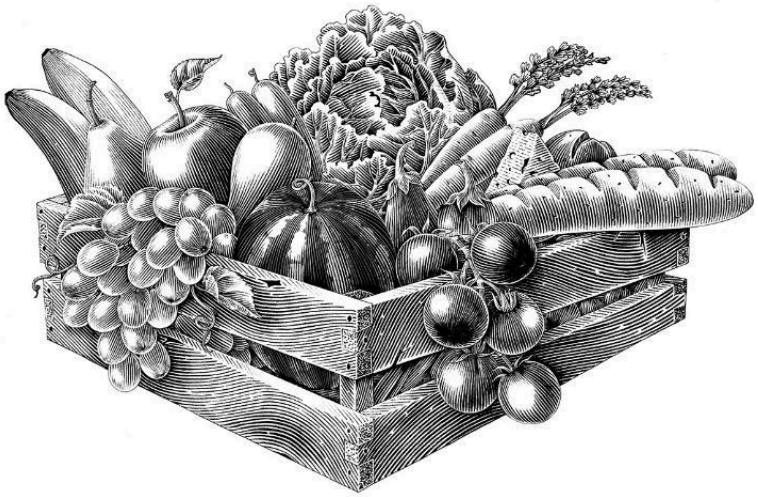
IMLEM Meet #1
October, 2026

# Intermediate Mathematics League of Eastern Massachusetts



**CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:**

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

**NEW FOR 2026-2027: The first five categories in ALL meets will include four problems. Questions #1, 2, 3, and 4 will be worth 2, 2, 1, and 1 points, respectively.**
**The success of this innovation will be evaluated in the spring, after all five meets have been completed.**

**Category 1**
**Mystery**
**Meet #1 - October, 2026**

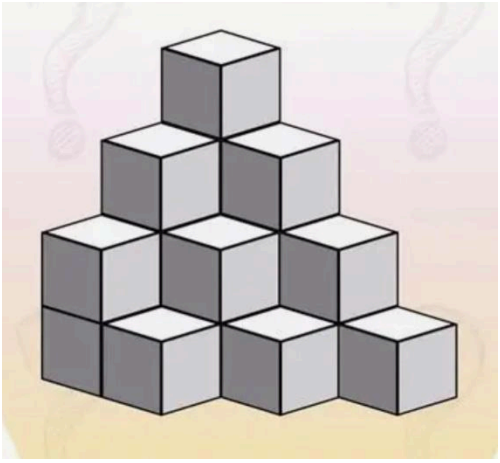


**1) (worth 2 points)** **A gymnasium has 450 spectators. If 175 leave and then 82 enter, then how many spectators are there now?**

**2) (worth 2 points)** **Fred is 49 years old and Barney is 19 years old. In how many years will Fred be twice as old as Barney?**

**3) (worth 1 point)** **Rearrange the four digits of the whole number 8354 to create a new 4-digit whole number that is as close as possible to the number 5000. It is possible that your answer is either less than or greater than 5000, as long as it is as close to 5000 as possible.**

**4) (worth 1 point)** **Some cubes of equal size are arranged on a tabletop, as shown. How many cubes are there? There are no "holes" in the structure.**



**Answers**

**1)** _____

**2)** _____

**3)** _____

**4)** _____

**Solutions to Category 1**
**Mystery**
**Meet #1 - October, 2026**

**1) (worth 2 points) 450 – 175 + 82 = 357.**

**2) (worth 2 points) Let X = the number of years required for Fred to be twice as old as Barney.**

$49 + X = 2(19 + X)$

$49 + X = 38 + 2X$

$11 = X$

**In 11 years, Fred will be 60 years old and Barney will be 30 years old and Fred will be twice as old as Barney.**

**3) (worth 1 point) The number less than 5000 that is closest to 5000 is 4853, or 147 less than 5000. The number greater than 5000 that is closest to 5000 is 5348, or 348 greater than 5000. So, 4853 is closest to 5000.**

**4) (worth 1 point) The bottom layer has 9 cubes. The layer above it has 6. The layer above that has 3 and the top layer has 1. Total: 9 + 6 + 3 + 1 = 19 cubes in all.**

