Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003
Round 1: Evaluation of Algebraic Expressions and Order of Operations

All answers must be in simplest exact form!

NO CALCULATOR ALLOWED!

---

1. Find the minimum value of $1 \circ 2 \circ 3 \circ 4 \circ 5 \circ 6 \circ 7 \circ 8 \circ 9$, if each "$\circ$" is either a "$+$" or a "$\times$".

2. Following the proper order of operations, evaluate:
$$10 + 10(10 + 10) - 10 \div 10 \cdot 10$$

3. Let $a \otimes b = 2a - b$ and $a \square b = b - ab$. Following the proper order of operations, evaluate:
$$\frac{\left(1 \otimes (-2)\right) \, \square \, 4\left((-2) \otimes 4\right)}{(-2) \cdot 4}$$

ANSWERS

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

QSC, Hudson, Algonquin

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003
Round 2: Solving Linear Equations

All answers must be in simplest exact form!

NO CALCULATOR ALLOWED!

---

1. Solve the following equation for x: $-2x-3=-\frac{1}{2}x+9$

2. If the cost of mailing a package weighing P pounds is \$2 for the first pound and 75 cents for each additional pound, write an equation for the cost *in dollars*, C, in terms of P.

3. If a number N is decreased by one-fourth of itself, then decreased by one-sixth of N, the result is $1\frac{2}{5}$. Find N.

ANSWERS

(1 pt.) 1._________________________

(2 pts.) 2._________________________

(3 pts.) 3._________________________

Doherty, Bromfield, Algonquin

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003
Round 3: Logic Problems

All answers must be in simplest exact form!

---

1. Use the following clues to discover the year of the completion of the transcontinental railroad in the United States:
   - The hundreds' digit and the tens' digit are even.
   - The ones' digit is one greater than the hundreds' digit.
   - The sum of all of the digits is 24.

2. The numbers 1, 2, 3,..., n are evenly spaced around a circle. If 15 is directly opposite 49, then find n.

3. Twenty-four people are gathered for a conference. Everyone at the conference shakes hands with everyone else. The conference begins promptly at 9 a.m., each handshake takes exactly 30 seconds, and all 12 pairs of people are shaking hands simultaneously during each 30-second interval. What time will all of the handshaking be completed? (Answer to the nearest second.)

<u>ANSWERS</u>

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Algonquin, Westboro

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003
Round 4: Ratio, Proportion and Variation

All answers must be in simplest exact form!

---

1. A blueprint of a house has a scale of $\frac{2}{3}$ cm = $1\frac{1}{2}$ ft. If the dimensions for the actual bathroom are 8 ft. by 6 ft., find the dimensions of the bathroom on the blueprint in centimeters.

2. A wise and generous investor purchased shares of the iCalc Company for \$6,000. Two years after the purchase, the value of the shares increased by 125% and the investor decided to give the shares to his three children. If he distributed the shares to his children in the ratio 4:3:2, find the value of the smallest gift.

3. The power in an electrical circuit varies directly as the square of the voltage and inversely as the resistance. If the resistance measures 45, and then the voltage is tripled and the power is quadrupled, find the new value for the resistance.

<u>ANSWERS</u>

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Bartlett, Hudson, Worc. Acad.

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003
TEAM ROUND

All answers must be in simplest exact form! (3 pts. each)

---

1. What is the sum of the perfect square factors of 4000? (Please include 1 as a factor)

2. Solve the following formula for *d* in terms of the other variables:

$$S = \frac{n}{2}\left[2a + (n-1)d\right]$$

3. Let Φ be defined by the equation $x \ \Phi \ y = x + xy + y$ for all real numbers *x* and *y*. Find the value of z such that $8 \ \Phi \ z = 3$.

4. Find the number of feet traveled by a car going 40 miles per hour for 3 seconds. (Note: there are 5,280 feet in 1 mile)

5. Find the remainder when 7^{2003} is divided by 100.

6. Two watches are set at 7:00 a.m. One watch runs 3 minutes fast every two hours. The other watch runs 1 minute slow every two hours. At what time the next day will the faster watch be exactly one hour ahead of the slower watch? (Indicate a.m. or p.m.)

