

IMLEM Meet #3
January, 2026

Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

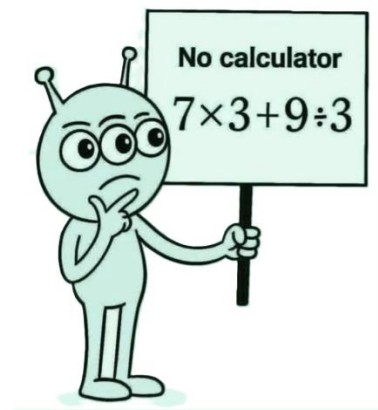
- No calculators (or only scientific calculators allowed for meets #4, #5)
- Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.
- **NEW FOR 2025-2026:** The October meet, Category 1, include an extra problem. Questions
- #1, 2, 3, and 4 were worth 2, 2, 1, and 1 points, respectively. The November meet will include
- A fourth question in category 2 as well as in category 1, with point values as just described. Meet #3 in January will include an extra question in categories 1, 2, and 3. Meet #4 will include an extra question in all of the first four categories. Meet #5 will include an extra question in each of the first five categories.
- The success of this innovation will be evaluated in the spring, after all five meets have been completed.

Category 1
Mystery
Meet #3 - January, 2026



- 1) (worth 2 points) I had my 13th birthday in the year 1964.
In what year did I have my 72nd birthday?

- 2) (worth 2 points) Employ the correct order of operations to evaluate this expression.



- 3) (worth 1 point) The Minutemen beat the Marauders by 7 points in the basketball game. If the total number of points scored was 79, then how many points did the Minutemen score?

- 4) (worth 1 point) In a row of 21 seats numbered consecutively from 1 to 21 at the movie theatre, I sit in seat #13. All seats in the row are occupied. The ratio of the number of people to my left to the number on my right is $A : B$ where $A < B$ and the ratio is in lowest terms. What is the value of B ?

ANSWERS

- 1) _____
2) _____
3) _____
4) _____

Solutions to Category 1
Mystery
Meet #3 - January, 2026

1) $72 - 13 = 59$ gives the number of years between birthdays. Add 59 to the year of 1964 to give the year of the 72nd birthday. $1964 + 59 = 2023$.

2) $7 \times 3 + 9 \div 3$
 $= 21 + 3$
 $= 24$

3) Find two numbers whose sum is 79 and whose difference is 7. One could quickly guess and check, starting at about half of 79 and adding and subtracting points until the correct answer is achieved. Doing so gives 43 points to the Minutemen and 36 points to the Marauders.

Students who are adept at algebra could write two equations in two variables: $X - Y = 7$
 $X + Y = 79$

Adding the equations yields $2X = 86$ and $X = 43$, the number of points scored by the Minutemen.

4) There are 8 seats to my left and 12 seats to my right. (Given all the data in the problem, the seats must be numbers consecutively from right to left.) The ratio of A:B is 8:12 or 2:3 when reduced to lowest terms. Therefore, $B = 3$.

Answers

1) 2023

2) 24

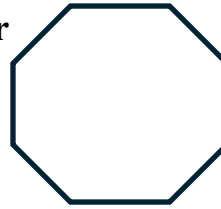
3) 43

4) 3

Category 2
Geometry
Meet #3 - January, 2026



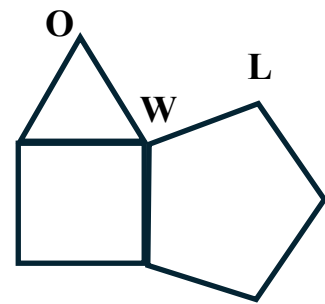
- 1) (worth 2 points) How many diagonals can be drawn from any one vertex of a regular octagon (8-sided polygon)? A side of a polygon is not a diagonal.



- 2) (worth 2 points) Rectangle EVAN has a width of 12 meters and a length of 16 meters. How many meters long is diagonal EA ?



- 3) (worth 1 point) A square, an equilateral triangle, and a regular pentagon meet at a common point W. How many degrees are in angle OWL ?