**ANSWERS**

**1) 357**

**2) 11**

**3) 4853**

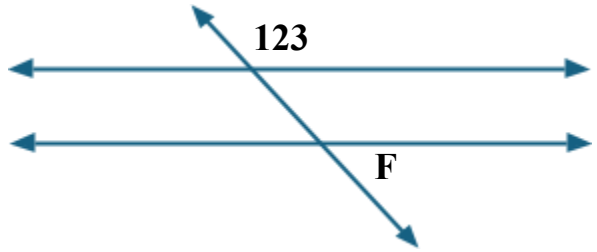
**4) 19**



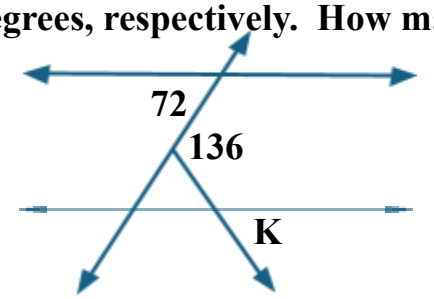
**Category 2**
**Geometry**
**Meet #1 - October, 2026**

**1) (worth 2 points)** Two angles are complementary if the sum of their angle measures is 90 degrees. Two angles are supplementary if the sum of their angle measures is 180 degrees. How many degrees are in the supplement of the complement of a 37 degree angle?

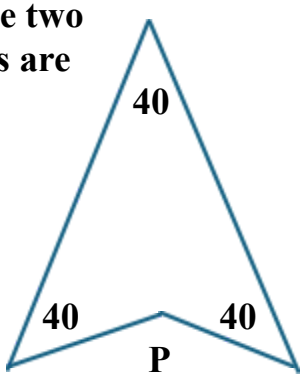
**2) (worth 2 points)** The two horizontal lines are parallel. One angle measure is given as 123 degrees. How many degrees are in the measure of angle F ?




**3) (worth 1 point)** The two horizontal lines are parallel. Two angles are given as 72 and 136 degrees, respectively. How many degrees are in angle K ?




**4) (worth 1 point)** The quadrilateral below has three angle measures, as marked. How many degrees are in the angle marked P ? The two longest segments are congruent.




**Answers**

1) _____

2) _____

3) _____

4) _____

**Solutions to Category 2**
**Geometry**
**Meet #1 - October, 2026**

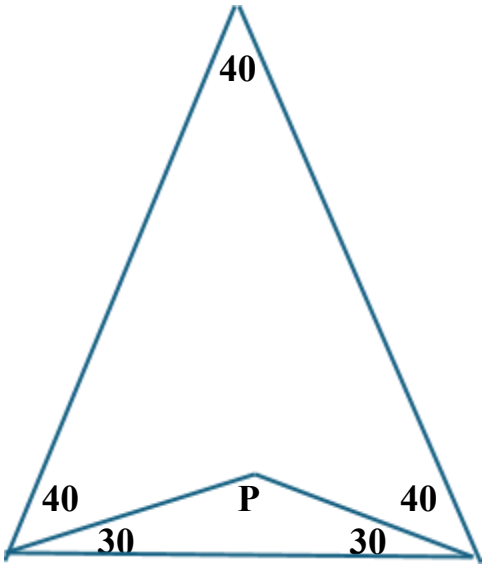
| Answers | |
|---|---|
| 1) | 127 |
| 2) | 57 |
| 3) | 64 |
| 4) | 120 |

**1) (worth 2 points)** The complement of a 37 degree angle is 90 – 37, or 53 degrees. The supplement of it is 180 – 53, or 127 degrees.

**2) (worth 2 points)** The external angles on the same side of the transversal are supplementary. F = 180 – 123 = 57 degrees.

**3) (worth 1 point)** Focus on the triangle. The left base angle is 72 degrees, as alternate interior angles are congruent. The angle adjacent to the 136-degree angle is its supplement, or 44 degrees. The measures of two of the three angles of the triangle are now known. The remaining angle, the right base angle, is 180- - (72 + 44), or 180 – 116, or 64 degrees.

**4) (worth 1 point)** Draw a line segment at the bottom of the shape, creating a large triangle and a small isosceles triangle. Each base angle measures 70 degrees. Subtract 40 from 70 to get the measure of each base angle of the small isosceles triangle, or 30 degrees. Two angles of the small isosceles triangle are now known, so P = 180 – (30 + 30), or 180 – 60, or 120 degrees.



