Level I MAML 2000

1. If n is an integer, then $\frac{n(n+1)}{2}$ is always

   A) positive B) an integer C) even D) greater than n E) odd

2. The absolute value of the difference between the square of any positive integer and the square of the integer formed by reversing its digits is always divisible by which whole number(s) greater than 1?

   A) 2 only B) 9 only C) 11 only D) 2 and 9 only E) 9 and 11 only

3. Choose any 3×3 square of nine dates from a month in a calendar. The absolute value of the difference of the products of the dates in the opposite corners is

   A) 0 B) 7 C) 14 D) 28 E) 35

4. When 60 minutes elapses on a correct clock, 62 minutes registers on clock *F* (fast), and only 56 minutes registers on clock *S* (slow). If later in the day, clock *F* reads eight o'clock and clock *S* reads seven o'clock, what was the correct time when the two clocks were originally set?

   A) 1:00 B) 7:20 C) 7:40 D) 9:20 E) 9:40

5. If $M > N > 0$, the percent that *M* is greater than *N* is

   A) $\frac{100(M-N)}{M}$ B) $\frac{100(M-N)}{N}$ C) $\frac{M-N}{100N}$ D) $\frac{M-N}{100M}$ E) $\frac{100M}{N}$

6. Point *P* lies on line $\ell, \{(x,y) | 3x+5y=15\}$. If *P* is also equidistant from the coordinate axes, *P* could be located in which of the four quadrants?

   A) None B) I only C) II only D) I or II only E) Any

7. A bookshelf is 36 inches long and contains *n* books each *w* inches thick. If each book were half an inch thinner, the shelf would hold 6 more of the same book. What is $\frac{n}{w}$?

   A) 1 B) $\frac{9}{4}$ C) 4 D) 9 E) 16

8. Let $P(x)$ be the fourth degree polynomial whose first two terms are $8x^4 + 4x^3$. If two of the roots are $\frac{1}{2}$ and $\frac{1}{2} + i\ (i = \sqrt{-1})$ and all the coefficients are real, the product of all the roots is

   A) -10 B) $-\frac{5}{4}$ C) $-\frac{1}{2}$ D) $\frac{5}{4}$ E) 10

9. If q and $(2q+50°)$ are acute angles in degrees, express q in radians if $\sin\left(\frac{1}{2}q\right)=\cos(2q+50°)$.

A) $\frac{p}{45}$ B) $\frac{p}{30}$ C) $\frac{2p}{45}$ D) $\frac{p}{15}$ E) $\frac{4p}{45}$

10. If $n=3^x+3^x+3^x$, then $n^2=$

A) 3^{3x} B) 9^{2x} C) 9^{x+1} D) 27^{2x} E) 27^{6x}

11. Given ΔABC with right angle C and a second right triangle ABD such that both triangles share the same hypotenuse. If $BC = 1$, $AC = b$ and $AD = 2$, then $BD =$

A) $\sqrt{b^2+1}$ B) $\sqrt{b^2-3}$ C) $\sqrt{b^2-1}$ D) b^2+5 E) $\sqrt{b^2+3}$

12. If $\tan\left(x+\frac{p}{4}\right)=a$, then $\sec^2 x=$

A) $1+a^2$ B) $\frac{4a}{(a+1)^2}$ C) $\frac{2(a^2+1)}{(a+1)^2}$ D) $\left(\frac{a-1}{a+1}\right)^2$ E) $\frac{a^2}{(a+1)^2}$

13. Starting at the point $P(x,y)$ on the coordinate plane, a pin can either be moved to point $A(x+1,y)$ or to point $B(x,y+1)$. If the pin starts at $(0,0)$ and is moved to $(4,4)$, what is the probability that it passed through $(2,2)$?