- 4) (worth 1 point) Moe and Larry race from point A to point S at a rectangular field. Moe runs from A to B to S at an average rate of 5 feet every second. Larry runs diagonally across the field from A to S at an average rate of 10 feet every 3 seconds. If they both leave point A at the same time, then by how many seconds does the winner finish ahead of the runner-up? Segment AB = 120 feet and segment BS = 50 feet.



Answers

- 1) _____
 2) _____ meters
 3) _____ degrees
 4) _____ seconds

Solutions to Category 2
Geometry
Meet #3 - January, 2026

<u>Answers</u>	
1)	5
2)	20
3)	102
4)	5

- 1) The number of diagonals is 3 fewer than the number of sides. $8 - 3 = 5$.
Or, the student can select a vertex and draw the diagonals from that vertex,

- 2) Apply the Pythagorean Theorem:

$$12^2 + 16^2 = D^2$$

$$144 + 256 = D^2$$

$$400 = D^2$$

$$20 = D$$

Therefore, diagonal EA measures 20 meters.

- 3) The interior angles of the three polygons meeting at point W are:

triangle: 60 degrees

square: 90 degrees

pentagon: 108 degrees ($3 \times 180 / 5$)

Angle OWL = $360 - (60 + 90 + 108)$, or $360 - 258$, or 102 degrees.

- 4) Moe runs $120 + 50$, or 170 feet. At a rate of 5 feet per second, it takes Moe $170 / 5$, or 34 seconds to reach point S.

Larry: Use the Pythagorean Theorem to find that Larry has run 130 feet.

At a rate of 10 feet every 3 seconds, it takes Larry $(130 / 10) \times 3$, or

39 seconds to reach point S. Moe, therefore, reaches point S ahead of

Larry by $39 - 34$, or by 5 seconds

Category 3
Number Theory
Meet #3 - January, 2026



- 1) (worth 2 points) When the whole number 360,000 is written in scientific notation as $3.6 \cdot 10^N$ then what is the value of N ?
- 2) (worth 2 points) Write the base 2 numeral 1011 as a base 10 numeral.
- 3) (worth 1 point) Write the base 3 numeral 2012 as a base 5 numeral.
- 4) (worth 1 point) Assume that a rocket can average a speed of 1.4×10^3 miles per hour in space. How many days would it take for this rocket to reach the moon if the moon is 2.38×10^5 miles from Earth on this date? Use the rounded figure that there are 24 hours in a day. Ignore fluctuations in distances due to the fact that the moon's orbit is not circular. Round your final answer to the nearest tenth of a day (as a decimal, but not in scientific notation).

Answers

- 1) _____
- 2) _____
- 3) _____
- 4) _____

Solutions to Category 3
Number Theory
Meet #3 - January, 2026

<u>Answers</u>	
1)	5
2)	11
3)	214
4)	7.1

1) The decimal point moves 5 places to the left from the right end (implied) to its final position. $N = 5$.

$$\begin{aligned}
 2) \quad & 1011 \text{ (base 2)} \\
 &= 1(2^3) + 0(2^2) + 1(2^1) + 1(2^0) \\
 &= 1(8) + 0(4) + 1(2) + 1(1) \\
 &= 8 + 0 + 2 + 1 \\
 &= 11
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & 2012 \text{ (base 3) converted to base 10 first:} \\
 &= 2(3^3) + 0(3^2) + 1(3^1) + 2(3^0) \\
 &= 2(27) + 0(9) + 1(3) + 2(1) \\
 &= 54 + 0 + 3 + 2 \\
 &= 59 \text{ (base 10)}
 \end{aligned}$$

59 (base 10) converts to base 5 by dividing by consecutive powers of 5:
 $59 / 25 = 2$ with remainder 9.
 $9 / 5 = 1$ with remainder 4.

Finally, $59 \text{ (base 10)} = 214 \text{ (base 5)}$

4) Divide the distance to the moon in miles by the average speed in mph to get the number of hours it takes to get to the moon. Divide that answer by 24 to get the number of days.

$$\frac{2.38 \times 10^5}{1.4 \times 10^3 \times 24} = \frac{1.7 \times 10^2}{24} = 7.083 \dots \approx 7.1$$

So, it would take about 7.1 days (rounded to the nearest tenth of a day) for the rocket to reach the moon.

