

Solutions to MAML Olympiad Level 2 2001

1. Fractionated

- a) The average (mean) of the two fractions is halfway between them:

$$\frac{1}{2} \left(\frac{p}{q} + \frac{r}{s} \right) = \frac{1}{2} \left(\frac{ps + rq}{qs} \right) = \frac{ps + rq}{2qs}$$

- b) The answer is yes. Assume without loss of generality that $\frac{p}{q} < \frac{r}{s}$. Then,

$$ps < qr \Rightarrow pq + ps < pq + qr \Rightarrow p(q+s) < q(p+r) \Rightarrow \frac{p}{q} < \frac{p+r}{q+s} \text{ and}$$

$$ps < qr \Rightarrow ps + rs < qr + rs \Rightarrow s(p+r) < r(q+s) \Rightarrow \frac{p+r}{q+s} < \frac{r}{s} \Rightarrow \frac{p}{q} < \frac{p+r}{q+s} < \frac{r}{s}$$

- c) It is possible as the following table shows and this is an example of **Simpson's Paradox**.

December	FT Made	FT Missed	Totals	Percent
Shaq	59	1	60	98%
Marcus	87	3	90	97%
January				
Shaq	21	19	40	53%
Marcus	3	7	10	30%
Dec. & Jan. Combined				
Shaq	80	20	100	80%
Marcus	90	10	100	90%

2. Divide and Conquer

- a) $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$; assume this sum equals the integer $e \Rightarrow \frac{ad + bc}{b} = de \Rightarrow \frac{ad}{b} + c = de \Rightarrow$

$\frac{ad}{b}$ is an integer; since a and b are relatively prime, then b must be a factor of d ;

also, $\frac{ad + bc}{d} = be \Rightarrow a + \frac{bc}{d} = be \Rightarrow \frac{bc}{d}$ is an integer; since c and d are relatively prime, then d must be a factor of b ; the only way b can be a factor of d and d can be a factor of b is if $b = d$.

- b) $223,020 = 2^2 \cdot 3^3 \cdot 5 \cdot 7 \cdot 59$; $5432 = 2^3 \cdot 7 \cdot 97$; since the greatest common factor of these two numbers is $2^2 \cdot 7$, then both numbers sought must be multiples of 28; therefore we can now consider both sought numbers divided by 28; the least common multiple of the two new numbers is now $3^3 \cdot 5 \cdot 59$ and their sum is $2 \cdot 97$; one of these new numbers is a multiple of 27, one of these new numbers is a multiple of 5, and one of these new numbers is a multiple of 59; since $27 \cdot 5 + 59 = 194 = 2 \cdot 97$, the two new numbers must be 135 and 59; multiplying each by 28 gives the two sought numbers: 1652 and 3780.

c) Assume $\sum_{k=1}^n \frac{1}{k} = m$, an integer and let L be the least common multiple of the set of numbers

$S = \{1, 2, 3, \dots, n\}$. Multiplying both sides of the equation by L gives: $L \sum_{k=1}^n \frac{1}{k} = Lm$.

By the nature of L , the left-hand side, when expanded will consist of integer terms. Let r be the highest power of 2 in the set S and consider the term on the left side formed by $L \left(\frac{1}{2^r} \right)$. Since L

is the LCM of S , $\frac{L}{2^r}$ must be odd since L does not contain any higher power of 2 than r .

However every other term in the equation, both on the left-hand and right-hand sides must be even. The reason why this is true is because if there were other odd terms, then they would have to be of the form $\frac{L}{k \cdot 2^r}$ where k is an integer greater than 1. This would imply that $2^{r+1} \leq k \cdot 2^2$

would also be in S . This contradicts that 2^r is the highest power of 2 in S .

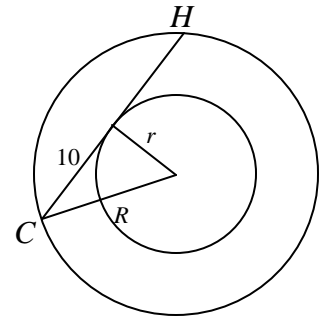
So there would be an equation such that a sum of all even integers except for one is an even integer. This is clearly impossible. Therefore the assumption that the original sum is an integer is false.

