WOCOMAL

Freshman Meet #1

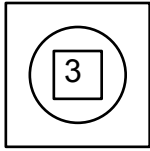### November 20, 2002

ROUND#1: **Evaluation of Algebraic Expressions & Order of Operations**

⟨ **No Calculators, and Simplest Forms Required** ⟩

1. If $\boxed{N}$ means "multiply N by 3 and add 4" and $\textcircled{N}$ means "square N and add 5" , then evaluate:

$$\boxed{\textcircled{3}}$$

2. If $a * b = a(a + b)$ and $a \# b = a(a - b)$, calculate the value of $(3 * 5)\#(6 * 5)$.

3. If $3 = x$, and $1 + \dfrac{x}{1 + \dfrac{x}{1 + \dfrac{x}{1 + x}}} = \dfrac{a}{b}$ in reduced form, find $a + b$.

Answer here: 1. (1 pt.) _____

                    2. (2 pts.) _____

                    3. (3 pts.) _____

St. John's, Shepherd Hill, Auburn

November 20, 2002   Freshman Meet#1

ROUND#2: **Solving Linear Equations**

⟨ **No Calculators, and Simplest Forms Required** ⟩

1. Solve for N: $\frac{2}{3}(7+N)=\frac{1}{5}N+5$

2. The sum of a number, one-half of the number, one-third of the number, and one-fourth of the number is 1000. What is the number ?

3. Solve for x: $0+1\cdot x - 2(3x+4)+5(6x+7)-8(9x-10)=1000$

Answer here: 1. (1 pt.) _____

                            2. (2 pts.) _____

                            3. (3 pts.) _____

Hopedale, Algonquin, Assabet

ROUND#3: **Logic Problems**

⟨ **Two points each question** ⟩

1. Each of the letters A thru F represents a different whole number from 0 to 5.
   Calculate the difference: 100F+10E+D.

$$
\begin{array}{cccc}
 & A & B & C & D \\
- & & F & E & B \\
\hline
 & & F & E & D \\
\end{array}
$$

2. Four men named Mr. Red, Mr. White, Mr. Blue, and Mr. Black went to a party where there were no other men, and each of them wore a shirt that was either red, white, blue, or black. But no two men wore the same color, and no man wore the same color as his name. Mr. Red and Mr. White arrived before the man in black. The man in red noted that Mr. White had a mustache. Which man wore blue?

3. There are three cups and beneath them three objects (a coin, a shell, a bean, but not necessarily in that order) arrayed left to right on a tabletop. 1) To the left of the white cup is the black cup. 2) To the left of the bean is the coin. 3) To the right of the shell is the gray cup. 4) To the right of the gray cup is the bean. Under which color cup is the coin?

Answer here: 1. (2 pts.) _____

                    2. (2 pts.) _____

                    3. (2 pts.) _____

Bromfield, Hudson Catholic, Tahanto

ROUND#4: **Ratio, Proportion, & Variation**

⟨ **Answers must be in Simplest Form** ⟩

1. If x varies directly as $y+1$, and $x=4$ when $y=7$,
   then what does y equal when $x=7$?

2. Jose took a sequence of examinations, some in Greek and some in Latin. His average
   (arithmetic mean) on all of the exams was 87.5. His average on the Greek exams
   alone was 80, and his average on the Latin exams was 92. What is the reduced ratio
   $\frac{G}{L}$ of the number of Greek exams he took to the number of Latin exams ?

3. Suppose $F = \dfrac{k\,A\,B}{D^2}$, where k is a constant of variation. If A increases by 10%,
   B increases by 25%, and D decreases by 5%, by what percent does F increase ?
   [Answer to the nearest tenth of a %.]