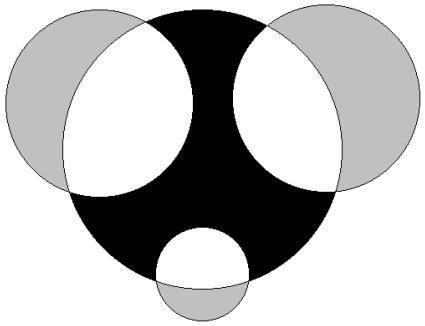
A) $\frac{6}{35}$ B) $\frac{8}{35}$ C) $\frac{12}{35}$ D) $\frac{16}{35}$ E) $\frac{18}{35}$

14. Let a be a positive real number. Find the area of the region of the solution set of the following inequality: $|x-1|+|y-2|\le a$

A) $\frac{1}{2}a^2$ B) a^2 C) $2(a+3)^2$ D) $2a^2$ E) $\frac{1}{2}\left(\frac{a-1}{a-2}\right)^2$

15. In the diagram shown below, two of the three partially shown, lightly shaded circles have diameters of length 4 and the third one has diameter of length 2. What must be the length of the diameter of the partially shown, darkly shaded circle to make the total of the lightly shaded areas equal to the darkly shaded area?

A) 4 B) 5 C) 6 D) 7 E) 8



16. The perimeter and area of a triangle are 20 inches and 12 square inches, respectively. What is the area of the inscribed circle in square inches?

A) 0.64π B) π C) 1.21π D) 1.44π E) 2.25π

17. What is the sum of all solutions for x, where $0 \le x < 2p$ and $\sin(5x) + \sin(x) = \sin(3x)$?

A) $\frac{9}{2}p$ B) $\frac{14}{3}p$ C) $8p$ D) $9p$ E) $11p$

18. Find the sum of all the possible real values of x which makes the following statement true.
$\log_6 54 + \log_x 16 = \log_{\sqrt{2}} x - \log_{36}\left(4/9\right)$

A) 2 B) $\frac{9}{4}$ C) 4 D) $\frac{9}{2}$ E) 8

19. The circumference of a circle circumscribing an equilateral triangle is 24π units. Find the number of square units in the area of the circle inscribed in the same triangle.

A) 12π B) 16π C) 36π D) 64π E) 324π

20. Mindy initially offers to sell Drew a tennis racket for p dollars and Drew makes a counter offer of q dollars. From this point they continue haggling, each making an offer equaling the average of the previous two amounts. On what amount will they settle?

A) $\frac{p+q}{3}$ B) $\frac{2p+q}{3}$ C) $\frac{p+2q}{8}$ D) $\frac{2p+3q}{5}$ E) $\frac{p+2q}{3}$

21. $S=\{s_1, s_2, s_3, ...\}$ and $T=\{t_1, t_2, t_3, ...\}$ are arithmetic sequences such that for all possible values of s_1, $s_1 = t_1 \neq 0$, $s_2 = 2t_2$, and $\sum_{i=1}^{10} s_i = \sum_{j=1}^{15} t_j$. Find $\frac{s_2 - s_1}{t_2 - t_1}$.