3. Round and Round

a) Let r = radius of the smaller circle and R = radius of the larger one.

By the Pythagorean Theorem $R^2 - r^2 = 10^2$. The area of the

annulus $A = \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = 100\pi$

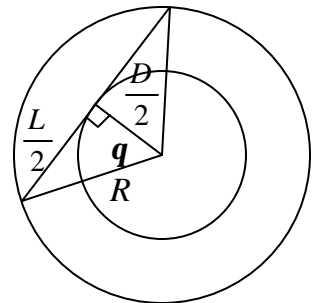


b) The area of the segment = area of the sector - area of the triangle

$$R^2 = \left(\frac{D}{2}\right)^2 + \left(\frac{L}{2}\right)^2 \Rightarrow R^2 = \frac{D^2 + L^2}{4}; \quad \mathbf{q} = \tan^{-1}\left(\frac{\frac{L}{2}}{\frac{D}{2}}\right) = \tan^{-1}\left(\frac{L}{D}\right)$$

(in radians); area of sector = $\frac{1}{2} \cdot R^2 (2\mathbf{q}) = \left(\frac{D^2 + L^2}{4}\right) \tan^{-1}\left(\frac{L}{D}\right)$;

area of triangle = $\frac{DL}{4}$; area of segment = $\left(\frac{D^2 + L^2}{4}\right) \tan^{-1}\left(\frac{L}{D}\right) - \frac{DL}{4}$



- c) If $L = \sqrt{3}D$ and the area of the segment = $4p - 3\sqrt{3}$, using the formula in part **b**, then

$$4p - 3\sqrt{3} = \frac{D^2 + (\sqrt{3}D)^2}{4} \tan^{-1} \left(\frac{\sqrt{3}D}{D} \right) - \frac{D(D\sqrt{3})}{4} \Rightarrow 4p - 3\sqrt{3} = D^2 \tan^{-1}(\sqrt{3}) - \frac{D^2\sqrt{3}}{4} \Rightarrow$$

$$4p - 3\sqrt{3} = D^2 \frac{p}{3} - \frac{D^2\sqrt{3}}{4} \Rightarrow 4p - 3\sqrt{3} = D^2 \frac{4p - 3\sqrt{3}}{12} \Rightarrow D^2 = 12 \Rightarrow D = 2\sqrt{3} \text{ and } L = 6.$$

4. Progressively Better

- a) Instead of the standard “text book” approach, you could use **Mathematical Induction**:

$$\sum_{k=1}^2 a_k = a_1 + a_2 = \frac{2}{2}(a_1 + a_2); \text{ assume it is true for } n, \text{ prove it is true for } n+1:$$

Using the formula $a_n = a_1 + (n-1)d \Rightarrow a_{n+1} = a_1 + ((n+1)-1)d = a_1 + nd$:

$$\sum_{k=1}^{n+1} a_k = \sum_{k=1}^n a_k + a_{n+1} = \frac{n}{2}(a_1 + a_n) + a_{n+1} = \frac{n}{2}(a_1 + a_1 + (n-1)d) + a_1 + nd =$$

$$\frac{n}{2}(2a_1 + (n-1)d) + \frac{1}{2}(2a_1 + 2nd) = \frac{n}{2}(2a_1 + nd - d) + \frac{1}{2}(2a_1 + 2nd) =$$

$$\frac{n}{2}(2a_1 + nd) - \frac{nd}{2} + \frac{1}{2}(2a_1 + 2nd) = \frac{n}{2}(2a_1 + 2nd) + \frac{1}{2}(2a_1 + 2nd - nd) =$$

$$\frac{n}{2}(2a_1 + 2nd) + \frac{1}{2}(2a_1 + nd) = \frac{n+1}{2}(2a_1 + nd) = \frac{n+1}{2}(a_1 + (a_1 + nd)) = \frac{n+1}{2}(a_1 + a_{n+1})$$

