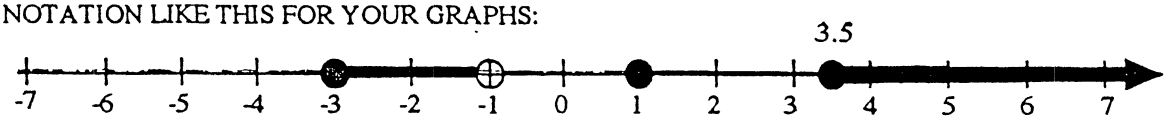
January 9, 2002 WOCOMAL Freshman Meet

ROUND I: Graphing on a number line - inequalities, absolute value

NO CALCULATOR USE

DRAW THE GRAPH FOR EACH PROBLEM ON THE NUMBER LINE PROVIDED.
SPECIFY ANY NON-INTEGER ENDPOINTS.
USE NOTATION LIKE THIS FOR YOUR GRAPHS:



$-3 \le x < -1$ or $x = 1$ or $x \ge 3.5$

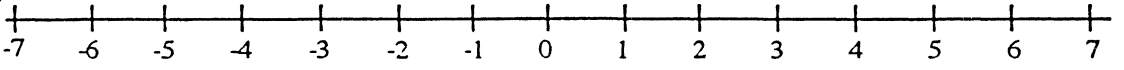
1. From the set that contains 3, -1, $\frac{7}{2}$, 0, $\sqrt{7}$, and -4, graph those numbers both less than 3 and greater than -4.

2. Graph the solution to this compound inequality: $4+2x \le 1$ or $3x+8>20$.

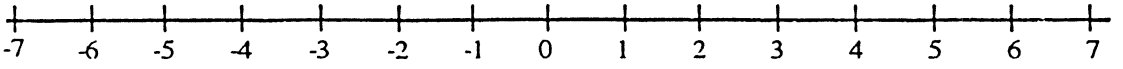
3. Graph $1<|x+1|\le 5$

ANSWERS

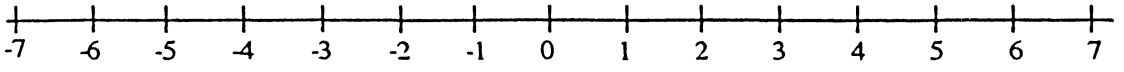
(1 pt.) 1.



(2 pts) 2.



(3 pts) 3.



Douglas, Notre Dame, St.John's

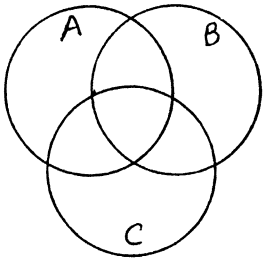
ROUND II: Set theory Note: $\overline{A}$ denotes the complement of set A

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Let A and B be subsets of U = { 0, 1, 2, 3, 4, 5, 6 }. Specify set B by a roster if A = { 2, 4, 6 }, $A \cap B = \{ 4, 6 \}$, and $A \cup B = \{ 0, 2, 3, 4, 6 \}$.

2. If U = { 1, 2, 3, 4, 5, 6 }, A = { 1, 2, 3 }, B = { 2, 3, 4 }, and C = { 2, 4, 6}, list the elements in $\overline{C} \cap \overline{(A \cup B)}$.

3. In the Venn diagram like this in the answer space, shade $(\overline{A} \cap C) \cup \overline{B}$ if the universe is $A \cup B \cup C$.

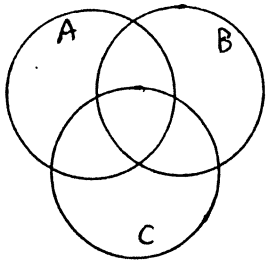


ANSWERS

1. (1 pt) { }

2. (2 pts) { }

3. (3 pts)



Auburn, Burncoat, Shrewsbury

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ROUND III: Operations on numerical fractions, decimals, percents, and percentage word problems

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. There are 180 days in a school year. After the 36th day is completed, the number of days remaining is what per cent of the 36 days completed?

2. Evaluate: $3+\dfrac{\dfrac{1}{0.2}}{3}-0.8\left(2+\dfrac{3}{\dfrac{8}{5}}\right)$.

3. Find the 40th decimal place in the decimal representation of the sum of $\frac{1}{3}$ and $\frac{1}{7}$.

ANSWERS

1. (1 pt) _____ %

2. (2 pts) _____

3. (3 pts) _____

Hudson, Quaboag, St.Peter-Marian

ROUND IV: Techniques of counting and probability

EACH ANSWER MUST BE A POSITIVE INTEGER OR A REDUCED FRACTION

1. Using the digits 1, 2, 3, and 4 , how many 4-digit numbers can be created if a digit can only be used once in a number?