7. How many two-digit numbers are there of the form *AB* such that *A* is more than twice as large as *B*.

8. The product of the digits of Ashley's age is the same nonzero number as it was 6 years ago. In how many years will that product be the same again?

Bromfield, Quaboag, Worc. Acad., Hudson, W. Boylston, St. John's, QSC

Worcester County Mathematics League

All answers must be in simplest exact form!

Freshman Meet 1 – November 5, 2003

ANSWER SHEET – TEAM ROUND

1.________________________________

2.________________________________

3.________________________________

4. ________________________________

5. ________________________________

6.________________________________

7.________________________________

8.________________________________

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003

ANSWERS

Round 1

1. 44

2. 200

3. –12

Round 2

1. –8

2. $C = 2 + 0.75(P - 1)$ or $C = 0.75P + 1.25$

3. $2.4 = 2\frac{2}{5} = \frac{12}{5}$

Round 3

1. 1869

2. 68

3. 9:11:30 a.m. (9:11 and 30 seconds)

Round 4

1. $3\frac{5}{9}$ cm by $2\frac{2}{3}$ cm (or $3.\overline{5}$ by $2.\overline{6}$)

2. $3,000

3. $101.25 = 101\frac{1}{4} = \frac{405}{4}$

Team Round

1. 546

2. $d = \frac{2(S - an)}{n(n-1)}$ or equivalent

3. $-\frac{5}{9}$

4. 176

5. 43

6. 1 p.m.

7. 25

8. 18

Worcester County Mathematics League

Freshman Meet 1 – November 5, 2003

SOLUTIONS

Round 1

1. For integers larger than 1, addition produces smaller values than multiplication. Make the first operator × and all of the others + to get the minimum value: $1\times 2+3+4+5+6+7+8+9=44$.

2. $10+10(10+10)-10\div 10\cdot 10$
$=10+10\cdot 20-10\div 10\cdot 10$
$=10+200-1\cdot 10$
$=210-10$
$=200$

3. $\dfrac{\big(1\otimes(-2)\big)\ \square\ 4\big((-2)\otimes 4\big)}{(-2)\cdot 4}=\dfrac{4\square 4\cdot(-8)}{-8}=\dfrac{4\square(-32)}{-8}=\dfrac{96}{-8}=-12$

Round 2

1. $-2x-3=-\frac{1}{2}x+9 \ \Rightarrow\ -\frac{3}{2}x=12 \ \Rightarrow\ -3x=24 \ \Rightarrow\ x=-8$

2. $C=2+0.75(P-1) \ \Rightarrow\ C=0.75P+1.25$

3. $x-\frac{1}{4}x-\frac{1}{6}x=\frac{7}{5} \ \Rightarrow\ 60x-15x-10x=84 \ \Rightarrow\ 35x=84 \ \Rightarrow\ x=2\tfrac{2}{5}$

Round 3

1. Guess that the railroad was built in the 1800's. Then, given the clues, the year will "look" like 18*x*9. Since the sum of the digits is 24, *x* must be 6. Thus, the transcontinental railroad was built in 1869. Note that any other year will create a contradiction of the given information.

2. Consider the numbers to be vertices of a polygon. There are 49 – 15 –1 = 33 vertices of the polygon on either side of the diameter connecting vertices #15 and #49. Thus, there are vertices on either side of the diameter plus those on the diameter for a total of $2\cdot 33+2=68$ vertices on the circle.

3. Since there are 24 people at the conference there are exactly 23 occasions where one person is shaking someone else's hand. These 23 occasions of handshaking occur every 30 seconds with no delay or overlap. Therefore all of the handshakes take $23 \cdot 30 = 690$ seconds. Since 690 seconds is equivalent to 11.5 minutes, the handshaking ends at 9:11:30 a.m.