- b) The n th set has n elements. If you recognize the subscripts of the last terms in each set are 1, 3, 6, 10, ..., which are the triangular numbers, you can consider the sets in reverse order in which case the constant difference between terms is now $-d$. The first element in the n th

“reversed set” is $a_{\frac{n(n+1)}{2}} = a + \left(\frac{n(n+1)}{2} - 1 \right) d = a + \left(\frac{n^2 + n - 2}{2} \right) d$; now use the formulas from

$$\text{part a: } S_n = \frac{n}{2} \left(2 \left(a + \left(\frac{n^2 + n - 2}{2} \right) d \right) + (n-1)(-d) \right) = \frac{n}{2} \left(2a + (n^2 + n - 2)d + (n-1)(-d) \right) =$$

$$\frac{n}{2} \left(2a + (n^2 - 1)d \right)$$

- c) Using the formula from part **a**, if the first integer is k , then the sum of r consecutive integers starting with k would be: $\frac{r}{2}(2k+r-1)$; if r is odd then the sum can not be a power of 2; if r is even, then $2k+r-1$ would be odd and so the sum is never a power of 2.
- d) If $c \neq 2^n$, then $c = 2^a(2b+1)$ where a is a non-negative integer and b is any positive integer. There are two cases to consider:
- (i) $b < 2^a$, then if the first integer is $2^a - b$ and the last integer is $2^a + b$, there would be $2b+1$ integers added and their sum would equal $\frac{2b+1}{2}(2^a - b + 2^a + b) = 2^a(2b+1)$.
- (ii) $b \geq 2^a$, then if the first integer is $b+1-2^a$ and the last integer is $b+2^a$, there would be $2 \cdot 2^a$ integers added and their sum would equal $\frac{2 \cdot 2^a}{2}(b+1-2^a + b+2^a) = 2^a(2b+1)$.

5. Does it Really Make a Difference?

- a) If $a^2 - b^2 = 28 \Rightarrow (a+b)(a-b) = 28$; to get integer solutions for a and b , let $a+b = 14$ and $a-b = 2 \Rightarrow a = 8, b = 6$; $\therefore 8^2 - 6^2 = 28$.
If $a^2 - b^2 = 48 \Rightarrow (a+b)(a-b) = 48$; to get integer solutions for a and b , let $a+b = 24$ and $a-b = 2 \Rightarrow a = 13, b = 11$; let $a+b = 12$ and $a-b = 4 \Rightarrow a = 8, b = 4$; let $a+b = 8$ and $a-b = 6 \Rightarrow a = 7, b = 1$; $\therefore 13^2 - 11^2 = 48, 8^2 - 4^2 = 48, \text{ and } 7^2 - 1^2 = 48$.
- b) $198 = 2 \cdot 3^2 \cdot 11$; if $a^2 - b^2 = 198 \Rightarrow (a+b)(a-b) = 198$; since 198 has only one factor of 2, if $a+b$ is even then $a-b$ is odd, or if $a+b$ is odd then $a-b$ is even; in either case if you added the equations a would not be an integer and so 198 cannot be written as a difference of squares.

c) Based on part **b**, set S cannot contain positive integers of the form $2(2k+1)$ where k is any non-negative integer. Furthermore these are the only positive integers not in set S . If the integer is odd or divisible by 4, it is always possible to express it as a difference of two squares of non-negative integers. In the case of the integer k being odd, then $a+b=k$ and $a-b=1$, which results in $a = \frac{k+1}{2}$ and $b = \frac{k-1}{2}$. In the case that $k = 4l$, where l is a positive integer, then $a+b = 2l$ and $a-b = 2$, which results in $a = l+1$ and $b = l-1$

d) Consider 3 types of integers in set S : (i) 2^r , where $r > 1$, (ii) $p_1^{s_1} \cdot p_2^{s_2} \dots$, where the p_i 's are odd primes, and (iii) $2^r \cdot p_1^{s_1} \cdot p_2^{s_2} \dots$, where $r > 1$ and the p_i 's are odd primes.

