WOCOMAL

Freshman Meet #2

January 8, 2003

WoCoMaL

January 8, 2003 Freshman Meet#2

ROUND#1: **Graphing on a Number Line**

<< No Calculators >>

1. How many integers are in the solution set of this inequality ?

$$|2x-7|<7$$

2. The graph of this double-sided inequality consists of two segments. What is the sum of the lengths of these two segments ?

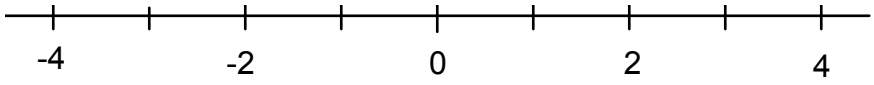
$$321 \le |5x-4| \le 876$$

3. Graph $3x+2|x-5| \le 10$ on the number line provided.

Answer here: 1. (1 pt.) _____

2. (2 pts.) _____

3. (3 pts.)



Holy Name, QSC, Assabet

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ROUND#2: **Set Theory**

1. How many subsets can be formed from the set $\{a\ ,\ b\ ,\ c\ ,\ d\ ,\ e\}$ if no subset has more than three elements ?

2. 104 freshmen own either a dog, a cat, a goldfish, or a combination of these. If 41 have a cat, 23 have a goldfish, no one has both a cat and a dog, 5 own a cat and a goldfish, and 12 own a dog and a goldfish, how many freshmen own only a dog ?

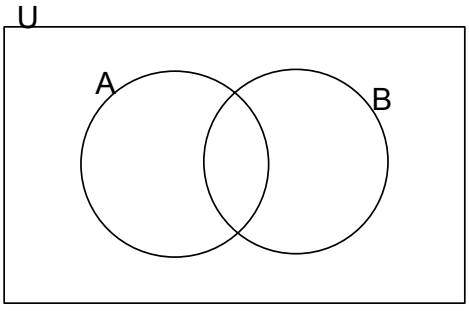
3. If $\overline{X}$ denotes the complement of set X, then shade the region of the Venn diagram which represents $\overline{(A \cap \overline{B}) \cup (\overline{A} \cap B)}$.

[Work here and answer cleanly below.]

Answer here: 1. (1 pt.) _____

2. (2 pts.) _____

3. (3 pts.)



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ROUND#3: **Operations on Numerical Fractions, Decimals, Percents, and Percentage Word Problems**

<< No Calculators >>

1. Compute N if $N = 1\left(1-\frac{1}{1}\right)-2\left(1+\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)-4\left(1+\frac{1}{4}\right)+\ \ldots\ +9\left(1-\frac{1}{9}\right)-10\left(1+\frac{1}{10}\right)$.

2. Stella pays cost plus 15 % for a food processor. If the store charges \$ 129.95 , what is the dollar amount of the markup ?

3. Mr. Jones left his entire estate to his wife, his daughter, his son, and his butler. His son and daughter shared in the ratio 3 to 7 and together received $33\frac{1}{3}\%$ of the total estate. His wife received twice as much as the daughter. If the butler received \$ 300,000 what was the total value of Mr. Jones' estate ?

Answer here: 1. (1 pt.) _____

2. (2 pts.) _____

3. (3 pts.) _____

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ROUND#4: **Techniques of Counting and Probability**

<< Simplify >>

1. If there are 262 students in your freshman class, how many different president/vice-president pairs could be elected from the class ?

2. In the tennis-crazed kingdom of BillieBob, where jeans are the norm and some say, "The games are rigged!", a game of mixed doubles involves two teams of players. Each team is composed of one man and one woman. One-half of the men from BillieBob are in the tournament, and two-thirds of the women are in it. What fraction of the population are <u>not</u> in the tournament ?

3. From the first 2000 counting numbers, one number is drawn. What is the probability that the number is either a multiple of 2 or a multiple of 5 ?

Answer here: 1. (1 pt.) _____

2. (2 pts.) _____

3. (3 pts.) _____

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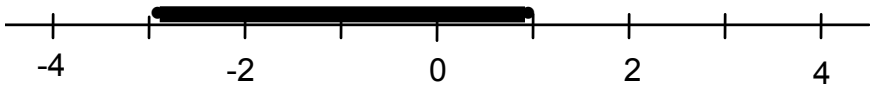
Team Round: Answers must be **exact** and **reduced fractions for probabilities**.