Category 4
Arithmetic
Meet #3 - January, 2026



1) Find the value of $5^2 + 6^0 + 2^3$

2) Find the value of $\sqrt{16} + \sqrt[3]{125} - \sqrt[5]{32} + \sqrt[8]{1} - \sqrt{0.09} + 2^{-3}$

Express your answer as a decimal.

3) What is the value of $\left(\sqrt[3]{2^6} \cdot \sqrt{3^4} \cdot \sqrt[6]{1^4}\right)^3$

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #3 - January, 2026

$$\begin{aligned} 1) \quad & 5^2 + 6^0 + 2^3 \\ &= 25 + 1 + 8 \\ &= 34 \end{aligned}$$

$$\begin{aligned} 2) \quad & \sqrt{16} + \sqrt[3]{125} - \sqrt[5]{32} + \sqrt[8]{1} - \sqrt{0.09} + 2^{-3} \\ &= 4 + 5 - 2 + 1 - 0.3 + 1/8 \\ &= 8 - 0.3 + 0.125 \\ &= 7.825 \end{aligned}$$

$$\begin{aligned} 3) \quad & \left(\sqrt[3]{2^6} \cdot \sqrt{3^4} \cdot \sqrt[6]{1^4} \right)^3 \\ &= \left(\sqrt{2^2} \cdot 3^2 \cdot 1 \right)^3 \\ &= (2 \cdot 9 \cdot 1)^3 \\ &= 18^3 \\ &= 5,832 \end{aligned}$$

Answers

1) 34

2) 7.825

3) 5,832

Category 5

Algebra

Meet #3 - January, 2026



- 1) What is the value of the following expression involving absolute values?

$$|(26)| + |(-17)| + |(0)|$$

- 2) What is the value of the smallest integer, N , that makes the following inequality true?

$$-3N + 7 < 15$$

- 3) The graph below represents all values of X that make the accompanying absolute value inequality true. What is the absolute value of A ?



Answers

1) _____

2) _____

3) _____

Solutions to Category 5

Algebra

Meet #3 - January, 2026

$$\begin{aligned} 1) \quad & |(26)| + |(-17)| + |(0)| \\ &= 26 + 17 + 0 \\ &= 43 \end{aligned}$$

$$\begin{aligned} 2) \quad & -3N + 7 < 15 \\ & -3N < 8 \end{aligned}$$

$N > -8/3$ Remember that multiplying both members of an inequality by a negative number reverses the sense of the inequality.

The smallest integer that makes this inequality true is -2.

- 3) Translation: The distance between X and A is more than 7 units. On the graph, find the midpoint of -20 and -6. That midpoint is -13. So, the distance between X and -13 is more than 7 units. The absolute value of -13 is 13.

Answers

1) 43

2) -2

3) 13

Category 6
Team Round
Meet #3 - January, 2026

Each of the following nine problems is worth four points.

- 1) The numbers 12, 6, 3, 10, 5, 16 ... form a sequence that follows these rules:
 - if a number is even, then divide it by 2 to get the next number.
 - if a number is odd, then multiply it by 3 and add 1 to get the next number.
 What is the value of the 900th number in the sequence?
- 2) The animals in the pasture are all either chickens or horses. There are 12 more chickens than horses. There is a total of 126 feet. How many animals are in the pasture?
- 3) In parallelogram ABCD, the measure of angle A is $\frac{1}{3}$ the measure of angle B. How many degrees are in angle B?
- 4) How many prime numbers between 0 and 100 have the digit 3 as its units digit (ones place)?
- 5) If $\sqrt{4 + \sqrt{X}} = 5$ then what is the value of X?

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

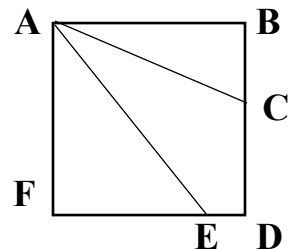
6) _____

7) _____

8) _____

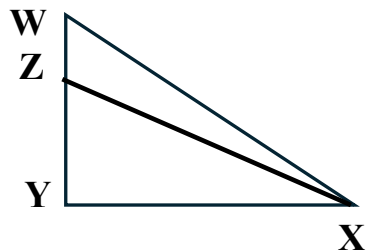
9) _____

- 6) In the square to the right, the area of quadrilateral ACDE is 28 square units. CD = 5 units and DE = 2 units. How many square units are in the area of square ABDF?