**Category 3**
**Number Theory**
**Meet #1 - October, 2026**



**1) (worth 2 points)** **The factors of a positive integer, A, consists of the set of positive integers by which A is divisible. For example, when A is 10, then its set of four factors is { 1, 2, 5, 10 }. What is the sum of the factors of 18 ?**

**2) (worth 2 points)** **A prime number, P, has exactly two distinct factors, namely, the number 1 and the number P. What is the sum of all prime numbers between 30 and 40 ?**

**3) (worth 1 point)** **What is the largest prime factor of the product of all the even numbers from 2 through 200, inclusive?**

**4) (worth 1 point)** **For how many positive whole number values of E will the expression below yield a positive whole number?**

$$\frac{72}{E}$$

**Answers**

**1)** _____

**2)** _____

**3)** _____

**4)** _____

**Solutions to Category 3**
**Number Theory**
**Meet #1 - October, 2026**

**1) (worth 2 points)** The factors of 18 are 1, 2, 3, 6, 9, 18. Their sum is 1+2+3+6+9+18 = 39.

**2) (worth 2 points)** The only prime numbers between 30 and 40 are 31 and 37. Their sum is 31 + 37 = 68.

**3) (worth 1 point)** The largest prime factor of such product would have to be 97 which is a factor of 194. The other even numbers greater than 194 have the following prime factorizations: 196 = 2 x 2 x 7 x 7, 198 = 2 x 3 x 3 x 11. The even numbers that are less than 194 would not have a prime factor greater than 97.

**4) (worth 1 point)** The answer consists of all of the positive factors of 72, which are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72, or a total of 12 factors.

**Answers**

1) 39
2) 68
3) 97
4) 12

**Category 4**
**Arithmetic**
**Meet #1 - October, 2026**



**1) (worth 2 points)** What is the value of the expression below? Each "x" is a symbol of multiplication.

**2 x 3 x 5 x 7 + 1 x 4 x 6 x 8 x 9**

**2) (worth 2 points)** If $\frac{7 \cdot 8}{3} = N$, then round N to the nearest tenth.

**3) (worth 1 point)** Consider the set W = { 3, 8, 5, 3, 6, 5, 14, 3, 7 }.
Let A = the median of W
and B = the mode of W
and C = the mean of W.
What is the value of the product ABC ?

**4) (worth 1 point)** Farmer Dell grows pumpkins and Hubbard squash. Dell harvested seven pumpkins today that had an average weight of 38 pounds and four Hubbard squash that had an average weight of 53 pounds. How many pounds was the average weight of all the vegetables? Round your answer to the nearest whole number of pounds.

**Answers**

1) _____

2) _____

3) _____

4) _____

**Solutions to Category 4**
**Arithmetic**
**Meet #1 - October, 2026**

| Answers |
|---|
| 1) 1938 |
| 2) 18.7 |
| 3) 90 |
| 4) 43 |

**1) (worth 2 points)** $2 \times 3 \times 5 \times 7 + 1 \times 4 \times 6 \times 8 \times 9$
$= 210 + 1728$
$= 1938$

**2)** $56 / 3 = 18.66666\ldots$, or $18.7$ when rounded to the nearest tenth.

**3)** Arrange the numbers in ascending (or descending) order to help locate the median, or middle number:
3 3 3 5 5 6 7 8 14
A = the median = the middle number = 5.
B = the mode = the most frequent number = 3.
C = the mean = the average = the sum of the numbers divided by the number of numbers = 54 / 9 = 6.
The product ABC = 5 x 3 x 6 = 90.

**4)** The average weight = the total weight divided by the number of vegetables
$= [ (7)(38) + (4)(53) ] / (7 + 4)$
$= [ 266 + 212 ] / 11$
$= 478 / 11$
$= 43.454545\ldots$
$= 43$ when rounded to the nearest whole number.
So, the average weight of the 1 vegetables is about 43 pounds.



**Category 5**
**Algebra**
**Meet #1 - October, 2026**

**1) (worth 2 points) If 7T + 13 = 69, then what is the value of 5T – 21 ?**

**2) (worth 2 points) Evaluate the expression below if**  **= 17 and**  **= 7.**

**3**  **+ 5**  **-**  **+ 2**  **+ 4**  **- 6** 

**3) (worth 1 point) For what value of G is the equation below true?**

$$\frac{5(G+2)}{3} = G + 10$$

**4) (worth 1 point) What value of X makes the following equation true?**

**4 ( 8X + 5 ) – 2 ( 6X + 1 ) – 3 ( 7X – 9) = 10 ( 2X + 5 ) + 58**

**Answers**

**1)** _____

**2)** _____

**3)** _____

**4)** _____

**Solutions to Category 5**
**Algebra**
**Meet #1 - October, 2026**

**1) (worth 2 points)** $7T + 13 = 69$, so $7T = 56$
and $T = 8$
So, $5T - 21 = 5(8) - 21 = 19$.

**2)** Replacing the pumpkin and ghost with C and D:
$3D + 5C - D + 2C + 4D - 6C$
$= 6D + C$
$= 6(7) + 17$
$= 42 + 17$
$= 59$

