

Worcester County Mathematics League

Freshman Meet 3 – March 3, 2004

Round 1: Graphing on a Number Line

NO CALCULATOR ALLOWED

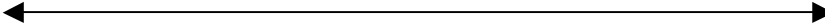
Draw the graph of each of the following problems on the corresponding number line provided below. Please specify all endpoints on your graph.

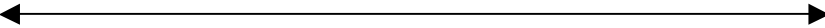
1. $|2x| < 10$

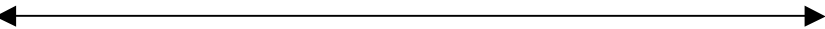
2. $2 < |x + 2| \leq 5$

3. $\frac{2x - 3}{x} < \frac{x + 5}{x}$

ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

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Round 2: Operations on Polynomials

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. Simplify the following expression as a single polynomial (DO NOT FACTOR YOUR ANSWER):

$$(5xy^3 - 7x^2y + 3x^4) - (2x^2y + xy^3)$$

2. Find the coefficient of the x^2 term when $3 - x(x - 2)^3$ is expanded and simplified.

3. Let x be an odd number. Find, as a single polynomial, the sum of the squares of the first two odd numbers which are greater than $3x + 1$. DO NOT FACTOR YOUR ANSWER.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Set Theory

All answers must be in simplest exact form!

1. Let R and S be subsets of the universal set $U = \{0,1,2,3,4,5,6\}$. If $R = \{1,3,5\}$, $R \cap S = \{5\}$ and $R \cup S = \{1,3,5,6\}$, specify S by listing its elements.

2. In a certain English class, each student is taking either Algebra I or Geometry or both. Twenty students are taking Algebra I, 10 are taking Geometry and 8 are taking both Algebra I and Geometry. How many students are there in the English class?

3. In the Venn diagram below, add numbers to each region so that the following is true:

$$A = \{1,3,5,7,9\}$$

$$A \cap B = \{5,7\}$$

$$A \cup B = \{1,2,3,4,5,7,9\}$$

$$A \cap C = \{7\}$$

$$B \cup C = \{2,4,5,6,7,8\}$$

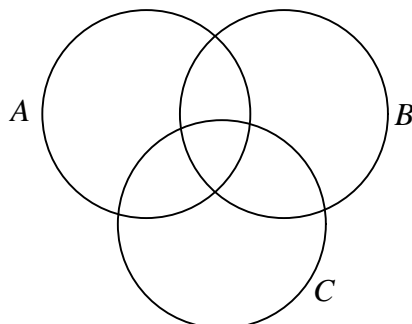
$$B \cap C = \{7\}$$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3.



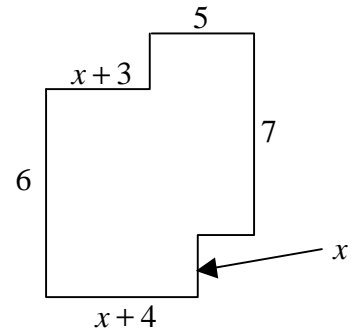
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Round 4: Perimeter, Area and Volume

All answers must be in simplest exact form!

1. A rectangle has a length of 25 centimeters and a width of 16 centimeters. A triangle has a base of 25 centimeters. What would the triangle's corresponding height be if the areas of the rectangle and triangle are equal?

2. In the octagon at the right, all angles are right angles along with the indicated side lengths. Find the octagon's perimeter in terms of x .



3. A rectangular solid is twice as long as it is wide or high. The solid's volume is 250. Find the solid's total surface area.

ANSWERS

(1 pt.) 1. _____ centimeters

(2 pts.) 2. _____

(3 pts.) 3. _____

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TEAM ROUND

All answers must *either* be in simplest exact form or as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. Solve the following equation for x :
$$\frac{\frac{x}{2} + 1}{\frac{2}{x} + 1} = 100$$
2. The width of a rectangle is three-fifths its length. If the length is decreased by 2 inches, the resulting rectangle has 12 square inches less area than the original rectangle. Find the dimensions of the original rectangle.
3. One day at Foxboro Stadium, the ratio of occupied seats to unoccupied seats was 13 to 2. If there were 9,000 unoccupied seats, how many seats are in the stadium?
4. On the number line provided on the answer sheet, graph the solution set for $-3 < 1 - \frac{1}{2}x \leq 4$. Please label the endpoints of your graph.
5. Using the Venn diagram on the answer sheet, shade the region represented by the set $(A \cup B) \cap (A \cup C)$.
6. Simplify the following expression to a single polynomial (DO NOT FACTOR YOUR ANSWER):
$$(3x^3 - 2)^2 - (2x^3 - 3)^2$$
7. One endpoint of a line segment is $(12, -8)$. The midpoint of the same segment is $(3, 18)$. Find the coordinates of the segment's other endpoint.
8. If $N = 2004^{2004}$, find the last two digits in the decimal expansion of N (that is, find the tens' digit and the units' digit).

