

April 11, 2001

WOCOMAL Freshman Meet

ROUND I: Algebraic Word Problems

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Jane had a collection of nickels, dimes, and quarters. She had 4 more quarters than dimes and 3 more dimes than nickels. Write a simplified algebraic expression which represents the number of cents Jane had in this collection in terms of the number of nickels, n .

2. The width of a rectangular window frame is 12 inches less than its length. If you add 30 inches to both the length and the width, you double the perimeter. Find the length and width of the original rectangle.

3. A man rows a boat 9 miles upstream and returns on a river that is flowing at a rate of 2 miles per hour. If his trip takes him 6 hours, at what rate would he be rowing in still water?

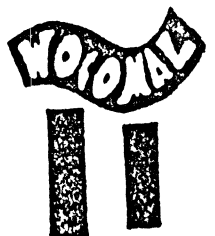
ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____ in. and _____ in.

(3 pts) 3. _____ mi/hr

Doherty, Holy Name, Westborough



ROUND II: Operations on polysomials

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Multiply $(x + 4)$ by $(x^2 - 4x + 16)$ and express the result as a simplified polynomial.

2. Subtract $(3x^2 - 4x + 10)$ from the average of $(x^2 + 5x + 21)$ and $(7x^2 - 7x - 15)$.

3. Factor completely: $3a^{9z-27} - 13a^{6z-18} - 10a^{3z-9}$

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

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ROUND III: Number theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the greatest common factor of 1776 and 2001.

2. Add : 432_5
 123_5
 214_5

Express the sum as a base 5 number.

3. Take the prime number that is between 47 and 59 and subtract the greatest common factor of 42 and 56. Then multiply that result by the negative reciprocal of -1.5. Take this answer and round it up to the nearest whole number divisible by 3. Finally express that result in base 2.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____₅

(3 pts) 3. _____₂

Bancroft, Hudson, St.Peter-Marian

ROUND IV: Perimeter, area volume

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM; except number 2.

1. A piece of wire 14 feet long is bent to form a rectangle whose length and width are in the ratio 3:2. Find the shorter dimension of the rectangle in feet.
2. A circular garden 18 feet in diameter is surrounded by a path 4 feet wide. Which has the bigger area, the garden or the path and, to the nearest integer, by how many square feet?
3. In a rectangular prism, the area of the base is 160 square inches, the area of the front is 200 sq in, and the area of the side is 80 sq in. What is the length of the longest of its three dimensions? Include units!

ANSWERS

(1 pt) 1. _____ ft

(2 pts) 2. _____ by _____ sq ft

(3 pts) 3. _____

TEAM ROUND: Topics of previous rounds and open-

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM and ON THE SEPARATE
TEAM ANSWER SHEET. 3 POINTS EACH

1. The sum of a number and 6 times its reciprocal is -5. How many such numbers are there?
2. Determine all values of a so that $x + 1$ is a factor of $x^4 - ax^3 + x - 3$
3. Find the product of the number of prime numbers less than 50 and the number of positive integers less than 50 which are not multiples of 2, 3, 5, or 7.
4. By what per cent is the volume of a cube increased when its surface area is made 4 times larger?
5. You have 99 cents in change. If you have the fewest possible coins, how many nickels do you have?
6. Find the least two positive numbers whose sum is an even integer and whose difference is an odd integer.
7. How many ordered pair solutions of the system

$$\begin{cases} x > 0 \\ y \geq 0 \\ 5x + 3y \leq 15 \end{cases}$$
 have only integer coordinates?

8. Find the total number of paths in any of 4 directions (right, left, down, or up) to an adjoining letter that you can follow to spell the word MAGIC in the following configuration.

M A G I C I G A M
M A G I G A M
M A G A M
M A M
M

- ROUND I
1. 1 pt $40n + 205$
- alc
word
prob
2. 2 pts $36 \text{ in and } 24 \text{ in}$
3. 3 pts 4 mi/hr

- ROUND II
1. 1 pt $x^3 + 64$
- ops on
polys
2. 2 pts $x^2 + 3x - 7$
3. 3 pts $a^{3z-9} (3a^{3z-9} + 2)(a^{3z-9} - 5)$

- ROUND III
- # thry
1. 1 pt 3
2. 2 pts 1324_5
3. 3 pts 11011_2

- ROUND IV
- perim
area
vol
1. 1 pt $2\frac{4}{5} \text{ ft or } 2.8 \text{ ft}$
2. 2 pts $\text{path, } 22 \text{ sqft}$
3. 3 pts 20 inches need
units

TEAM ROUND 3 pts each

1. 2
2. $a = 3$
3. 180
4. 700%
5. $\text{none or } 0 \text{ or zero}$
6. $\frac{1}{2}$ and $1\frac{1}{2}$ either
order,
decimals
OK
7. 7
8. 31

ROUND I

1 n nickels, n+3 dimes, n+3+4 quarters

$$\begin{aligned} \text{Number } \phi &= 5n + 10(n+3) + 25(n+7) \\ &= 40n + 205 \end{aligned}$$

