

IMLEM Meet #5
April, 2026

Intermediate Mathematics League of Eastern Massachusetts



Calculator Meet



- **CALCULATORS:** only *scientific calculators* allowed for meets #4 & #5 • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in disqualification.
- **NEW FOR 2025-2026:** The October meet, Category 1, include an extra problem. Questions
- #1, 2, 3, and 4 were worth 2, 2, 1, and 1 points, respectively. The November meet will include
- A fourth question in category 2 as well as in category 1, with point values as just described. Meet #3 in January will include an extra question in categories 1, 2, and 3. Meet #4 will include an extra question in all of the first four categories. Meet #5 will include an extra question in each of the first five categories.
- The success of this innovation will be evaluated in the spring, after all five meets have been completed.

Category 1

Mystery

Meet #5 - April, 2026



Calculator Meet

1) (worth 2 points)

If the letters in the word **STONEHAM** are repeated over and over again, what will be the value of the 100th letter if **A = 1**, **B = 2**, **C = 3**, and so on?

2) (worth 2 points)

The average of three numbers is 8. The average of five numbers, including the first three numbers, is 11. What is the sum of the two numbers in the second set that were not included in the first set?

3) (worth 1 point)

What is the sum of all the prime factors of the number 2026 ?

4) (worth 1 point)

At the school nurse's office, 120 boxes of tissues were used during a certain 3-day period. There are 144 tissues in each box, The nurse's office is open each day from 8:00 A.M. to 2:00 P.M. On average, how many tissues were used each minute?

Answers

1) _____

2) _____

3) _____

4) _____

Solutions to Category 1

Mystery

Meet #5 - April, 2026

- 1) There are 8 letters in the word STONEHAM.
Dividing 100 by 8 yields 12 blocks of letters of the complete word, with a remainder of 4 letters: S T O N. N is the 100th letter and has a value of 14, being the 14th letter of the alphabet.
- 2) If the average of three numbers is 8, then their sum is (3)(8), or 24. If the average of five numbers, including the first three, is 1, then their sum is (5)(11), or 55. The difference of the two sums is $55 - 24$, or 31.
- 3) $2026 = 2 \times 1013$
Both 2 and 1013 are prime.
So, $2 + 1013 = 1015$.
- 4) $120 \text{ boxes} \times 144 \text{ tissues/box} = 17,280 \text{ tissues total.}$
 $17,280 \text{ tissues} / 3 \text{ days} = 5,760 \text{ tissues/day.}$
 $5,760 \text{ tissues} / 6 \text{ hours/day} = 960 \text{ tissues per hour.}$
 $960 \text{ tissues/hour} / 60 \text{ minutes per hour} = 16 \text{ tissues/minute.}$

Answers

- | | |
|----|-------|
| 1) | 14 |
| 2) | 31 |
| 3) | 1,015 |
| 4) | 16 |

Category 2
Geometry
Meet #5 - April, 2026



Calculator Meet

1) (worth 2 points)

A rectangular box of Graham crackers measures 2 inches by 8 inches by 10 inches. How many square inches of cardboard are required to create the box?

2) (worth 2 points)

Refer to problem #1. If the box were filled with honey, how many cubic inches of honey would fill the box?



3) (worth 1 point)

Including face (surface) diagonals and space diagonals, how many diagonals does a cube have?

4) (worth 1 point)

Four equivalent tennis balls fit tightly inside a cylindrical cannister, touching each other as well as the appropriate inner surfaces of the cannister, including the upper and bottom circular lids. The number of cubic centimeters inside the cannister that lie outside of the four tennis balls is $X\pi$. What is the value of X, rounded to the nearest whole number ? The diameter of each ball is 6 centimeters.



