ROUND I: Algebraic word problems

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If you are paid $12 an hour for up to 40 hours per week and time and a half for overtime, how many hours would you have to work in a week to earn $624?

2. Marsha spent two-thirds of her money and misplaced three-quarters of the remainder, leaving herself with $18   With how many dollars did Marsha start?

3. A square piece of remnant meterial is on sale. A rectangular piece of the same material whose length is one yard longer than a side of the square and whose width is $5/9$ yard shorter than a side of the square is also on sale. If the square and rectangle have the same area and you buy both pieces, how much material will you get? Answer in square yards.
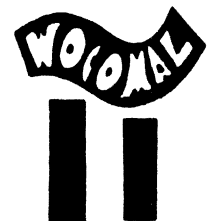
ANSWERS

1. (1 pt) _____ hours

2. (2 pts) $ _____

3. (3 pts) _____ sq yds

Hudson, Quaboag, St.John's

ROUND II: Operations on polynomials

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $(x+1)^{20}$ is multiplied out and all combinable terms are combined, how many terms will there be?

2. Express $(2x+3)^2 +(2x-3)^2 +(2x+3)(2x-3)$ in simplified polynomial form.

3. Factor completely: $x^3y - 63y^2 + 7x^2 - 9xy^3$

ANSWERS

1. (1 pt) _____

2. (2 pts) _____

3. (3 pts) _____

Bartlett, Notre Dame, Shepherd Hill

ROUND III: Number theory

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the sum of 111101_2 and 1001100_2. Give your answer in base 10.

2. Find the sum of the greatest common factor and the least common multiple of 476 and 539

3. Let a prim prime be defined as a prime that can be expressed as the sum of two other primes. What is the sum of the biggest prim prime less than 100 and the smallest prim prime that exists?

ANSWERS

1. (1 pt) _____
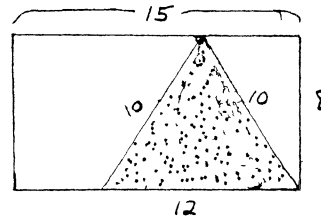
2. (2 pts) _____

3. (3 pts) _____

St.John's, Shepherd Hill, Shrewsbury

ROUND IV: Perimeter, area, volume

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What is the perimeter of a square it its area is .0729 square kilometers?

2. What percent of the rectangular region is shaded?

3. A 4 by 4 by 4 cubical box contains 64 identical cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

ANSWERS

1. (1 pt) _____

2. (2 pts) _____ % _____

3. (3 pts) _____

Algonquin, Bancroft, South

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE
TEAM ANSWER SHEET   3 points each

1. Two planes leave A at the same time to fly to B. One plane averages 720 km/hr and the other 750 km/hr. If the faster plane arrives 4 minutes before the other and they both travel the same distance, what is that distance?

2. Factor completely: $x^2 - y^2 - 2yz - z^2$

3. The makers of Cooltreat Ice Cream put a coupon for a free ice cream bar in every 80th bar they make. They put a coupon for 2 free bars in every 180th bar and a coupon for 3 free bars in every 300th bar. How often do they put all three coupons in a single bar?

4. Find the area of one of the smaller sections of the wooden frame if the picture itself is 16 inches by 8 inches and the overall dimensions are 20 inches by 12 inches. The upper and lower sections are congruent as are the two side sections.
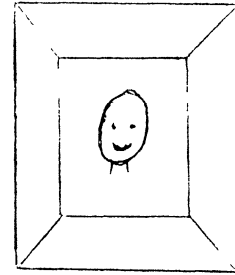
5. The average of a set of 10 numbers is 10. The average of a set of 20 numbers is 20. The average of a set of 30 numbers is 30. The average of a set of 40 numbers is 40. If the four sets of numbers are combined into a single set, find the average of that set of numbers.

6. Write the formula for expressing the charge C in dollars for a telephone call of n minutes, n being greater than 3, if the charge for the first 3 minutes or less is A cents and the charge for each additional minute in excess of 3 minutes is B cents.

7. If t is the tens digit of a 2-digit number and u is the units digit, by how much does the number exceed the sum of its digits?

8. If a = 3b + 2c and b = 5a - 3c, express c in terms of a only.

Algonquin, Assabet Valley, Bartlett, South, Tantasqua Jr., Westboro, Worcester Academy

ROUND I   1. 1 pt   **48** hours

word prob   2. 2 pts   **$216**

3. 3 pts $3\frac{1}{8}$ sq yds   **3.125** or $\frac{25}{8}$ OK

ROUND II   1. 1 pt   **21**

poly ops   2. 2 pts $12x^2 + 9$

3. 3 pts $(7 + xy)(x + 3y)(x - 3y)$

ROUND III   1. 1 pt   **137**

thry   2. 2 pts   **36 659**

3. 3 pts   **78**

ROUND IV   1. 1 pt   **1.08 Km**   Need Units!

perim area vol   2. 2 pts   **40 %**

3. 3 pts   **52**

)

TEAM ROUND   3 pts each

1. **1200** km

2. $(x + y + z)(x - y - z)$

3. In every **3600** th bar

4. **20** sq in

5. **30**

6. $c = \dfrac{A + B(n-3)}{100}$ or equivalent

7. $9t$

8. $C = 2a$

ROUND I

1. $12/hr \cdot 40\ hrs = \$480$. $\$624 - \$480 = \$144$

   $\$18/hr \cdot x\ hrs = \$144 \Rightarrow x = \frac{144}{18} = 8\ hrs$

   $40 + 8 = 48\ hours$

2. $x - \frac{2}{3}x - \frac{3}{4}\left(x - \frac{2}{3}x\right) = 18$

   $\frac{1}{3}x - \frac{3}{4}\left(\frac{1}{3}x\right) = 18$

   $4x - 3x = 12 \cdot 18$

   $x = \$216$

3.