- 7) If $3^R + 3^R + 3^R = 3^5$ then what is the value of R?

- 8) Triangle WXY is a right triangle. These angles have the following degree measures: $W = 50$; $WXZ = 2n$; $ZXY = 3n$; $Y = 90$; $YZX = 3k$. What is the value of k?



- 9) $\sqrt{X} + \sqrt[3]{Y} = 9$. Find the sum of all possible values of $Y - X$ if both X and Y are positive integers.

Solutions to Category 6
Team Round
Meet #3 - January, 2026

- 1) Extend the sequence until the answer is obvious: 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1 . . .
 The number 900 is a multiple of 3.
 Every third number will be the number 2.
 So, the 900th number is 2.

- 2) $4H + 2C = 126$. $H + 12 = C$
 So, $4H + 2(H + 12) = 126$, $6H + 24 = 126$,
 $6H = 102$. Finally, $H = 17$ and $C = 29$.
 The total number of animals is $17 + 29 = 46$.

- 3) Any two consecutive angles of a p-gram have a sum of 180 degrees.
 $A = \frac{1}{3} B$ and $A + B = 180$.
 So, $\frac{1}{3} B + B = 180$, $\frac{4}{3} B = 180$,
 $B = \frac{3}{4} (180) = 135$ degrees.

- 4) The prime numbers are 3, 13, 23, 43, 53, 73, and 83 for a total of seven.

- 5) Square both sides to get $4 + X^{(1/2)} = 25$,
 then $X^{(1/2)} = 21$. Square both sides to get $X = 441$.

- 6) The sum of the three interior areas is equal to the area of the square. Let X be the length of one side of the square.

Area of top triangle: $(1/2)(X)(X - 5)$

Area of bottom triangle: $(1/2)(X)(X - 2)$

Area of quadrilateral: 28

Area of square: $(X)(X)$, or X^2

Total area of square: $(1/2)(X)(X - 5) + (1/2)(X)(X - 2) + 28 = X^2$

$$\frac{1}{2} X^2 - \frac{5}{2} X + \frac{1}{2} X^2 - X + 28 = X^2$$

$$X^2 - \frac{7}{2} X + 28 = X^2$$

$$-\frac{7}{2} X + 28 = 0$$

$$\frac{7}{2} X = 28$$

$$X = (28)(2/7)$$

$$X = 8$$

The area of the square is $(8)(8)$, or 64 square units.

ANSWERS

- 1) 2
 2) 46
 3) 135
 4) 7
 5) 441
 6) 64
 7) 4
 8) 22
 9) 1,092

$$7) \quad 3^R + 3^R + 3^R = 3^5$$

$$3(3^R) = 3^5$$

$$3^{R+1} = 3^5$$

$$R + 1 = 5$$

$$R = 4$$

8) The angles labelled $2n$ and $3n$ combine to make a single angle of $5n$.

The sum of the angles of a triangle is 180 degrees.

$$\text{So, } 50 + 90 + 5n = 180$$

$$140 + 5n = 180$$

$$5n = 40$$

$$n = 8$$

$$\text{Then } 3n = 24$$

$$\text{Bottom triangle: } 24 + 90 + 3k = 180$$

$$114 + 3k = 180$$

$$3k = 66$$

$$k = 22$$

9) The following ordered pairs, (X, Y) , make the equation true: $(1, 512)$, $(4, 343)$, $(9, 216)$, $(16, 125)$, $(25, 64)$, $(36, 27)$, $(49, 8)$, and $(64, 1)$. The corresponding values of $Y - X$ are:

$$512 - 1 = 511$$

$$343 - 4 = 339$$

$$216 - 9 = 207$$

$$125 - 16 = 109$$

$$64 - 25 = 39$$

$$27 - 36 = -9$$

$$8 - 49 = -41$$

$$1 - 64 = -63$$

The sum of these differences of $Y - X$ is

$$511 + 339 + 207 + 109 + 39 + (-9) + (-41) + (-63) = 1092.$$