Answers

1) _____

2) _____

3) _____

4) _____

Solutions to Category 2
Geometry
Meet #5 - April, 2026

- 1) Surface area of a rectangular solid:**

$$\begin{aligned} SA &= 2LW + 2LH + 2WH \\ &= (2)(2)(8) + (2)(2)(10) + (2)(8)(10) \\ &= 32 + 40 + 160 \\ &= 232 \text{ square inches.} \end{aligned}$$

- 2) Volume of a rectangular solid:**

$$\begin{aligned} V &= LWH \\ &= (2)(8)(10) \\ &= 160 \text{ cubic inches.} \end{aligned}$$

- 3) Six surfaces: 2 diagonals per surface x 6 surfaces**

$$= 12 \text{ diagonals}$$

Space diagonals: one from each of the upper vertices

$$= 4 \text{ diagonals}$$

$$\text{Total number of diagonals} = 12 + 4 = 16.$$

- 4) Volume of empty space inside the cannister but outside the tennis balls:**

Volume of cannister - volume of the tennis balls

$$= (\text{area of circular top surface})(\text{height of cannister}) - (4)(\frac{4}{3})(\pi)(\text{cube of radius})$$

$$= (2)(\pi)(\text{square of radius})(\text{height of cannister}) - (4)(\frac{4}{3})(\pi)(\text{cube of 3})$$

$$= (\pi)(\text{square of 3})(24) - (4)(\frac{4}{3})(\pi)(27)$$

$$= (24)(9)(\pi) - (144)(\pi)$$

$$= (216)(\pi) - (144)(\pi)$$

$$= (72)(\pi)$$

$$\text{Therefore, } X = 72.$$

Answers

1) 232

2) 160

3) 16

4) 72

Category 3
Number Theory
Meet #5 - April, 2026



Calculator Meet

1) (worth 2 points)

Set A = { 2, 4, 6, 8, 10, 12, 14 }

Set B = { 2, 5, 8, 11, 14, 17 }

How many numbers are in the intersection of sets A and B?

2) (worth 2 points)

**Refer to problem #1. How many numbers are in the union of sets A and B?
(There are fewer than 13.)**

3) (worth 1 point)

In a class of 28 students,

*** 16 study French,**

*** 21 study Spanish,**

*** 4 do not study any language.**

How many students study both French and Spanish?

4) (worth 1 point)

Sabatino arranged some of the toys that the school donated to Toys for Tots into roped areas, as shown. All 912 trucks are in the rectangle. The 328 orange toys are in the smaller circle while the 507 red toys are in the larger circle. There are 69 red trucks and 123 orange trucks. If 2,453 toys were donated in all, then how many of the toys were outside of the areas that Sabatino had roped off?