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All answers must be in simplest exact form!

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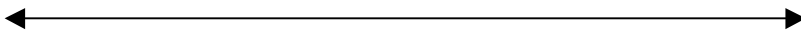
ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

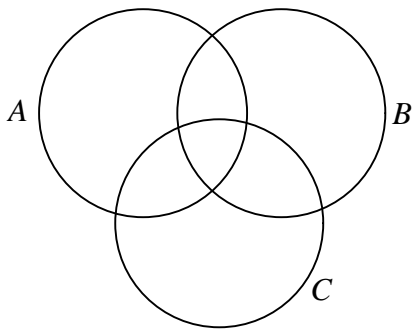
1. _____

2. _____ inches by _____ inches

3. _____

4. 

5.



6. _____

7. _____

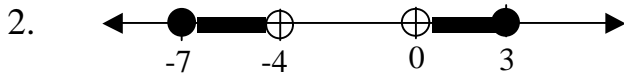
8. _____

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ANSWERS

Round 1



Round 2

1. $4xy^3 - 9x^2y + 3x^4$

2. -12

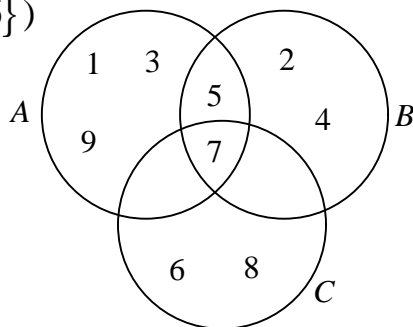
3. $18x^2 + 36x + 20$

Round 3

1. 5, 6 (or $\{5, 6\}$)

2. 22

3. 



Round 4

1. 32

2. $4x + 30 = 2(2x + 15)$

3. 250

Team Round

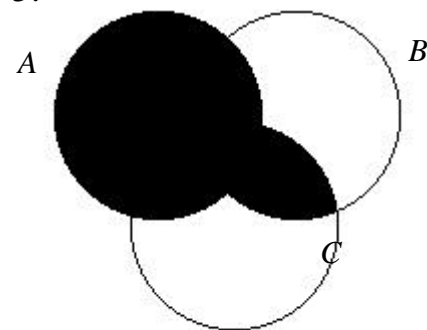
1. 200

2. 10 by 6 (or 6 by 10)

3. 67,500



5.



6. $5x^6 - 5$

7. $(-6, 44)$

8. 56

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SOLUTIONS

Round 1

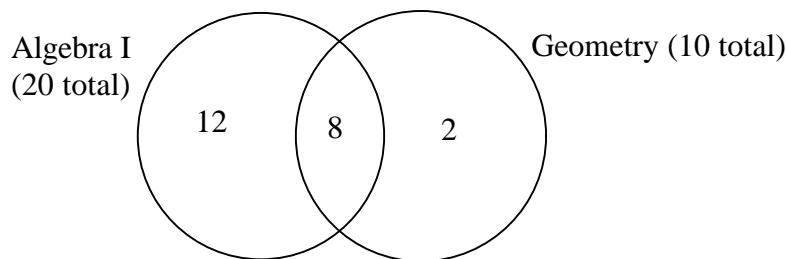
- $|2x| < 10 \Rightarrow -10 < 2x < 10 \Rightarrow -5 < x < 5$
- $2 < |x+2| \leq 5 \Rightarrow 2 < |x+2|$ and $|x+2| \leq 5$. Then, $2 < |x+2| \Rightarrow x+2 > 2$ or $x+2 < -2 \Rightarrow x > 0$ or $x < -4$. Also, $|x+2| \leq 5 \Rightarrow -5 \leq x+2 \leq 5 \Rightarrow -7 \leq x \leq 3$.
- $\frac{2x-3}{x} < \frac{x+5}{x} \Rightarrow 2x^2 - 3x < x^2 + 5x \Rightarrow x^2 - 8x < 0 \Rightarrow x(x-8) < 0 \Rightarrow 0 < x < 8$.

Round 2

- Combine like terms: $(5xy^3 - 7x^2y + 3x^4) - (2x^2y + xy^3) = 4xy^3 - 9x^2y + 3x^4$.
- Expand and simplify:
 $3 - x(x-2)^3 = 3 - x(x^3 - 6x^2 + 12x - 8) = -x^4 + 6x^3 - 12x^2 + 8x + 3$
- Since x is odd, $3x+1$ is even. The next two odd numbers are $3x+2$ and $3x+4$. So,
 $(3x+2)^2 + (3x+4)^2 = (9x^2 + 12x + 4) + (9x^2 + 24x + 16) = 18x^2 + 36x + 20$.