Answer here: 1. (1 pt.) _____

                    2. (2 pts.) _____

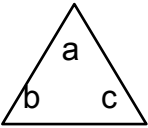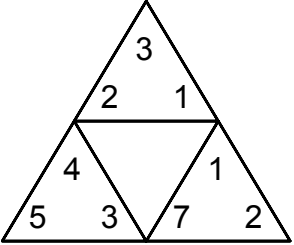   3. (3 pts.) _____%

Quaboag, Worcester Acad., QSC

Team Round:

⟨ **Answers must be exact and in simplest form, or rounded to 3 decimal places**. ⟩

1. If $= a \cdot b - c$, compute the value of   .

2. Solve: $\frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}x - \frac{1}{5}x + \frac{1}{6}x - \frac{1}{7}x = \frac{1}{4}$.

3. Three friends, each with some cash, redistribute their money as follows: Ami gives enough money to Jan and Troy to double the amount that each has. Jan then gives enough to Ami and Troy to double their amounts. Finally, Troy gives Ami and Jan enough to double their amounts. Troy has $36 when they begin and $36 when they end. What is the total amount that all three have?

4. The interest rate required to yield a given income is inversely proportional to the invested principle. Mr. Doherty has an annual income from a $25,000 investment at a rate of 6%. If the rate had been 8%, how much less money would he have had to invest to obtain the same yield?

5. Jack and Jill are each to mow a rectangular lawn, 160 feet by 100 feet. Each uses a mower which has a blade 20 inches wide. Jack mows his strips 160 feet long, and Jill mows in the other direction with strips 100 feet long. Each walks at the same speed, cutting paths that do not overlap. Jack's "turn-around time" is 10 seconds, and Jill's is 8 seconds. Who finishes first, and by how many seconds? [1 foot = 12 inches]

6. If three positive integers are added two at a time, the sums obtained are 180, 208, and 222. Find the largest of these integers.

7. I have a digital clock that does not display a colon between the hours and minutes. In a 12-hour period, how many perfect squares are displayed ?

8. If $\dfrac{a+b}{a-b} = \dfrac{7}{3}$, compute the ratio $\dfrac{a^2 + b^2}{a^2 - b^2}$.

Bromfield-Hudson, Bartlett, Bancroft, Doherty, Assabet, Hudson, Northbridge, QSC

3 Points Each

Answers must be **exact** or
rounded to **three decimal** places,
except where stated otherwise.

Answers here ⬇ :

1._____

2._____

3._____

4._____

5._____

6._____

7._____

8._____

School:_____

Team#: _____

Players' Names ⬇ :

#1:_____

#2:_____

#3:_____

#4:_____

#5:_____

WOCOMAL Answers Freshman Meet #1 November 20, 2002

R#1: 1. 526

2. -1008

3. 59

R#2: 1. $N = \dfrac{5}{7} \approx 0.714$

2. 480

3. -19

R#3: 1. 523

2. Mr. White

3. Gray

R#4: 1. 13

2. $\dfrac{3}{5}$

3. 52.4 %

Team:

1. 80

2. $\dfrac{105}{101}$

3. $ 252

4. $ 6,250

5. Jack by 170 sec.
(or 2 min., 50 sec.)

6. 125

7. 16

8. $\dfrac{29}{21}$

WoCoMaL

F1 - Solutions   Nov. 20, 2002

Round#1 1. $\boxed{3}$ = 13 , $\bigcirc\!\!\!\!13$ = 174 , and $\boxed{174}$ = 526

2. $(3*5) = 24$, $(6*5) = 66$, $24 \# 66 = -1008$.

3. $1+3/(1+3/(1+3/(1+3)))=\frac{40}{19}$; so ans. is 59 .

Round#2 1. $N = \frac{5}{7} = 0.\overline{714285} \approx .714$ by our rules.

2. Solve $N + \frac{1}{2}N + \frac{1}{3}N + \frac{1}{4}N = 1000$ gives $N = 480$.

3. $0 + x - 6x - 8 + 30x + 35 - 72x + 80 = 1000$ or $-47x + 107 = 1000$ or $x = -19$.

Round#3 1. Since B=0 and A can be no larger than 1, F+F=5+5=10. The only remaining way that E+E=C is 2+2=4. Therefore, D=3, and ans. is 523.

2. No man wore the same color as his name. And since Red and White arrived before the man wearing black, Blue had to be wearing black. Therefore, either White or Black wore red. But since the man in red noticed a mustache on White, Black must have worn red. This leaves Red and White to wear blue and white. It can only happen that <u>White wears blue</u>.