Let $f(n)$ be the number of ways a number in set S can be written as a difference of squares of positive integers.

(i) 2^r , where $r > 1$: In this case you can "split" 2^r into $2^t \cdot 2^{r-t}$ where $0 < t \leq r-t \Rightarrow$

$0 < t \leq \frac{r}{2}$. If you let $a+b = 2^{r-t}$ and $a-b = 2^t$, you will get a difference of squares of non-

negative integers for each pair of values of $r-t$ and t . If $t = \left\lfloor \frac{r}{2} \right\rfloor$ where $\lfloor x \rfloor$ is the greatest

integer less than or equal to x , this will give you the number of cases. $\therefore f(2^r) = \left\lfloor \frac{r}{2} \right\rfloor$.

(ii) $p_1^{s_1} \cdot p_2^{s_2} \dots$: In this case n has $(s_1+1)(s_2+1) \dots$ factors. If n is not a perfect square, there will be $\frac{(s_1+1)(s_2+1) \dots}{2}$ pairs of factors, each pair producing a difference of two squares of non-

negative integers. If n is a perfect square there will be $\frac{1+(s_1+1)(s_2+1) \dots}{2}$ pair of factors

$\left\lfloor \frac{1+(s_1+1)(s_2+1) \dots}{2} \right\rfloor$ will handle both situations. $\therefore f(p_1^{s_1} \cdot p_2^{s_2} \dots) = \left\lfloor \frac{1+(s_1+1)(s_2+1) \dots}{2} \right\rfloor$.

(iii) $2^r \cdot p_1^{s_1} \cdot p_2^{s_2} \dots$: In this case n has $(r+1)(s_1+1)(s_2+1) \dots$ factors. You want to eliminate all pairs of factor in which 2^r is contained in one of them. From case (ii) there are

$(s_1+1)(s_2+1) \dots$ factors that are odd. The factor 2^r could be multiplied by any one of these, which would then be paired with a second odd factor so that both multiply to n . Therefore you

want to eliminate $(s_1 + 1)(s_2 + 1) \dots$ pairs of factors. The total number of pairs can be arrived at in a similar way to case (ii).

$$\therefore f(2^r \cdot p_1^{s_1} \cdot p_2^{s_2} \dots) = \left[\frac{1 + (r+1)(s_1+1)(s_2+1)\dots}{2} \right] - (s_1+1)(s_2+1)\dots$$

A second, more general, formula for $f(n)$ is presented here without explanation:

Let $n = 2^r \cdot p_1^{s_1} \cdot p_2^{s_2} \dots$ with $r, s_1, s_2 \dots$ non-negative integers and $p_1, p_2 \dots$ odd primes, then

$$f(n) = \begin{cases} \left[\frac{r}{2} \right] (s_1+1)(s_2+1)\dots & \text{if } r \text{ is odd} \\ \frac{(r-1)(s_1+1)(s_2+1)\dots}{2} & \text{if } r \text{ and } (s_1+1)(s_2+1)\dots \text{ are both even} \\ \left[\frac{(r-1)(s_1+1)(s_2+1)\dots}{2} \right] + 1 & \text{if } r \text{ is even and } (s_1+1)(s_2+1)\dots \text{ is odd} \end{cases}$$

6. I'm Lost and You R2

- a) Beginning at the origin, $(0,0)$, R2 proceeds to the point $(10,0)$ and then changes direction by 45° . For the next 5 miles his x and y coordinates change by the amounts $(5\cos 45^\circ, 5\sin 45^\circ)$ making his new location $(10 + 5\cos 45^\circ, 0 + 5\sin 45^\circ)$. On the next leg of his trip the change in the x and y coordinates is $(2.5\cos 90^\circ, 2.5\sin 90^\circ)$ bringing him to the point $(10 + 5\cos 45^\circ + 2.5\cos 90^\circ, 0 + 5\sin 45^\circ + 2.5\sin 90^\circ)$, and so on. You want to find the limiting values for the x and y coordinates of the position. You should treat each coordinate separately.