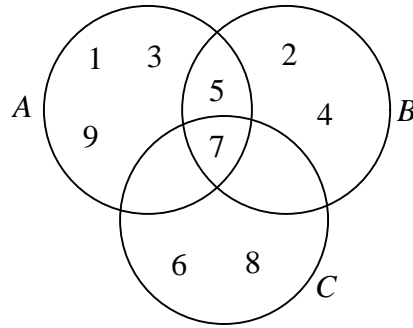
Round 3

- Since $R \cap S = \{5\}$, S contains 5. Since $R = \{1,3,5\}$ and $R \cup S = \{1,3,5,6\}$, S must contain 6. So, $S = \{5,6\}$.
- Use a Venn diagram:



$\therefore 12 + 8 + 2 = 22$ students.

3. In the Venn diagram, work from "inside-out."



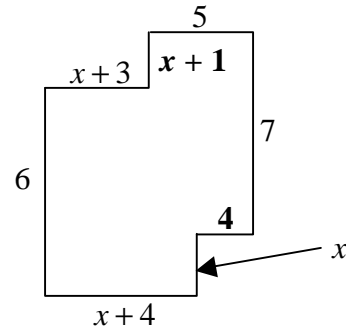
Round 4

1. The area of the rectangle is $A = bh = 16 \cdot 25 = 400$. The area of a triangle is $A = \frac{1}{2}bh$.

So, $400 = \frac{1}{2}25h \Rightarrow 800 = 25h \Rightarrow h = 32$.

2. The missing sides are 4 and $x + 1$. The perimeter is:

$$5 + 7 + 4 + x + x + 4 + 6 + x + 3 + x + 1 = 4x + 30 \text{ or } 2(2x + 15).$$



3. Let the dimensions of the solid be x , x and $2x$. Then the solid's volume is

$$2x^3 = 250 \Rightarrow x = 5. \text{ Hence, the dimensions are } 5, 5 \text{ and } 10. \text{ The total surface area is } 2(25) + 4(50) = 250.$$

Team Round

$$1. \frac{\frac{x}{2} + 1}{\frac{x}{2} + 1} = 100 \Rightarrow \frac{x + 2}{x} = 100 \Rightarrow x^2 + 2x = 100(2x + 4) \Rightarrow x^2 - 198x - 400 = 0$$

$\Rightarrow (x - 200)(x + 2) = 0$. However, $x = -2$ is an extraneous root, causing the denominator on the left-hand side of the original equation to equal zero. Hence, the only solution is $x = 200$.

2. Let the dimensions of the rectangle be x and $\frac{3}{5}x$. Then, setting up and solving an equation: $(\frac{3}{5}x) \cdot x - \frac{3}{5}x(x - 2) = 12 \Rightarrow x = 10$. The original dimensions are 6 and 10.

3. Set up and solve a proportion: $\frac{x}{9,000} = \frac{15}{2} \Rightarrow x = 67,500$.

4. $-3 < 1 - \frac{1}{2}x \leq 4 \Rightarrow -4 < -\frac{1}{2}x \leq 3 \Rightarrow 8 > x \geq -6$.

5. Lightly shade $(A \cup B)$ and $(A \cup C)$. The regions where the two shadings overlap represents $(A \cup B) \cap (A \cup C)$.

6. $(3x^3 - 2)^2 - (2x^3 - 3)^2 = (9x^6 - 6x^3 - 6x^3 - 2) - (4x^6 - 6x^3 - 6x^3 + 9) = 5x^6 - 5$.

7. The midpoint of a line segment given its endpoints, (a, b) and (c, d) , is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. Letting the second endpoint be (c, d) and using this "midpoint formula," we have $\frac{12+c}{2} = 3$ and $\frac{-8+d}{2} = 18 \Rightarrow c = -6$ and $d = 44$.

8. Investigate powers of 4:

$$\begin{aligned}4^1 &= 4 \\4^2 &= 16 \\4^3 &= 64 \\4^4 &= 256 \\4^5 &= 1024 \\4^6 &= 4096 \\4^7 &= 16384 \\4^8 &= 65536 \\4^9 &= 262144 \\4^{10} &= 1048576 \\4^{11} &= 4194304 \\4^{12} &= 16777216\end{aligned}$$

Notice that the tens' and units' digits of the powers of 4 cycle with a cycle length of 10. Since 2004 divided by 10 leaves a remainder of 4, the last two digits of 2004^{2004} are 56.