1. Write a single inequality, simplified and having the pictured graph, from -3 to +1.

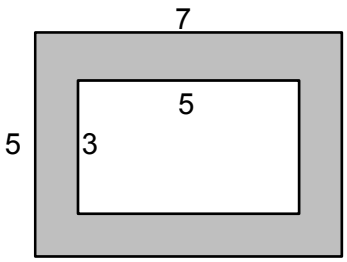


2. Suppose the universal set is $\{1\ ,\ 2\ ,\ 3\ ,\ 4\ ,\ 5\ ,\ 6\ ,\ 7\ ,\ 8\ ,\ 9\ ,\ 11\}$.
If $A=\{1\ ,\ 3\ ,\ 7\ ,\ 9\ ,\ 11\}$, $B=\{2\ ,\ 5\ ,\ 7\ ,\ 8\ ,\ 11\}$, and $C=\{1\ ,\ 4\ ,\ 6\}$, list the elements in $\overline{C\cup\left(\overline{A}\cap B\right)}$.
Note: $\overline{X}$ is the symbol for the complement of set X.

3. A pair of pants retails for \$ D. During a sale, the pants are 20 % off with a rebate of \$ 5 from the manufacturer. A sales tax of 5 % must be paid on the price after the 20 % discount but before the \$ 5 rebate. The total cost for the pants is \$ 16. Find D.

4. Pauline's ATM card must have a pin # with no more than 6 but no fewer than 3 alphanumeric characters $(1\to 9\ ,\ A\to Z)$. If no character is allowed to repeat within a pin number, how many different pin numbers are possible ?

5. What is the probability that a point selected randomly within the outer rectangle will <u>not</u> lie within the inner rectangle ?
[Dimensions are as shown.]



6. A set that has N subsets is itself a subset of a set that has N+1984 subsets. How many elements has the set that has N subsets ?

7. Two normal hexahedral dice are rolled. What is the probability that either the sum or the product is divisible by 3 ?

8. A, B, and C are towns in that order along a highway. On Monday, a driver goes from A to B at 36 mph and from B to C at 24 mph, taking a total of 2 hours and 50 minutes. On Tuesday, she covers the entire distance from A to C in 2 hours and 48 minutes at an average speed of 30 mph. How far is it from B to C ?

QSC, Douglas, Southbridge, Assabet, Shrewsbury, QSC, Burncoat & QSC, St. John's

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January 8, 2003 Team Round Freshman Meet#2

3 Points Each

Answers must be **exact** and **reduced fractions for probabilities**.

Answers here ↓ :

1._____

2._____

3._____

4._____

5._____

6._____

7._____

8._____miles__

School:_____

Team#: _____

Players' Names ⬇ :

#1:_____

#2:_____

#3:_____

#4:_____

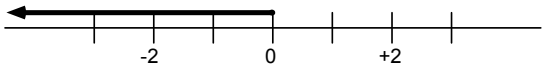
#5:_____

WOCOMAL Answers Freshman Meet #2 January 8, 2003

R#1: 1. 6

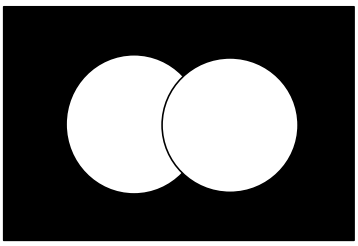
2. 222

3.



R#2: 1. 26

2. 45

3. 

R#3: 1. - 15

2. $ 16.95

3. $ 1,500,000

R#4: 1. 68,382

2. $\frac{3}{7}$

3. $\frac{3}{5}$

Team: 1. $|x+1| \leq 2$

2. {3 , 7 , 9 , 11}
in any order, with or without { }

3. 25
with or without $

4. 1,208,926,950

5. $\frac{4}{7}$

6. 6

7. $\frac{7}{9}$

8. 36 miles

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F2 - Solutions Jan. 8, 2003

Round#1 1. The inequality is equivalent to $0 < x < 7$. So, ans. is 6 .

2. 5 times the length of one of the two segments is 876 - 321 = 555 .
Therefore, the two segments add up to 222 .

3. Both $3x + 2(x-5) \le 10$ *and* $3x + 2(5-x) \le 10$ must hold.
ie. $x \le 4$ *and* $x \le 0$. This is only true when $x \le 0$.