For the x coordinate, the terminal value =

$$10 + 5\cos 45^\circ + 2.5\cos 90^\circ + 1.25\cos 135^\circ + 0.625\cos 180^\circ + 0.3125\cos 225^\circ + 0.15625\cos 270^\circ +$$

... This series can be split into 4 infinite geometric series $\left(S = \frac{a}{1-r} \right)$ with the first series

consisting of the 1st term, 5th term, 9th term, ..., the second series being the 2nd term, 6th term,

10th term, ... etc. For each of these series $r = -\frac{1}{16}$. Note the 3rd series of terms are all 0.

$$x_{\text{terminal}} = \frac{10}{1 - \left(-\frac{1}{16}\right)} + \frac{5\cos 45^\circ}{1 - \left(-\frac{1}{16}\right)} + 0 + \frac{1.25\cos 135^\circ}{1 - \left(-\frac{1}{16}\right)} = \frac{10 + 5\cos 45^\circ + 1.25\cos 135^\circ}{\frac{17}{16}} =$$

$$\frac{160 + 30\sqrt{2}}{17} \rightarrow X \approx 11.907 \text{ miles (Storing the value in } X \text{ to use in part b.)}$$

For the y coordinate, the terminal value =

$$0 + 5\sin 45^\circ + 2.5\sin 90^\circ + 1.25\sin 135^\circ + 0.625\sin 180^\circ + 0.3125\sin 225^\circ + 0.15625\sin 270^\circ + \dots$$

Note the 1st series of terms are all 0.

$$y_{\text{terminal}} = 0 + \frac{5\sin 45^\circ}{1 - \left(-\frac{1}{16}\right)} + \frac{2.5\sin 90^\circ}{1 - \left(-\frac{1}{16}\right)} + \frac{1.25\sin 135^\circ}{1 - \left(-\frac{1}{16}\right)} = \frac{5\sin 45^\circ + 2.5\sin 90^\circ + 1.25\sin 135^\circ}{\frac{17}{16}} =$$

$$\frac{40 + 50\sqrt{2}}{17} \rightarrow Y \approx 6.512 \text{ miles (Storing the value in } Y \text{ to use in part b.)}$$

- b) The total distance that R2 travels is $10 + 5 + 2.5 + 1.25 + \dots = \frac{10}{1-0.5} = 20$ miles. At a constant rate of 4mph, R2 should arrive at the terminal point (X, Y) in 5 hours. Since R2 left 2 hours ago he has 3 more hours before he reaches the terminal point. Your distance from the origin to the terminal point = $\sqrt{X^2 + Y^2}$. Since you need to reach the point the same time as R2, your rate should be $\frac{\sqrt{X^2 + Y^2}}{3}$ mph ≈ 4.524 mph.

- c) Note that in the complex plane $10e^{0i} = 10 + 0i$, $5e^{45i} = 5\cos 45^\circ + 5i\sin 45^\circ$, etc.

To find the terminal point in the complex plane with the direction altered by q degrees each time would amount to adding the infinite series: $10e^{0i} + 5e^{qi} + 2.5e^{2qi} + 1.25e^{3qi} + \dots$. This is an infinite geometric series with $r = 0.5e^{qi}$. Using the same formula as part a:

Terminal point in the complex plane =

$$\frac{10}{1 - .5e^{qi}} = \frac{10(1 - .5e^{-qi})}{(1 - .5e^{qi})(1 - .5e^{-qi})} = \frac{10 - 5e^{-qi}}{1.25 - .5(e^{qi} - e^{-qi})} = \frac{10 - 5(\cos q - i \sin q)}{1.25 - \cos q} =$$

$$\frac{10 - 5\cos q}{1.25 - \cos q} + \frac{\sin q}{1.25 - \cos q}i.$$

The coordinates of the terminal point in the Cartesian plane are $\left(\frac{10 - 5\cos q}{1.25 - \cos q}, \frac{\sin q}{1.25 - \cos q} \right)$

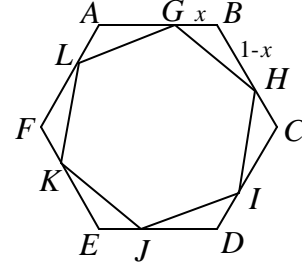
7. What the Hex?