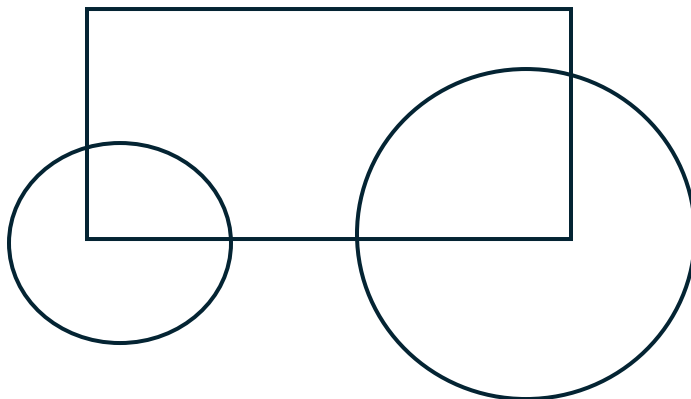
Answers

1) _____

2) _____

3) _____

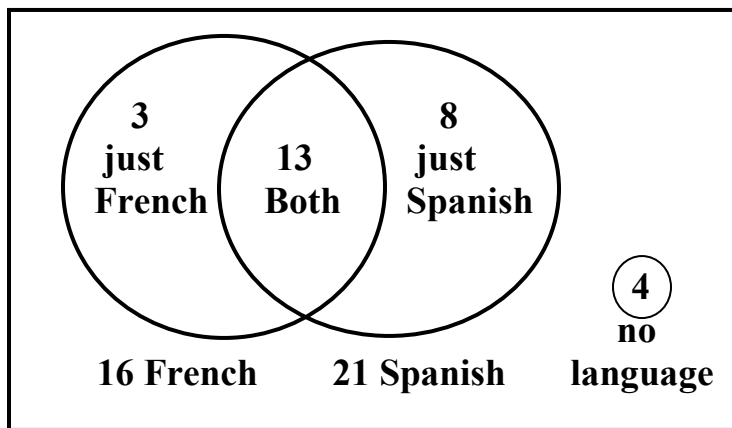
4) _____



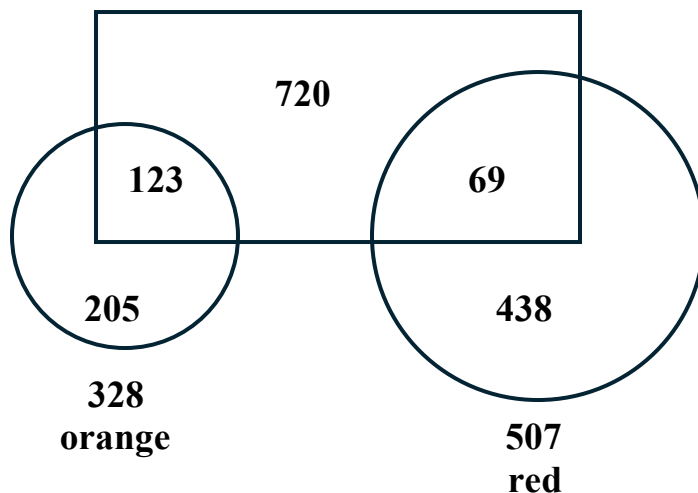
Solutions to Category 3
Number Theory
Meet #5 - April, 2026

- 1) The intersection of the two sets is the collection of elements that the two sets have in common. Those numbers are { 2, 8, 14 } for a total of three numbers.
- 2) The union of two sets is the collection of elements that are in either set, without repetition. Those numbers are { 2, 4, 5, 6, 8, 10, 11, 12, 14, 17 } for a total of 10 numbers.

3)



4)



$$\begin{aligned}
 \text{Toys outside the roped areas} &= 2,453 - (720 + 123 + 69 + 205 + 438) \\
 &= 2,453 - 1,555 \\
 &= 898
 \end{aligned}$$

ANSWERS

- 1) 3
 2) 10
 3) 13
 4) 898

Category 4
Arithmetic
Meet #5 - April, 2026



Calculator Meet

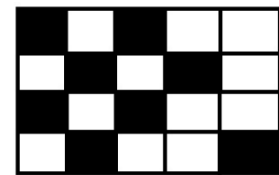
1) (worth 2 points)

To create a delicious sandwich, Rachael chooses one of four types of bread, one of three possible cold cuts, and one of six possible cheeses. If Rachael were to make one sandwich a day, then for how many consecutive days could Rachael make a different sandwich?



2) (worth 2 points)

The rectangular grid below consists of 20 congruent squares. Some are shaded and some are not. What is the probability that one of the 20 squares, selected at random, is not shaded? Express your answer as a decimal.



3) (worth 1 point)

In a new lottery game, a player must choose four different numbers from the set of integers from 1 through 30, inclusive. If every player selects a different set of numbers (order does not matter), then what is the greatest number of people who can play?

4) (worth 1 point)

A bag of colored candies contains seven red, eleven green, and four blue candies. Maxibillion selects three candies at random and eats each one after selecting it from the bag. What is the probability that the three candies are the same color? Express your answer as a decimal, rounded to the nearest hundredth.

ANSWERS

1) _____

2) _____

3) _____

4) _____

Solutions to Category 4

Arithmetic

Meet #5 - April, 2026

1) $(4)(3)(6) = 72$ possible sandwiches

2) Twelve of the twenty squares are not shaded.
 $12 / 20 = 0.6$

3) ${}_{30}C_4 = 27,405$

4) (red, red, red) = $(7/22)(6/21)(5/20) = 210/9,240$
(green, green, green) = $(11/22)(10/21)(9/20)$
= $990/9,240$

(blue, blue, blue) = $(4/22)(3/21)(2/20) = 24/9,240$

Add the probabilities to get the probability that any of these three

outcomes occurs: $210/9,240 + 990/9,240 + 24/9,240 = 1,224/9,240$

= $0.132467 \dots$

= 0.13 when rounded
to the nearest hundredth.