2. Suppose your dog has 4 puppies. What is the probability that all 4 will be the same sex? Assume that male and female puppies are equally likely.

3. Mr. and Mrs. Smith are going out to eat with their two sons. The two boys cannot be trusted to sit across from each other because they kick each other under the table. If the family is seated in a booth for four, two on each side, how many different seating arrangements are acceptable? Individual seat location does matter.

ANSWERS

1. (1 pt) _____

2. (2 pts) _____

3. (3 pts) _____

Algonquin, Burncoat, St.Peter-Marian

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TEAM ROUND: Topics of previous round and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 3 points each

1. Graph over the set of integers on the number line provided: $|1-3x|<11$

2. $U=\{0,1,2,3,...,9\}$. $A=\{1,2,5,8\}$. $B=\{2,3,6,8\}$. $C=\{5,6,7\}$. If $\overline{A\cup B}\cup C=\overline{D}$, list the elements of D in increasing order.

3. At Adams H. S., 30% of the students in the math club are in the science club, and 80% of the students in the science club are in the math club. There are 15 students in the science club. How many students are in the math club?

4. If Meghan tosses a pair of dice, what is the probability that the sum of the top face dots is exactly 8?

5. If the length of a rectangular field is twice the width and the field is enclosed by x yards of fencing, what is its area in square yards in terms of x?

6. Two identical jars are filled with mixtures of water and vinegar in the ratio of 2 to 1 and 3 to 1, water to vinegar, respectively. If both jars are emptied into another container, find the ratio of water to vinegar in the combined mixture.

7. Each of the letters w, x, y, and z represents a different integer in the set $\{1,2,3,4\}$, but not necessarily in that order. If $\frac{w}{x}-\frac{y}{z}=1$, find the sum of w and y.

8. If the American baseball League standings read:

	wins	losses	games behind
New York	84	62	-
Boston	76	70	8

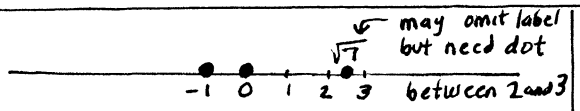
what combination of New York wins and/or Boston losses will mathematically eliminate the Boston team? There are 162 games in the season.

Algonquin, Bancroft, Bromfield, Holy Name, Hudson, Quaboag

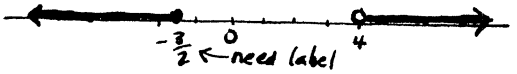
January 9, 2002 WCCOMAL Freshman Meet ANSWERS

ROUND I — ½ line graphs

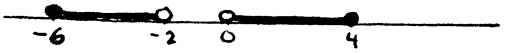
1. 1 pt



2. 2 pts



3. 3 pts



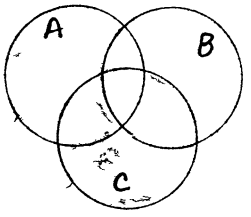
[or] OK for ● (or) OK for ○ endpts

ROUND II — sets

1. 1 pt $\{0, 3, 4, 6\}$ any order

2. 2 pts $\{5\}$

3. 3 pts



ROUND III — fract dec %

1. 1 pt 400 %

2. 2 pts $1\frac{17}{30}$ OR $\frac{47}{30}$ OR $1.5\overline{6}$

3. 3 pts 1

ROUND IV — count prob

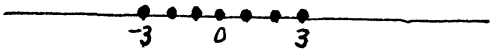
1. 1 pt 24

2. 2 pts $\frac{1}{8}$

3. 3 pts 16

TEAM ROUND 3 pts each

1.



2. $\{1, 2, 3, 8\}$ in this order

3. 40

4. $\frac{5}{36}$ OR $.13\overline{8}$ OR $13.\overline{8}\%$

5. $\frac{x^2}{18}$

6. 17 to 7 OR 17:7 OR $\frac{17}{7}$

7. 7

8. 9

WOCOMAL FRESHMAN MEET Jan 9, 2002 BRIEF SOLUTIONS

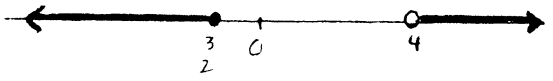
ROUND I

1 Test each number to see if it is strictly between -4 and 3, not inclusive. The dot for $\sqrt{7}$ needs to be somewhere between 2 and 3

