

IMLEM Meet #2
November, 2025

Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- No calculators (or only scientific calculators allowed for meets #4, #5)
- Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

NEW FOR 2025-2026: The October meet, Category 1, include an extra problem. Questions #1, 2, 3, and 4 were worth 2, 2, 1, and 1 points, respectively. The November meet will include A fourth question in category 2 as well as in category 1, with point values as just described. Meet #3 in January will include an extra question in categories 1, 2, and 3. Meet #4 will include an extra question in all of the first four categories. Meet #5 will include an extra question in each of the first five categories.

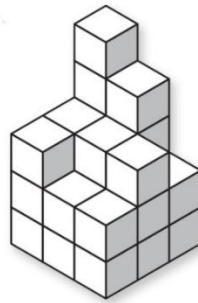
The success of this innovation will be evaluated in the spring, after all five meets have been completed.

Category 1
Mystery
Meet #2 - November, 2025



- 1) (worth 2 points) How many odd integers (odd whole numbers) are there between 50 and 70 ?
- 2) (worth 2 points) A young maple tree sapling was 5 years old when the older tree that dropped the seed from which it grew was 48 years old. The older tree is now 72 years old. How many years old is the sapling?

- 3) (worth 1 point) Same-sized cubes are stacked on a plane, as shown. The bottom cube of each column of cubes touches the plane, much like on a table. How many cubes are there in all?



**How
many
cubes?**

- 4) Sadie is 3 years younger than Beth.
Leila is 2 years older than James.
Nathaniel is 4 years older than Sadie.
Leila is 4 years younger than Beth.
How many years older is Nathaniel than Beth?

Answers

1) _____

2) _____

3) _____

4) _____

Solutions to Category 1
Mystery
Meet #2 - November, 2025

- 1) The odd integers between 50 and 70 are
51, 53, 55, 57, 59, 61, 63, 65, 67, and 69,
or TEN in total.
- 2) The older tree is 43 years older than the sapling,
as $48 - 5 = 43$. Subtract 43 from 72 to get the
current age of the sapling, or 29 years old.
- 3) The number of cubes on each level, starting with the
bottom level, or level 1:
level 1 = $3 \times 3 = 9$
level 2 = $3 \times 3 = 9$
level 3 = 6
level 4 = 2
level 5 = 1
Total = $9 + 9 + 6 + 2 + 1 = 27$ cubes.
- 4) We only need the first and third statements to solve this problem. If Sadie is
3 years younger than Beth, then Beth is 3 years older than Sadie. Nate is
4 years older than Sadie, so Nate must be $4 - 3 = 1$ year older than Beth.

Answers

1) 10

2) 29

3) 27

4) 1

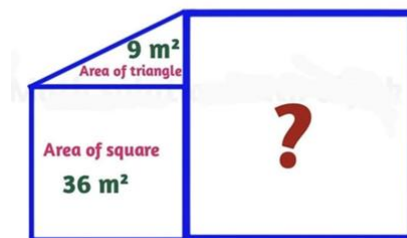
Category 2
Geometry
Meet #2 - November, 2025



- 1) (worth 2 points) A square has an area of 25 square inches. How many inches are in its perimeter?
- 2) (worth 2 points) A rectangle has an area of 24 square meters. Its perimeter is 22 meters. If the rectangle's length is longer than its width, then how many meters are in its length?

Area of Big square = ?

- 3) (worth 1 point) The figure to the right was spotted on a mathematics page on the Internet. How many square meters are in the area of the large square?



- 4) (worth 1 point) A square, a regular hexagon, and a regular octagon have equal perimeters. If
T = the perimeter of the octagon, and
H = half the perimeter of the hexagon, and
the area of the square is 169 square feet,
then find the value of T - H.