Answers

1) 72

2) 0.6

3) 27,405

4) 0.13

Category 5

Algebra

Meet #5 - April, 2026



Calculator Meet

1) (worth 2 points)

There are two different values of N that make the equation below true.
What is the sum of those two values?

$$(N + 3)(N - 8) = 0$$

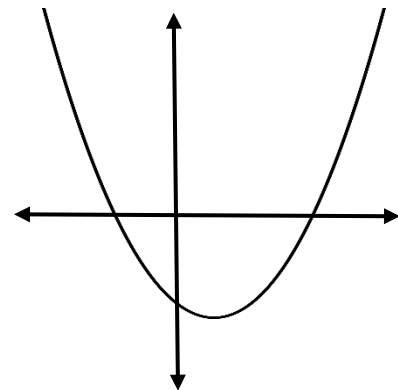
2) (worth 2 points)

A quadratic equation is a second-degree equation of the form $0 = ax^2 + bx + c$ where a is not zero, while b and c are real numbers. When the values of a , b , and c are known, the equation can be solved in a variety of ways. One such way is to employ the quadratic formula, as posted below. If $x^2 - 8x + 12 = 0$ then what is the sum of the two possible values of x that make the equation true?

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

3) (worth 1 point)

The quadratic equation, $y = ax^2 + bx + c$ when graphed, is a parabola that passes through the points $(3, 11)$ and $(-1, -1)$ and has a Y -intercept of $(0, -4)$, as shown. What is the value of $a + b + c$?



4) (worth 1 point)

A rectangle has a perimeter of 110 meters and an area of 750 square meters. The length of the pasture is longer than the width. How many meters are in the length of the rectangle?

ANSWERS

1) _____

2) _____

3) _____

4) _____

Solutions to Category 5

Algebra

Meet #5 - April, 2026

- 1) If the product of two numbers is zero, then at least one of those two numbers has a value of zero.
So, $(N + 3) = 0$ or $(N - 8) = 0$.
Then $N = -3$ or $N = 8$. Finally, $(-3) + 8 = 5$,

- 2) Although students may wish to employ the quadratic formula, factoring can be much quicker: $(X - 2)(X - 6) = 0$, so $X = 2$ or 6 .
The sum of the solutions is 8 . For students who learned that the sum of the solutions is $-b/a$,
 $-(-8/1) = 8$.

<u>Answers</u>	
1)	5
2)	8
3)	-3
4)	30

- 3) Use the value of the y-intercept to get the value of $c = -4$.
Then substitute the values of the other two known point to determine the values of a and b :

$$a(3^2) + b(3) - 4 = 11$$

$$a((-1)^2) + b(-1) - 4 = -1$$

$$9a + 3b = 15$$

$$a - b = 3 \text{ implies } 3a - 3b = 9$$

Adding the two equations now eliminates b : $12a = 24$, so $a = 2$.

Then $b = -1$.

Finally, $a + b + c = 2 + (-1) + (-4) = -3$.

- 4) If the rectangle has a perimeter of 110 meters, then half the perimeter is 55 meters.

Let $X =$ one dimension and let $55 - X =$ the other dimension.

Then the product of the two dimensions is equal to the area.

$$(X)(55 - X) = 750$$

$$55X - X^2 = 750$$

$$X^2 - 55X + 750 = 0$$

$$(X - 25)(X - 30) = 0$$

$$X = 25 \text{ or } X = 30.$$

Since the problem states that the length is longer than the width, then the length is 30 meters.

Category 6
Team Round
Meet #5 - April, 2026

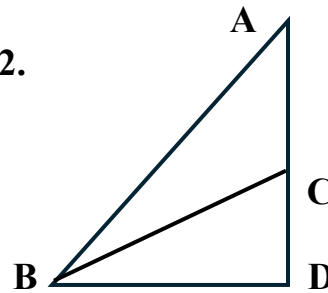
Each of the following nine problems is worth four points.

1) Arrange the digits of the number 2,549 to create a 4-digit number that is as close as possible to the number 5,000.

2) For how many pairs of integers, (X, Y) , is $X^2 - Y^2 = 21$ true?

3) The angles of a triangle are in the ratio 2: 3: 4. How many degrees are in the largest angle?

4) In this right triangle, $AB = 15$, $AC = 4$, and $BD = 12$.
How many units long is BC ?



5) A palindrome is a number that reads the same forwards as backwards. What is the value of the largest prime palindrome that is less than 1000 ?

6) A cube's length, width, and height are each increased by 50%. By what percent is its volume increased? If the answer is N percent , then give the value of N .

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

7) One natural number is two more than another. If the sum of their squares is 580, then what is the larger of the two natural numbers ?

8) How many feet are in the perimeter of a rectangle whose diagonal is 75 feet if it is similar to a rectangle that is 36 by 48 feet ?