Round 4

1. Setting up two proportions: $\dfrac{\frac{2}{3}\text{ cm}}{x\text{ cm}} = \dfrac{1.5\text{ ft}}{8\text{ ft}} \Rightarrow x = 3\frac{5}{9}$ and

$$\dfrac{\frac{2}{3}\text{ cm}}{y\text{ cm}} = \dfrac{1.5\text{ ft}}{6\text{ ft}} \Rightarrow y = 2\tfrac{2}{3}$$

2. If the initial investment increased by 125%, then it became $(\$6000)(1+1.25)$ $= (\$6000)(2.25) = \$13{,}500$. Since the gifts were in the ratio 4:3:2, the smallest gift received was $\dfrac{2}{9}$ of the total value of all the gifts $\Rightarrow \dfrac{2}{9}(\$13{,}500) = \$3000$.

3. Call P the power, V the voltage, R the resistance and k the constant of proportionality. Then, $P = \dfrac{kV^2}{R} \Rightarrow R = \dfrac{kV^2}{P}$. If the voltage triples and the power is quadrupled, then the resistance increases by a factor of $\dfrac{3^2}{4} = \dfrac{9}{4} = 2.25$. If the original resistance was 45, the new resistance is $45 \cdot 2.25 = 101.25$.

Team Round

1. The prime factorization of 4000 is $2^5 \cdot 5^2$. Therefore, the perfect square factors are $1^2, 2^2, 2^4, 5^2, 2^2 5^2$, and $2^4 5^2$. The sum is: 1 + 4 + 16 + 25 + 100 + 400 = 546.

2. $S = \dfrac{n}{2}\left[2a + (n-1)d\right] \Rightarrow \dfrac{2S}{n} = 2a + (n-1)d \Rightarrow \dfrac{2S}{n} - 2a = (n-1)d$

$\Rightarrow \dfrac{2S - 2an}{n(n-1)} = d$ (This is equivalent to $d = \dfrac{2(S-an)}{n(n-1)}$)

3. $8\ \Phi\ z = 8 + 8z + z = 8 + 9z$. Then, $8 + 9z = 3 \Rightarrow 9z = -5 \Rightarrow z = -\dfrac{5}{9}$.

4. Changing the units we have:

$$\frac{40 \ \cancel{\text{miles}}}{\cancel{\text{hour}}} \times 3 \ \cancel{\text{seconds}} \times \frac{1 \ \cancel{\text{hour}}}{60 \ \cancel{\text{minutes}}} \times \frac{1 \ \cancel{\text{minutes}}}{60 \ \cancel{\text{seconds}}} \times \frac{5280 \text{ feet}}{1 \ \cancel{\text{mile}}} = 176 \text{ feet}.$$

5. Explore powers of 7:

$7^1 = 7$
$7^2 = 49$
$7^3 = 343$
$7^4 = 2401$
$7^5 = 18,807$ etc.

Hence, the last two digits in the decimal expansion of the powers of 7 cycle by "fours." Now, note that $2003 \div 4 = 500 \text{ r } 3$. Therefore, the last two digits of 7^{2003} are 43 and the remainder is 43.

6. With one watch running 3 minutes fast every two hours and the other watch running 1 minute slow every 2 hours, the "net" difference in the watches every two hours is 4 minutes. The watches will show the same time after the net differences accumulate to 60 minutes. It will take $15 \cdot 2$ hours = 30 hours for this to happen. Thus, the real time the watches show the same time will be 7 a.m. + 30 hours = 1 p.m. the next day.

7. If B is 0, then A can be any value 1 through 9. If B is 1, then A can be any value 3 through 9. If B is 2, then A can be any value 5 through 9. If B is 3, then A can be any value 7, 8 or 9. Finally, if B is 4, then A is 9. Therefore, there are 9 + 7 + 5 + 3 + 1 = 25 such two digit numbers.

8. Suppose Ashley is $10A + B$ years old. Then, $AB = (A-1)(B+4) \Rightarrow A = 2$ and $B = 4$ (note that any other combination will create contradictions). So, Ashley is 24. It will be another 18 years, when Ashley is 42, before the product of the digits of her age is again 8.