of my lying days." What day of the week did they say this?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Quabog, QSC, Algonquin

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Round 4: Ratio, Proportion and Variation

All answers must be in simplest exact form

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1. Three numbers are in the ratio 3:7:9. If their sum is 760, find the value of the largest of the three numbers.

2. On a map, $\frac{3}{16}$ of an inch represents 10 miles. Find the length of a line segment on the map (in inches) if it represents 96 miles.

3. The quantity K varies directly as the square of P and inversely as the product of R and the square root of W. Also, $K = 9$ and $R = 3$ when $W = 16$ and $P = 4$. Find the value of W when $K = 3$, $R = 9$ and $P = 6$.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____ inches

(3 pts.) 3._____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

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1. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (for example, 8 is Beprisque, but 12 is not). Find the number of two-digit Beprisque numbers.

2. The ratio of boys to girls at a school is 3 to 5. There are 144 students at the school, and last Friday, 5 out of every 6 boys wore sneakers. How many boys did not wear sneakers last Friday?

3. If $a * b$ represents the larger of a and b, and $a \# b$ represents the smaller of a and b, evaluate $(1\#(2*(3\#4)))+(1*(2\#(3*4)))$.

4. If $\frac{1}{4}$ of 2^{30} is 4^x, find the value of x.

5. Don starts jogging at a constant rate of five miles per hour. Half an hour later, Russ starts running along the same route at seven miles per hour. For how many minutes must Russ run to catch Don?

6. Solve for x: $\frac{1}{2}\left[x-\frac{1}{3}(4-x)\right]=\frac{3}{4}(2x-1)$

7. The sum of four positive integers is 180. If each successive integer is double the one before it, find the sum of the smallest and the largest of the four integers.

8. The width of a rectangle is two-thirds of its length. If each dimension of the rectangle is decreased by 3 cm, the area is decreased by 66 cm^2. Find the original dimensions of the rectangle (in centimeters).

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ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

1. _____

2. _____

3. _____

4. _____

5. _____ minutes

6. _____

7. _____

8. _____cm by _____ cm

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ANSWERS

Round 1

1. 29

2. 765

3. $\frac{-13}{5} = -2\frac{3}{5} = -2.6$

Round 2

1. 14

2. –54

3. 6

Round 3

1. April 10 (OK to accept 4/10)

2. 4

3. Thursday

Round 4

1. 360

2. $1.8 = 1\frac{4}{5} = \frac{9}{5}$

3. 81

Team Round

1. 5

2. 9

3. 3

4. 14

5. 75

6. $\frac{1}{10} = 0.1$

7. 108

8. 15 by 10 (or 10 by 15)

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SOLUTIONS

Round 1

1. $2^4 - 3^2 \div 3 + 4 \cdot \sqrt{36} - 2 \cdot 4 = 16 - 3 + 24 - 8 = 29$

2. $(2 * 6) @ (4 * -3) = (12 - 6) @ (-12 + 3) = 36 + 729 = 765$

3. $\left[\frac{\frac{1}{4} + \frac{2}{3} + \frac{1}{6}}{\frac{1}{4} - \frac{2}{3}} \right] \cdot \frac{12}{12} = \frac{3 + 8 + 2}{3 - 8} = \frac{-13}{5}$

Round 2

1. $55 = \frac{5}{2}(b + 8) \Rightarrow 22 = b + 8 \Rightarrow b = 14$

2. $\frac{3}{8}(x + 2) = \frac{1}{4}x - 6 \Rightarrow 3(x + 2) = 2x - 48 \Rightarrow 3x + 6 = 2x - 48 \Rightarrow x = -54$

3. $\frac{2}{3}x - \frac{x + 3}{2} + \frac{3}{4}(x + 5) = \frac{x + 1}{4} + \frac{3x - 6}{2}$
   $\Rightarrow 8x - 6(x + 3) + 9(x + 5) = 3(x + 1) + 6(3x - 6)$
   $\Rightarrow 8x - 6x - 18 + 9x + 45 = 3x + 3 + 18x - 36$
   $\Rightarrow 11x + 27 = 21x - 33$
   $\Rightarrow x = 6$

Round 3

1. Realize that 2004 is a leap year, so there are 29 days in February. Then, carefully count the days remaining in January, the days in February, March and April. There are 20 days remaining in January, 29 in February, 31 in March. This is a total of 80 days. There are ten days remaining in the sentence, so John will have to wait until April 10th for his privileges to be reinstated.

2. The italicized items can be filled in using the condition that each row and each column contains each of the numbers 1, 2, 3 and 4 exactly once. This implies that x must be 4.

2	*4*	*3*	1
4	2	*1*	*3*
3	*1*	*x = 4*	*2*
1	*3*	*2*	4

3. Politician A could only have said that, "yesterday was one of my lying days," if it was Monday or Thursday. However, politician B could not have said this on Monday since Sunday and Monday are truth-telling days for him (or her). Although, politician B could have said this on Thursday, giving us the answer.

Round 4

1. Let x be the common ratio factor. So, $3x + 7x + 9x = 760 \Rightarrow x = 40$. Hence, the largest of the three numbers is $9 \cdot 40 = 360$.

2. Set up and solve a proportion: $\frac{\frac{3}{16}}{10} = \frac{x}{96} \Rightarrow 10x = 18 \Rightarrow x = 1.8$.

3. The relationship in question is $K = \frac{mP^2}{R\sqrt{W}}$ where m is the constant of proportionality. Using the first pieces of given information, we have $9 = \frac{16m}{3\sqrt{16}} \Rightarrow m = \frac{27}{4}$. Now, using the second bits of given information, we have $3 = \frac{27}{4} \cdot \frac{36}{9\sqrt{W}} \Rightarrow W = 81$.

Team Round

1. Do a careful search: 9, 10, 11, …23, 24, 25,…,47, 48, 49,…,79, 80, 81, 82, 83. There are 5.

2. Let x be the common ratio factor. Then, $3x + 5x = 144 \Rightarrow x = 18 \Rightarrow$ there are 54 boys. We can then set up a proportion with s being the number of boys who wore sneakers: $\frac{5}{6} = \frac{s}{54} \Rightarrow s = 45$. So, 45 boys wore sneakers.

3. $(1\#(2*(3\#4))) + (1*(2\#(3*4))) \Rightarrow (1\#(2*3)) + (1*(2\#4))$
$\Rightarrow (1\#3) + (1*2) \Rightarrow 1+2=3.$

4. $2^{30} = (2^2)^{15} = 4^{15}$ and $\frac{1}{4} \cdot 4^{15} = 4^{14} \Rightarrow x = 14$

5. Let t represent the number of minutes Russ must run to catch up with Don. Then, using "distance = rate times time," we have $5(t+30) = 7t \Rightarrow t = 75$.

6. $\frac{1}{2}\left[x - \frac{1}{3}(4-x)\right] = \frac{3}{4}(2x-1) \Rightarrow \frac{4}{3}x - \frac{4}{3} = 3x - \frac{3}{2} \Rightarrow x = \frac{1}{10}$

7. Let x be the smallest of the four integers. Then, $x + 2x + 4x + 8x = 180 \Rightarrow x = 12$. So, the smallest of the four integers is 12 and the largest is 96, hence 12 + 96 = 108.

8. Let l be the rectangle's original length. Then the rectangle's original width is $\frac{2}{3}l$, and so the original area is $\frac{2}{3}l^2$. Then, $(l-3)(\frac{2}{3}l - 3) = \frac{2}{3}l^2 - 66 \Rightarrow l = 15$. Hence, the original dimensions were 15 and 10.