Answers

- 1) _____ inches
- 2) _____ meters
- 3) _____ square meters
- 4) _____

Solutions to Category 2
Geometry
Meet #2 - November, 2025

- 1) $5 \times 5 = 25 =$ area of square.
(4) (5) $= 20 =$ perimeter of square
- 2) Half the perimeter is half of 22, or 11 meters.
Find two numbers whose sum is 11 and whose product is 24. Those two numbers are 3 and 8, the width and length of the rectangle. The length is longer than the width, according to the wording of the problem. So, the length is 8 meters.
- 3) The square is a 6 by 6 to yield an area of 36 square meters.
The base of the triangle is, therefore, 6 meters, so the height (altitude) must be 3 meters for its area to be 9 square meters.
Finally, one side of the large square is $6 + 3$, or 9 meters and its area is 9×9 , or 81 square meters.
- 4) One side of the square is equal to the square root of 169, or 13 feet.
Its perimeter is 4×13 , or 52 feet.
The hexagon has the same perimeter of 52 feet, so half its perimeter, H, is equal to $(1/2)(52)$, or 26.
T = the perimeter of the octagon = 52 feet.
So, $T - H = 52 - 26 = 26$.

Answers

1) 20

2) 8

3) 81

4) 26

Category 3
Number Theory
Meet #2 - November, 2025



- 1) Let A = the greatest common factor of 8 and 12.
 B = the lowest common multiple of 8 and 12.
 What is the value of $A + B$?
- 2) The prime factorization of 4200 is $3 \times 2 \times 7 \times C \times 2 \times C \times D$ where
 C and D are different prime numbers.
 What is the value of $C + D$?
- 3) Remiggio has a rectangular piece of poster board that measures 36 inches by 42 inches. Remiggio wants to cut this entire board into some number of squares of equal size with no waste. How many inches are in the side length of the largest possible squares that Remiggio can cut from the poster board?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - November, 2025

- 1) factors of 8: 1, 2, 4, 8
factors of 12: 1, 2, 3, 4, 6, 12
GCF = 4
multiples of 8: 8, 16, 24, 32, 40, ...
multiples of 12: 12, 24, 36, 48, ...
LCM = 24
 $A + B = 4 + 24 = 28.$

- 2) $4200 = 3 \times 2 \times 7 \times C \times 2 \times C \times D$
 $= 84 \times C \times C \times D$
Divide 4200 by 84 = 50 = $C \times C \times D$
 $50 = 2 \times 5 \times 5$
Therefore $C = 5$ and $D = 2$
So, $C + D = 5 + 2 = 7.$

- 3) The largest possible squares that Remiggio can cut from his 36 inch by 42 inch poster board have a side length equal to the greatest common factor of 36 and 42, which is 6 inches.
 $6 \times 6 = 36$ and $7 \times 6 = 42$

Answers

1) 28

2) 7

3) 6

Category 4
Arithmetic
Meet #2 - November, 2025



- 1) When reduced to lowest terms, the fraction $\frac{36}{84} = \frac{E}{F}$.
What is the value of $E + F$?

- 2) If the following numbers are arranged in order from smallest to largest, what is the decimal value of the largest, rounded to the nearest hundredth?

$$34.2\% \quad \frac{1}{3} \quad 0.304 \quad \frac{35}{111}$$

- 3) When simplified, the following expression is a positive whole number value.
What is that value?

$$\frac{(3.75)\left(1\frac{7}{9}\right)}{\frac{2}{3} - 0.25}$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - November, 2025

1) Reduce the fraction by the GCF of 36 and 84, or by 12. $36 / 12 = 3$ and $84 / 12 = 7$. So, $36 / 84 = 3 / 7$.
 $A + B = 3 + 7 = 10$.

2) $34.2\% = 0.342$
 $1/3 = 0.33333 \dots$
 $35 / 111 = 0.315315 \dots$
 Then there's 0.304.
 The largest is 0.342. Rounding to the nearest hundredth yields a value of 0.34 or just .34.

$$\begin{aligned}
 3) & \quad \frac{(3.75)\left(1\frac{7}{9}\right)}{\frac{2}{3} - 0.25} \\
 &= \frac{\left(3\frac{3}{4}\right)\left(\frac{16}{9}\right)}{\frac{2}{3} - \frac{1}{4}} \\
 &= \frac{\left(\frac{15}{4}\right)\left(\frac{16}{9}\right)}{\frac{8}{12} - \frac{3}{12}} \\
 &= \frac{\frac{20}{3}}{\frac{5}{12}} \\
 &= \frac{20}{3} \cdot \frac{12}{5} \\
 &= 16
 \end{aligned}$$