9) There are two different values of X that make the equation $(X - 3)^{(X^2 - 9)} = 1$ true. What is the product of those two values of X ? (The number 3 is not one of those two numbers.)

Solutions to Category 6
Team Round
Meet #5 - April, 2026

ANSWERS

1) 4,952

2) 8

3) 80

4) 13

5) 929

6) 237.5

7) 18

8) 210

9) - 12

1) 4,952 comes closest to 5,000.

2) The following ordered pairs satisfy the equation:
 (5, 2), (-5, 2), (5, -2), (-5, -2), (11, 10), (-11, 10),
 (11, -10), (-11, -10) for a total of eight pairs.

3) $2x + 3x + 4x = 180$, so $9x = 180$ and $x = 20$.
 So, the largest angle is $(20)(4)$, or 80 degrees.

4) Pythagoras: $(BD)^2 + (AD)^2 = (AB)^2$
 $12^2 + (AD)^2 = 15^2$
 $144 + (AD)^2 = 225$
 $(AD)^2 = 81$
 $AD = 9$

$AD - AC = DC$, so $9 - 4 = 5$.

The smaller right triangle in the bottom portion of the diagram is a 5-12-13 Pythagorean triple, so $BC = 13$ units.

5) If the number is a palindrome, try three-digit numbers with a hundreds digit of 9 and a ones digit of 9. Start as high as possible for the tens digit, testing each candidate by using divisibility tests or the calculator. Eliminating 999, 989, 979, etc, we finally arrive at the correct value: 929, the largest prime palindrome that is less than 1000.

6) $(1.5L)(1.5W)(1.5H) = 3.375LWH$, where LWH is the volume of the original cube. So, the enlarged cube is 237.5% larger than the original cube. Therefore, $N = 237.5$.

7) A non-algebra approach would be to find either two consecutive even integers or two consecutive odd integers, such that the sum of their squares is 580. A student with sharp number sense skills may arrive at the answer rather quickly. An algebra solution would involve writing and solving a quadratic equation: $X^2 + (X + 2)^2 = 580$. Then $X^2 + X^2 + 4X + 4 = 580$.

NEXT PAGE FOR THE REMAINDER OF THE SOLUTIONS

7) (continued) $2X^2 + 4X - 576 = 0$
 $X^2 + 2X - 288 = 0$

Factor or solve by the quadratic formula. Here, we solve by factoring:

$$(X + 18)(X - 16) = 0$$

The original question requires us to only consider natural numbers, aka positive integers.

$$X = -18 \text{ or } X = 16.$$

We reject -18 as a possible solution because it is not a natural number.

The number 16 is a natural number (required) and 18 is two more than 16. The sum of their squares, $256 + 324$, is 580.

So, the larger natural number is 18.

Note: Employing the quadratic equation as a strategy comes with issues, namely, the production of a possible solution that is a negative number that violates one of the original conditions set forth in the problem – that the answer must be a natural number (positive integer). That leaves 16 to be the only natural number to consider, and 18 as the natural number that is two greater.

8) Shrink the 36 by 48 rectangle by a factor of 12 to get a 3 by 4 rectangle.

The diagonal is the hypotenuse of a 3-4-5 right triangle. Scale up by a factor of 12 to get a diagonal of 60 for the original rectangle.

The ratio of 75 : 60 is 5 : 4. The perimeters of the two rectangles are in the same ratio. $5 : 4 = X : 168$

Cross products are equal, so $4X = (5)(168)$ and $X = (5)(168) / 4 = 210$ ft.

Alternatively, scale the 3-4-5 right triangle up by a factor of 15 to get the diagonal of 75. Then scale up the original perimeter of 14 by that factor of 15 to get 210 feet.

9) Utilize the concepts that the number 1, raised to any power, is equal to 1 and any non-zero number, raised to the power of 0, is equal to 1. The text eliminates the possibility of 0 raised to the 0 power.

So, case 1: Let the base of $(X - 3) = 1$, so that $X = 4$.

Then case 2: Let the exponent, of $X^2 - 9 = 0$, so that $X^2 = 9$ and $X = 3$ or -3 . But we must eliminate 3, as it produces the expression 0^0 which was eliminated in the text.

The product of the two solutions is $(-3)(4) = -12$.