Round#2 1. Either list them, or subtract from $2^5 = 32$ 1 subset of five elements and 5 subsets of four elements, to obtain 32 - 1 - 5 = 26 .

2. Use the ole Ballantine Ale three rings method. The only real thinking occurs when one realizes that if "no one has both a cat and a dog," then it must also be true that no one has a cat and a dog and a goldfish.

3. Make three or four preliminary drawings.

Round#3 1. N = 1 - 2 + 3 - 4 + ... + 9 - 10 - 10 again. So, N = -15 .

2. It is easiest to compute the cost first: solve $(1+0.15)\cdot C = 129.95$.
So, C = 113 , and Markup = 129.95 - 113 = 16.95 .

3. The son and daughter shared in the ratio 3 to 7 one-third of the estate. Therefore, the son's share was $\frac{3}{10}$ *of* $\frac{1}{3} = \frac{3}{30}$ and the daughter's was $\frac{7}{10}$ *of* $\frac{1}{3} = \frac{7}{30}$. The wife then received $\frac{14}{30}$ for a total, so far, allotted of $\frac{24}{30}$. So, the butler received $\frac{6}{30} = \frac{1}{5}$ and 300,000 is $\frac{1}{5}$ of 1,500,000 .

Round#4 1. $262 \times 261 = 68,382$.

2. Suppose there are M men and W women in the kingdom. Since teams contain one man and one woman, $\frac{1}{2}M = \frac{2}{3}W$ *or* $M = \frac{4}{3}W$. The fraction not in the tournament is $\dfrac{\frac{1}{2}M + \frac{1}{3}W}{M+W} = \dfrac{\frac{1}{2}\cdot\frac{4}{3}W + \frac{1}{3}W}{\frac{4}{3}W + W} = \dfrac{1}{\frac{7}{3}} = \dfrac{3}{7}$.

3. The answer is the same for every ten consecutive counting numbers.
So, do it for 1 thru 10, and you see the result is $\frac{6}{10}$ or $\frac{3}{5}$.

Team 1. Notice that a dog tied to a post at -1 would be on a leash 2 units long. Therefore, $|x-(-1)| \le 2$ *or* $|x+1| \le 2$.

2. $\overline{A} = \{2,\ 4,\ 5,\ 6,\ 8\}$, $\overline{A} \cap B = \{2,\ 5,\ 8\}$, $C \cup (\overline{A} \cap B) = \{1,\ 2,\ 4,\ 5,\ 6,\ 8\}$, and the result follows.

3. $(D - 0.20 \cdot D) \times 1.05 - 5 = 16 \qquad (D = 25)$.

4. There are 9+26 = 35 allowable characters, from which 3 thru 6 must be chosen without repetition. The answer is ${}_{35}P_3 + {}_{35}P_4 + {}_{35}P_5 + {}_{35}P_6 = 1,208,926,950$.

5. The answer is a ratio of two areas: $\dfrac{5 \times 7 - 3 \times 5}{5 \times 7} = \dfrac{20}{35} = \dfrac{4}{7}$.

6. Suppose the set that has *N* subsets has e elements. Then $2^e = N$. Suppose also that the set of which it is a subset has a more elements. Then, $2^{e+a} = 1984 + 2^e$. Subtracting 2^e from both sides and factoring out 2^e , we obtain $2^e \cdot (2^a - 1) = 1984$. The problem reduces, therefore, to one of determining how many factors of 2 the number 1984 has. Since $1984 = 2^6 \cdot 31$, the answer is 6 .

7. To solve this, I wrote out the 6 by 6 tables for the sum and for the product of two rolled dice. Then I placed one on top of the other. If either result in a box was a multiple of 3, then that box was counted. The total of counted boxes was 28. So, the probability was $\dfrac{28}{36} = \dfrac{7}{9}$.

8. Suppose the distance from B to C is D. Since the entire distance from A to C was covered in 2 hours, 48 minutes, or $2\frac{4}{5}$ hours, at an average of 30 mph, the entire distance must be $2\frac{4}{5} \times 30 = 84$ miles. Thus, $\dfrac{84 - D}{36} + \dfrac{D}{24} = 2\frac{50}{60}$ hours. And D = 36 miles.