   $x^2 = (x+1)\left(x - \frac{5}{9}\right)$

   $x^2 = x^2 + \frac{4}{9}x - \frac{5}{9}$

   $\frac{5}{9} = \frac{4}{9}x \Rightarrow x = \frac{5}{4}\ yd$

   Amt material $= 2x^2 = 2 \cdot \frac{25}{16} = 3\frac{1}{8}\ sq\ yds$

ROUND II

1. Starts with x^{20} and ends with 1. Every integer exponent from 19 through 1 is used on x in between. $\therefore$ 21 terms

2. $4x^2 + 6x + 9 + 4x^2 - 6x + 9 + 4x^2 - 9$

   $= 12x^2 + 9$

3. Rearrange to

   $x^3 y + 7x^2 - 9xy^3 - 63y^2$

   $= x^2(xy + 7) - 9y^2(xy + 7)$

   $= (xy + 7)(x^2 - 9y^2)$

   $= (xy + 7)(x + 3y)(x - 3y)$

ROUND III

1. $\begin{array}{r} 111101 \\ 1001100 \\ \hline 10001001_2 \end{array} = 2^7 + 2^3 + 1 = 128 + 8 + 1 = 137$

   OR from $61 + 76$

ROUND III cont.

2. $476 = 2 \cdot 2 \cdot 7 \cdot 17$
   $539 = 7 \cdot 7 \cdot 11$

   GCF $= 7$
   LCM $= 2 \cdot 2 \cdot 7 \cdot 17 \cdot 7 \cdot 11 = 476 \cdot 77$
   $\qquad\qquad = 36652$

   Sum $= 36659$

3. Smallest $= 5$ $(= 2 + 3)$
   Biggest is odd, hence not a sum of two odd numbers, but must be prime + 2.
   Biggest $= 73 = 71 + 2$, by looking at sequence of primes.
   Sum $= 5 + 73 = 78$
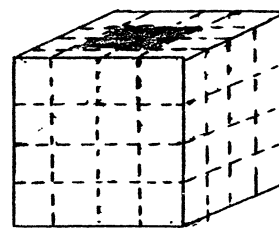
ROUND IV

1. side $= \sqrt{.0729} = .27\ km$

   perim $= 4(.27) = 1.08\ km$

2. $\triangle$ area $= \frac{1}{2} \cdot 12 \cdot 8 = 48$

   rect. area $= 8 \cdot 15 = 120$

   $\frac{48}{120} = \frac{2}{5} = 40\%$

3. All but those 12 in a 2 by 2 by 3 rectangular solid inside except for the top.
   $64 - 12 = 52$

TEAM ROUND

1. Let $t =$ time of faster plane, in hours
   $750t = 720\left(t + \frac{1}{15}\right)$
   $30t = \frac{720}{15} = 48$
   $t = \frac{8}{5}\ hrs$
   Dist $= 750 \cdot \frac{8}{5} = 1200\ km$

TEAM ROUND cont.

2. Change to
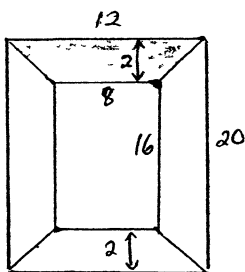$$x^2 - (y^2 + 2yz + z^2)$$
$$= x^2 - (y+z)^2$$
$$= (x+y+z)(x-y-z)$$

3. Need least common multiple of
80, 180, and 300.

$80 = 2^4 \cdot 5$ LCM is
$180 = 2^2 \cdot 3^2 \cdot 5$ $2^4 \cdot 3^2 \cdot 5^2 = 3600$
$300 = 2^2 \cdot 3 \cdot 5^2$

4. Assume trapezoid.
Upper and lower
sections are smaller.

Area $= \frac{1}{2}(2)(8+12)$

$= 20$ sq in

5.
$10 \cdot 10 = 100$
$20 \cdot 20 = 400$
$30 \cdot 30 = 900$
$40 \cdot 40 = 1600$
$\overline{\hspace{2cm} 3000}$

100 numbers
in all

Ave $= \dfrac{3000}{100} = 30$

6. A cents for the first 3 minutes.
Beyond 3 min, the time is $n-3$ at B ¢ each.
Charge $= B(n-3)$. Add these and
divide by 100 to get dollars
$$C = \frac{A + B(n-3)}{100}$$

7. $(10t + u) - (t + u) = 9t$

8. $a = 3b + 2c$ and $b = 5a - 3c$ (
$\therefore a = 3(5a - 3c) + 2c$
$a = 15a - 9c + 2c$
$7c = 14a$
$c = 2a$