3. From statements 3 and 4, it is clear that the gray cup must be in the middle, with the shell on the left and the bean on the right. So, the coin is in the middle under the gray cup.

Round#4 1. $x = k(y+1)$ or $\dfrac{x_1}{y_1 + 1} = \dfrac{x_2}{y_2 + 1}$. Therefore, $\dfrac{4}{7+1} = \dfrac{7}{y+1}$ and $y = 13$.

2. If he took G Greek exams and L Latin exams, then total points can be computed in two ways: $TP = 87.5(G + L)$ and $TP = 80G + 92L$. Sinplifying $87.5(G + L) = 80G + 92L$ we get $7.5G = 4.5L$ or G/L=4.5/7.5 = 3/5 .

3. In the given expression for F, replace A with 1.10A, B with 1.25B, and D with .95D Then look at the factors $\dfrac{1.10 \times 1.25}{.95^2}$. By calculator, this equals 1.5235 , and represents a 52.4 % increase in F's value, to the nearest hundredth.

<u>Team</u> 1. The only way this makes sense is if the small corner triangles are evaluated first,
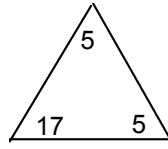
leaving a problem which looks like (a triangle with 5 at top, 17 and 5 at bottom corners), and itself equal to 80 .

2. The LCD $= 420$. Mult. by it we get $210x - 140x + 105x - 84x + 70x - 60x = 105$ or $101x = 105$, from which $x = \frac{105}{101} = 1.\overline{0396} \approx 1.040$, to 3 decimal places.

3. Suppose we start with the amounts: A and J and T . We eventually learn that T $= 36$, but for now let's stay with T. After one round of giving the amounts are: A - J - T and 2J and 2T . After two rounds they have: 2A - 2J - 2T and 2J -A + J + T - 2T and 4T . After the third round of giving Troy has 4T- 2A + 2J + 2T - 2J + A - J - T + 2T . This latter simplifies to 7T- A - J and equals T . So, A + J + T = 7T = 7(36) = 252. What luck!

4. $r = \dfrac{k}{P}$ or $rP = k$. So, $r_1 P_1 = r_2 P_2$, from which $.06 \times 25000 = .08 \times P$. This means that $P = 18,750$, which is a drop of $ 6,250 .

5. Jack and Jill spend the same amount of time mowing the total area of lawn. So, the only difference in their times derives from their total time spent turning around. Jack mows $\dfrac{100 \times 12}{20} = 60$ strips, and therefore turns 59 times, spending $59 \times 10 =$ 590 seconds. Jill, on the otherhand, mows $\dfrac{160 \times 12}{20} = 96$ strips. Since she turns 95 times, she spends $95 \times 8 = 760$ seconds turning. Thus, he finishes first by 170 seconds.

6. Suppose the largest integer is L . Then $a + b = 180$, $a + L = 208$, and $b + L = 222$. Add the second and third and subtract the first to obtain $2L = (a + L) + (b + L) - (a + b) = 250$ or $L = 125$.

7. This is a tricky problem, best done by listing the possibilities in order. Between 1200 and 0100, the only solution is $35^2 = 1225$. Then the solutions are 0100, 0121, 0144, 0225, 0256, 0324, 0400, 0441, 0529, 0625, 0729, 0841, 0900, 1024, and 1156 . This makes a total of sixteen perfect squares displayed.

8. First flatten out the equation in the hypothesis: $3(a + b) = 7(a - b)$. Distribute and collect like terms to obtain: $10b = 4a$. From which $a = \frac{5}{2}b$. Now substitute this to obtain: $\dfrac{a^2 + b^2}{a^2 - b^2} = \dfrac{\frac{25}{4}b^2 + b^2}{\frac{25}{4}b^2 - b^2} = \dfrac{\frac{25}{4} + 1}{\frac{25}{4} - 1} = \dfrac{25 + 4}{25 - 4} = \dfrac{29}{21}$.