2. $4+2x \le 1$ OR $3x+8 > 20$

$2x \le -3$ $\quad$ $3x > 12$

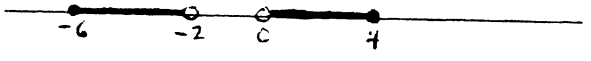
$x \le -\frac{3}{2}$ $\quad$ $x > 4$



3 $1 < x+1 \le 5$ OR $1 < -x-1 \le 5$

$0 < x \le 4$ $\quad$ $2 < -x \le 6$

$-2 > x \ge -6$



ROUND II

1. $A=\{2,4,6\}$ and $A \cap B = \{4\ 6\}$ means that B includes 4 and 6, but not 2

$A=\{2,4,6\}$ and $A \cup B = \{0\ 2\ 3\ 4\ 6\}$ means that B also includes 0 and 3

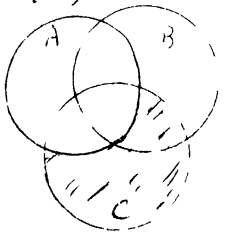
$\therefore B = \{0, 3, 4, 6\}$

2 $\bar{C} = \{1,3,5\}$

$A \cup B = \{1,2,3,4\}$ so $\overline{A \cup B} = \{5,6\}$

Then $\bar{C} \cap \overline{A \cup B} = \{5\}$

3 $\bar{A} \cap C$ gets



This $\cup \bar{B}$ gets

There is nothing outside because of the stated universe

ROUND III

1 $180-36 = 144$ days remaining

$$\frac{144 \text{ days left}}{36 \text{ days completed}} = 4 \Rightarrow 400\%$$

2 $$3 + \frac{\frac{1}{0.2}}{3} - 0.8\left(2 + \frac{3}{\frac{8}{5}}\right)$$

$$= 3 + \frac{5}{3} - \frac{4}{5}\left(2 + \frac{15}{8}\right)$$

$$= 3 + \frac{5}{3} - \frac{4}{5}\left(\frac{31}{8}\right)$$

$$= \frac{14}{3} - \frac{31}{10}$$

$$= \frac{140-93}{30} = \frac{47}{30} \text{ or } 1\frac{17}{30} \text{ or } 1.56\overline{6}$$

3 $\frac{1}{3} = 0.\overline{3}$ and $\frac{1}{7} = 0.\overline{142857}$

$\frac{1}{3} + \frac{1}{7} = 0.\overline{476190}$

In this 6 digit repeating block, the digit 4 occurs 1st, 7th, 13th, ..., 37th, making the 40th digit a 1

ROUND IV

1 Ways to choose digits, in order

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

2 $P(\text{all M}) + P(\text{all F})$

$$= \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

3 There are $4! = 24$ possible seating arrangements, 8 of which are not acceptable $24-8=16$

Mr or Mrs	boy 1
Mrs or Mr	boy 2

Mr or Mrs	boy 2
Mrs or Mr	boy 1

boy 1	Mr or Mrs
boy 2	Mrs or Mr

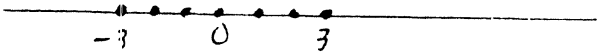
boy 2	Mr or Mrs
boy 1	Mrs or Mr

TEAM ROUND

1. $-11 < 1-3x < 11$

$-12 < -3x < 10$

$4 > x > -\frac{10}{3}$, but integers only



2. $\overline{A \cup B} \cup C = \overline{\{1,2,3,5,6,8\}} \cup \{5,6,7\}$

$= \{0,4,5,6,7,9\} = D$

Then $D = \{1,2,3,8\}$

3. 80% of science club $= .8(15) = 12$ are in math club and this is 30% of the math club, M.

$.30M = 12$

$M = \frac{12}{.3} = 40$

4

	1	2	3	4	5	6
1						
2				•		
3				•		
4				•		
5		•				
6	•					

Of 36 equally likely outcomes, 5 have a sum of 8

$\therefore \frac{5}{36}$

5. Perimeter $= 2L + 2w = x$

But $L = 2w$, so $6w = x$

and $w = \frac{x}{6}$. Thus $L = \frac{x}{3}$

and area $= \frac{x^2}{18}$ sq yds

6. Use easy numbers. Let vol of each jar = 12.

One jar has $\frac{2}{3}$ water and $\frac{2}{3}(12) = 8$.

Other jar has $\frac{3}{4}$ water and $\frac{3}{4}(12) = 9$.

Total water $= 8+9 = 17$

Total vinegar $= 24-17 = 7$

Ans: $\frac{17}{7}$

7. Find $\frac{3}{1} - \frac{4}{2} = 1$ by trial and error. (Try values for w first).

More trial finds no other solutions.

Thus $w+y = 3+4 = 7$

8. Both teams have 16 games left. If Boston wins all of them, its record will be 92 W, 70 L. To finish ahead, NY must then win at least 9 more to get 93 W.

Ans: 9