A) $\frac{8}{3}$ B) $\frac{8s_1}{3}$ C) $\frac{3}{2}$ D) $\frac{19}{8}$ E) 2

22. If r is a complex number and $\left(r + \frac{1}{r}\right)^2 = 3$, then $r^3 + \frac{1}{r^3} =$

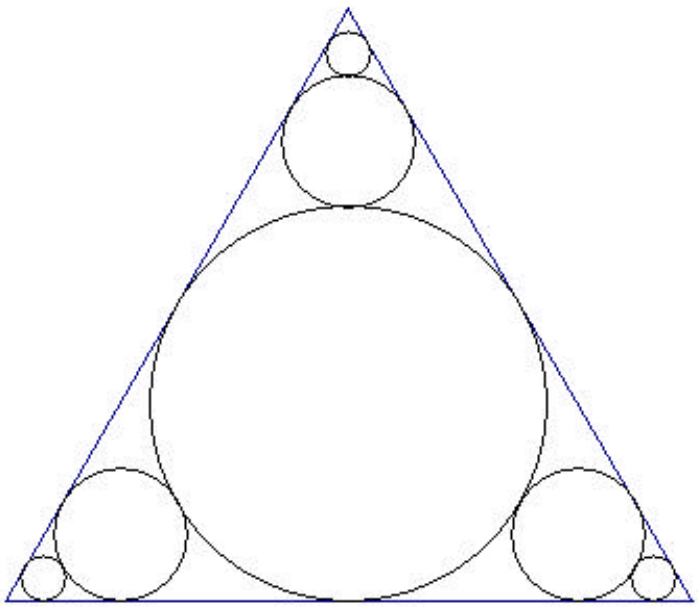
A) 0 B) 2 C) 4 D) 6 E) 8

23. What is the sum of all the positive 3-digit integers that can be formed from the digits 2, 3, 5, 6, and 7? Note that the same digit can appear more than once.

A) 7659 B) 12765 C) 15318 D) 30636 E) 63825

24. A circle is inscribed in an equilateral triangle with a side of length 2. Three circles are drawn externally tangent to this circle and internally tangent to two sides of the triangle. Three more circles are drawn externally tangent to these circles and internally tangent to two sides of the triangle as shown in the diagram below. If this process is continued forever, find the sum of the areas of all the circles.

A) $\frac{p}{24}$ B) $\frac{13p}{24}$ C) $\frac{p}{3}$ D) $\frac{p}{8}$ E) $\frac{11p}{24}$



25. Suppose the height of an ice cream cone is three times the diameter of the cone. Two spherical scoops of ice cream with the same diameter as the cone are packed into the cone, filling it completely. After the cone is completely filled, there is some ice cream bulging above the top of the cone. What part of the scoops remain outside the cone? (Assume no dripping and no compression from the packing.)

A) 25% B) 33% C) 50% D) 67% E) 75%

26. Calculate the area of ΔABC with the following vertices: $A(1,3,0)$, $B(0,2,5)$, and $C(-1,0,2)$

A) $3\sqrt{26}$ B) $\frac{3}{2}\sqrt{26}$ C) $\frac{1}{2}\sqrt{6}$ D) $\sqrt{6}$ E) $\frac{3}{2}\sqrt{6}$

27. Using principal values for $\tan^{-1}$ and $\cos^{-1}$, the expression $\cos\left(\tan^{-1}\left(\sin\left(\cos^{-1}(x)\right)\right)\right)$ is equivalent to

A) x B) $2-x^2$ C) $\sqrt{2-x^2}$ D) $\sqrt{\frac{1-x^2}{2-x^2}}$ E) $\frac{\sqrt{2-x^2}}{2-x^2}$

28. If two points are selected at random from the interval $[0,1]$, what is the probability that the distance between them is less than one-fourth?

A) $\frac{3}{8}$ B) $\frac{7}{16}$ C) $\frac{1}{2}$ D) $\frac{4}{7}$ E) $\frac{3}{4}$

29. Shown below is part of a *Binary Pascal Triangle*. Each row has a 1 on each end, but each interior value is found by taking the absolute value of the difference of the two numbers immediately above it. If the number of ones and zeros in a row n is $g(n)$ and $f(n)$ respectively, what is the value of $g(96)-f(96)$?

Row 1						1					
Row 2					1		1				
Row 3				1		0		1			
Row 4			1		1		1		1		
Row 5		1		0		0		0		1	
Row 6	1		1		0		0		1		1

A) 8 B) 24 C) 32 D) 84 E) 92

30. Determine the sum of the radii of all the circles which pass through the points $A(1,9)$ and $B(8,8)$ and are tangent to the x-axis.

A) 700 B) 750 C) 800 D) 850 E) 900

Solutions to Level I MAML 2000

1.	B	16.	D
2.	E	17.	D
3.	D	18.	D
4.	E	19.	C
5.	B	20.	E
6.	D	21.	D
7.	D	22.	A
8.	B	23.	E
9.	E	24.	E
10.	C	25.	A
11.	B	26.	B
12.	C	27.	E
13.	E	28.	B
14.	D	29.	C
15.	C	30.	D