- a) Let
- s
- = side of the inner hexagon. Use the Law of Cosines:

$$s^2 = x^2 + (1-x)^2 - 2x(1-x)\cos 120^\circ \Rightarrow$$

$$s^2 = x^2 + x^2 - 2x + 1 - 2x(1-x)\left(-\frac{1}{2}\right) \Rightarrow$$

$$s^2 = x^2 + x^2 - 2x + 1 + x - x^2 \Rightarrow s^2 = x^2 - x + 1$$

Use the formula for area of a regular hexagon in terms of its side s :

$$\text{area} = \frac{3s^2\sqrt{3}}{2} = \frac{3(x^2 - x + 1)\sqrt{3}}{2}$$

- b) If the area
- $< \frac{31\sqrt{3}}{24}$
- , then
- $\frac{3(x^2 - x + 1)\sqrt{3}}{2} < \frac{31\sqrt{3}}{24} \Rightarrow x^2 - x + 1 < \frac{31}{36} \Rightarrow x^2 - x + \frac{1}{4} < \frac{31}{36} - 1 + \frac{1}{4} \Rightarrow$

$$\left(x - \frac{1}{2}\right)^2 < \frac{1}{9} \Rightarrow \left|x - \frac{1}{2}\right| < \frac{1}{3} \Rightarrow -\frac{1}{3} < x - \frac{1}{2} < \frac{1}{3} \Rightarrow \frac{1}{6} < x < \frac{5}{6}. \text{ The probability that } x \text{ is selected}$$

$$\text{from this interval} = \frac{2}{3}$$

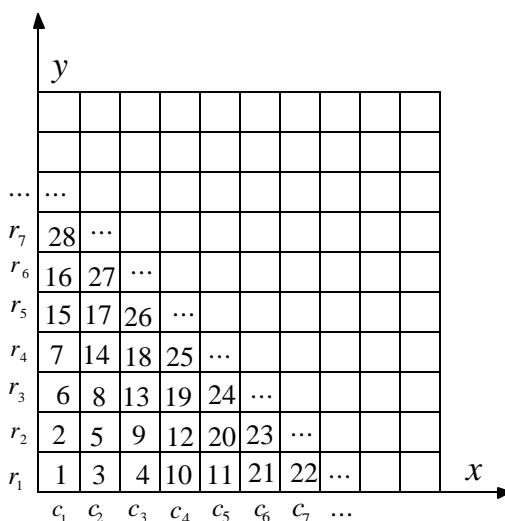
- c) If probability that the area of the hexagon is greater than
- a
- square units
- $= \frac{1}{2} \Rightarrow$

$$\frac{3(x^2 - x + 1)\sqrt{3}}{2} > a \Rightarrow x^2 - x + 1 > \frac{2a}{3\sqrt{3}} \Rightarrow x^2 - x + \frac{1}{4} > \frac{2\sqrt{3}a}{9} - 1 + \frac{1}{4} \Rightarrow$$

$$\left(x - \frac{1}{2}\right)^2 > \frac{2\sqrt{3}a}{9} - \frac{3}{4} \Rightarrow \left|x - \frac{1}{2}\right| > \frac{\sqrt{8\sqrt{3}a - 27}}{6}.$$

This inequality should represent intervals which are $\frac{1}{2}$ of the interval $[0,1]$. Therefore

$$\frac{\sqrt{8\sqrt{3}a - 27}}{6} = \frac{1}{4} \Rightarrow \frac{8\sqrt{3}a - 27}{36} = \frac{1}{16} \Rightarrow 8\sqrt{3}a = \frac{117}{4} \Rightarrow a = \frac{117\sqrt{3}}{96} = \frac{39\sqrt{3}}{32}$$

8. Lower Left Corner

- a) Consider the following partition of the natural numbers: $\{1\}, \{2,3\}, \{4,5,6\}, \{7,8,9,10\}$, etc.