**3)** Cross multiply: $5(G + 2) = 3(G + 10)$
$5G + 10 = 3G + 30$
$2G = 20$
$G = 10$.

**4)** $4(8X + 5) - 2(6X + 1) - 3(7X - 9) = 10(2X + 5) + 58$
$32X + 20 - 12X - 2 - 21X + 27 = 20X + 50 + 58$
$-X + 45 = 20X + 108$
$-21X = 63$
$X = -3$

| Answers | |
|---|---|
| 1) | 19 |
| 2) | 59 |
| 3) | 10 |
| 4) | - 3 |

**Category 6**
**Team Round**
**Meet #1 - October, 2026**

***Each of the following NINE problems is worth <u>four</u> points.***

**1) What is the remainder when the following factorial sum is divided by 5? The expression is the sum of the factorials of the first ten positive integers.**

**1! + 2! + 3! + 4! + . . . + 10!**

**2) A taxi charges $100 as base fare and $30 per kilometer. If the total bill is $520, then how many kilometers was the trip?**

**3) The sum of three different prime numbers is 40. What is their product?**

**4) If the sum of the measures of the interior angles of a convex polygon is 1620 degrees, then how many sides does the polygon have?**

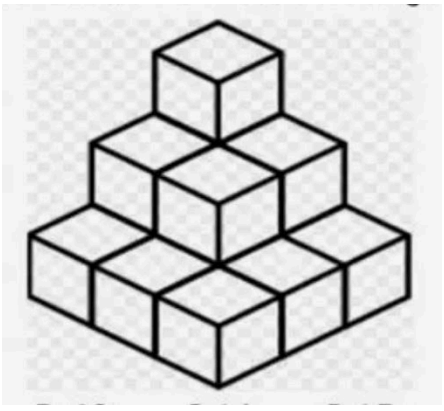
**5) If the degree measure of an angle's supplement is three times that of its complement, then how many degrees are in the angle's measure?**

**6) The five-digit number 4056N is divisible by nine. What is the remainder when that five-digit number is divided by 7 ?**

**7) Let R be the largest 5-digit number whose digits have a product of 280. What is the sum of those five digits of R?**

**8) Trav shared a secret with seven friends. Each of the seven friends shared the secret with nine other friends. How many people now know the secret if no one was shared more than once?**

**9) The configuration below consists of equivalent cubes. How many cubes are there? Assume that there are no "holes" in the interior of the configuration.**



**<u>ANSWERS</u>**

**1) _____**

**2) _____**

**3) _____**

**4) _____**

**5) _____**

**6) _____**

**7) _____**

**8) _____**

**9) _____**

**Solutions to Category 6**
**Team Round**
**Meet #1 - October, 2026**

| ANSWERS | |
|---|---|
| 1) | 3 |
| 2) | 14 |
| 3) | 434 |
| 4) | 11 |
| 5) | 45 |
| 6) | 5 |
| 7) | 22 |
| 8) | 71 |
| 9) | 14 |

1) 1! = 1. 2! = 2x1 = 2. 3! = 3 x 2 x 1 = 6. 4! = 4x3x2x1 = 24. The rest of the factorials each contain 2x5, or 10. So, The sum of the rest of the factorials has a units digit of 0. The sum of the first four factorials is 33 which, when added to the rest, has a units digit of 3. When divided by 5, the full sum has a remainder of 3.

2) $520 - $100 = $420. Divide $420 by $30 to calculate the number of kilometers travelled = 14.

3) For the sum of three primes to be 40, an even number, one of the factors must be 2. The two odd primes must have a sum of 38. Guessing and checking should yield 7 and 31. Finally, the product of the digits is 2x7x31, or 434.

4) The formula for the sum of the interior angles of a convex polygon is S = 180 (N – 2), where S is the sum and N is the number of sides. 1620 = 180(N – 2), 9 = N – 2, so N = 11. The polygon has 11 sides.

5) Let X = the angle measure. So, 180 – X = 3(90 – X), 180 – X = 270 – 3X, 2X = 90, and X = 45. The angle measures 45 degrees.

6) For the 5-digit number to be divisible by 9, the sum of the digits must be divisible by 9. So, the only possibility for N is 3 so that the sum of the digits is 18. When the 5-digit number is divided by 7, the quotient is 5794 and the remainder is 5.

7) The prime factorization of 280 is 2x2x2x5x7. To maximize the product of the 5-digit number, utilize the 2x2x2 to make 8 the first digit, followed by 7, 7, 1, 1 to make 87511. The sum of the digits is 8+7+5+1+1 = 22.

8) 1 + 7 + (9)(7) = 1 + 7 + 63 = 71.

9) Bottom layer: 9 Middle layer: 4 Top layer: 1 So, 9 + 4 + 1 = 14.