2. L = length, L-12 = width

new perimeter = 2(orig perim)

$$2(L+30) + 2(L-12+30) = 2(2L + 2(L-12))$$

$$2L + 48 = 4L - 24$$

$$72 = 2L$$

$$L = 36 \text{ in and } W = 24 \text{ in}$$

3 R = still water rate, time = $\frac{\text{dist}}{\text{rate}}$

time up + time down = 6 hours

$$\frac{9}{R-2} + \frac{9}{R+2} = 6$$

$$9(R+2) + 9(R-2) = 6(R-2)(R+2)$$

$$18R = 6(R^2 - 4)$$

$$3R = R^2 - 4$$

$$0 = R^2 - 3R - 4 = (R-4)(R+1)$$

$$R = 4 \text{ mi/hr } (R \neq -1)$$

ROUND III

1 $1776 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 37$
 $2001 = 3 \cdot 23 \cdot 29$ GCF = 3

2
$$\begin{array}{r} 432_5 \\ 123_5 \\ \hline 214_5 \\ \hline \end{array}$$

 ($\bar{79}$) Think, base 10 $\left[\begin{array}{l} 9=5+4 \\ 7=5+2 \\ 8=5+3 \end{array} \right]$
 1324_5 Write, base 5

3 prime between 47 and 51 is 53
 GCF of 42 = $7 \cdot 3 \cdot 2$ and 56 = $7 \cdot 2 \cdot 2 \cdot 2$
 is $7 \cdot 2 = 14$

Subtract, get 39

Mult $39 \left(\frac{2}{3}\right) = 26$ $-1.5 = -\frac{3}{2}$

Round up to 27

For base 2, $27 = 16 + 8 + 2 + 1$

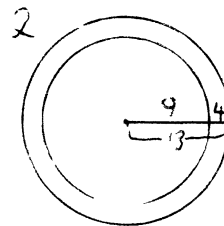
In base 2 get 11011.

ROUND IV

1 Length = 3x, Width = 2x

$$3x + 2x = 7 = \frac{1}{2} \text{ perim}$$

$$5x = 7, x = \frac{7}{5}, \text{ width} = 2x = \frac{14}{5} = 2.8 \text{ ft}$$

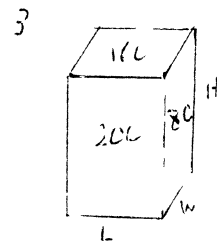


Area garden = 81π

Area path = $169\pi - 81\pi = 88\pi$

Path bigger by 7π sq ft

For nearest integer: $7 \cdot \frac{22}{7} = 22 \text{ ft}^2$



$LW = 160 \Rightarrow w = \frac{160}{L}$

$LH = 200 \Rightarrow H = \frac{200}{L}$

$WH = 80$

Longest would be involved with .60 and 200 but 80 so is L

$$WH = 80 = \frac{160}{L} \cdot \frac{200}{L}$$

$$80L^2 = 160 \cdot 200$$

$$L^2 = 400$$

$$L = 20 \text{ inches}$$

ROUND II

1 $(x+4)(x^2 - 4x + 16)$

$$\begin{aligned} &= x^3 - 4x^2 + 16x \\ &\quad + 4x^2 - 16x + 64 \\ &= x^3 + 64 \end{aligned}$$

2
$$\text{AVC} = \frac{x^2 + 5x + 21 + 7x^2 - 7x - 15}{2}$$

$$= 4x^2 - x + 3$$

Summ $3x^2 - 4x - 12$

Get $x^2 + 3x - 7$

3. See comm, factor first, then use exp rules

$$a^{3z-9} (3a^{6z-12} - 13a^{3z-9} - 16)$$

$$= a^{3z-9} (3a^{3z-9} + 2)(a^{3z-9} - 5)$$

TEAM ROUND

1 $x + \frac{6}{x} = -5, \quad x \neq 0$

$x^2 + 6 = -5x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$ has 2 roots

2 $x+1$ is a factor means that the equation $x^4 - ax^3 + x - 7 = 0$ could be written as $(x+1)(x^3 + \dots) = 0$

so that $x = -1$ makes the polynomial = 0

Subst -1 for x and get

$1 + a - 1 - 7 = 0$ and $a = 7$

3 15 primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

12 non-multiples of 2, 3, 5 or 7.

1 and the primes 11 through 47

$15 \cdot 12 = 180$

4 % change, so choose easy numbers

Orig side = 1, area = 6, vol = 1

For area = 24 and side = x

$6x^2 = 24$ and $x = 2$

Then vol = 8

$\frac{\text{inc case in vol}}{\text{orig vol}} = \frac{7}{1}$ or 700%

5

3	254	or maybe	1	504
2	164		1	254
4	14		2	164
			4	14

Either way no tickets

6. Since we want the "least", they are one apart say x and $x+1$
Make their sum the smallest positive even integer

$x + (x+1) = 2$

which makes $x = \frac{1}{2}$

The least two pos num... sought are $\frac{1}{2}$ and $\frac{3}{2}$

7

These seven	Value of $5x+3y$	
(1,0)	5	
(1,1)	8	
(1,2)	11	
(1,3)	14	
(2,0)	10	
(2,1)	13	
(3,0)	15	

8

MAGICIGAM (2)

MACIGAM (8)

MALAM (12)

MAM (8)

M (6)

31 ways

and start at right

and start at right