These correspond to the cell numbers of the “diagonals” in the lattice as you move away from the origin. Notice that, as such, the sum of the coordinates of the lower left corner point of each of the cells along any diagonal is constant. For example the third diagonal consists of cells numbered 4, 5, and 6. The coordinates of the lower left corner of these cells are respectively $(2,0), (1,1)$, and $(0,2)$. Each ordered pair’s coordinates add to 2. In general the n th diagonal will have lower left corner points each of whose coordinates add to $n - 1$.

Next note that the largest number on each of the diagonals is, naturally, a triangular number.

This means they have the form $\frac{k(k+1)}{2}$, $k = 1, 2, 3, \dots$. Cell 100 falls between 91 and 105, the 13th and 14th triangular numbers. Therefore cell 100 is on the 14th diagonal. The sum of the coordinates of the lower left corner of cell 100 must add to 13. Cell 100 is the 9th one in the list 92, 93, ..., 105. Since even numbered diagonals have cell numbers that increase with increasing x coordinates, beginning with 0, the x coordinate of the cell 100 must be 8. Therefore the lower left hand coordinates of cell 100 are $(8, 5)$.

- b) Suppose we set $N = \frac{k(k+1)}{2}$ and solve for k . This gives the positive root from the quadratic formula: $k = \frac{-1 + \sqrt{1 + 8N}}{2}$. Clearly this will be an integer only if N is a triangular number.

However from the value of k , you can determine between which two triangular numbers N falls.

In particular, N lies between $[k]$ th and the $([k]+1)$ st triangular numbers, where $[k]$ is the greatest integer less than or equal to k . Based on the argument in part **a**, the sum of the coordinates of the lower left corner point of cell N must be $[k]$. As with part **a**, the next consideration is the parity of $[k]$ and the relative position of cell N on the $[k]$ th diagonal. In general, the number N is the $\left(N - \frac{[k]([k]+1)}{2}\right)$ th one on the diagonal. If $[k]$ is odd, then, as in part **a**, the x coordinates increase with the cell numbers and the lower left corner's x coordinate would be $N - 1 - \frac{[k]([k]+1)}{2}$. On the other hand, if $[k]$ is even, then the situation is reversed

and the x coordinate of the lower left corner of the cell would be

$$[k] - \left(N - 1 - \frac{[k]([k]+1)}{2}\right) = 1 - N + \frac{2[k]}{2} + \frac{[k]([k]+1)}{2} = (1 - N) + \frac{[k]([k]+3)}{2}.$$

Putting all of this together: Given N , determine $k = \frac{-1 + \sqrt{1 + 8N}}{2}$;

if k is odd, the coordinates of the lower left corner of cell $N =$

$$\left((N-1) - \frac{[k]([k]+1)}{2}, [k] - \left((N-1) - \frac{[k]([k]+1)}{2} \right) \right)$$

if k is even, the coordinates of the lower left corner of cell $N =$

$$\left((1-N) + \frac{[k]([k]+3)}{2}, [k] - \left((1-N) + \frac{[k]([k]+3)}{2} \right) \right)$$

c) In the first quadrant the only integer coordinate points through which the circle

$\{(x, y) | x^2 + y^2 = 442\}$ passes are $(1, 21)$, $(21, 1)$, $(9, 19)$, and $(19, 9)$. The first two points have a

coordinate sum equal to 22 and therefore $[k] = 22$ for the number N . Since 22 is even, the x

coordinate equals $1 - N + \frac{22 \cdot 25}{2} = 276 - N$, using the formula in part **b**. Therefore

$N = 276 - 1 = 275$ for the first point, and $N = 276 - 21 = 255$ for the second.

Similarly, for the second pair of points $[k] = 28$ and the x coordinate equals

$1 - N + \frac{28 \cdot 31}{2} = 435 - N$. Thus, the cell numbers corresponding to these would be

$N = 435 - 9 = 426$ and $N = 435 - 19 = 416$. The cell numbers are 255, 275, 416, 426.