Answers

1) 10

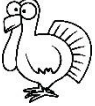
2) 0.34
 or .34

3) 16

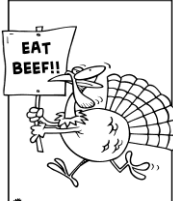
Category 5
Algebra
Meet #2 - November, 2025

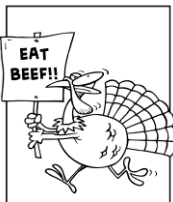


November 1

1) If $3 \text{  + 19 = 34$ then what
is the value of $\text{ ?$

2) If $\text{ = \text{ + 3$ and $\frac{3}{5} \text{  = 30$

and $6 \text{  = \text{ + 1$

then what is the value of $\text{ + \text{ + \text{ ?$

3) In the sport of baseball, the statistic called the batter's SLUGGING AVERAGE

is denoted by the formula $\frac{S + 2D + 3T + 4H}{A}$ where S is the number of

Answers

1) _____

2) _____

3) _____

singles, D is the number of doubles, T is the number of triples, H is the number of homeruns, and A is the number of official at bats for those phenomena. For Shohei Ohtani, $D = 25$, $T = 9$, $H = 55$, and $A = 611$. These stats are from his 2024 season. If his slugging average was 0.622, rounded to the nearest thousandth, then how many singles did he hit? Your answer will be a whole number.

Solutions to Category 5

Meet #2 - November, 2025

1) Use X to represent the turkey.

$$3X + 19 = 34, \text{ so } 3X = 15 \text{ and } X = 5.$$

2) Substituting capital letters for the turkeys,

$$A = B + 3 \quad 6C = A + 1 \quad \text{and} \quad \frac{3}{5}B = 30$$

starting with the third equation, $B = (\frac{5}{3})(30)$ or 50.

Then go to the first equation, substituting 50 for B :

$$A = 50 + 3 = 53.$$

Then, using the second equation, substituting

$$53 \text{ for } A: \quad 6C = 53 + 1, \quad 6C = 54, \quad \text{and } C = 9.$$

Finally, $A + B + C = 53 + 50 + 9 = 112$

$$3) \quad \frac{S + 2D + 3T + 4H}{A}$$

$$\frac{S + 2(25) + 3(9) + 4(55)}{611} \approx 0.622$$

$$\frac{S + 50 + 27 + 220}{611} \approx 0.622$$

$$\frac{S + 297}{611} \approx 0.622$$

$$S + 297 \approx (0.622)(611)$$

$$S + 297 \approx 380.042$$

$$S \approx 83.042$$

Since the slugging average of 0.622 was given as a rounded number, the number of singles must have been a whole number, then the number of singles was 83.

Answers

1) 5

2) 112

3) 83

Category 6
Team Round
Meet #1 - November, 2025

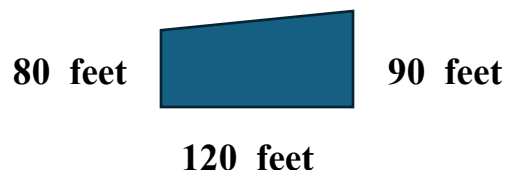
Each of the following nine problems is worth four points.

- 1) What is the units digit (ones place) of 3^{53} ?
- 2) A machine can fill 240 boxes of thumbtacks in 8 minutes. Each box contains 90 thumbtacks. How many boxes can be filled in one hour?
- 3) The perimeter of a regular octagon is 72 centimeters. How many square centimeters are in the area of a square where each side of the square is 3 centimeters longer than a side of the octagon?
- 4) A Mersenne prime is a prime number of the form $2^N - 1$ where N is a prime number. Although not a prime number, what is the value of the only “Mersenne” type number between 300 and 3000 ?
- 5) Each side of a scalene triangle has a length which is an even integer number of units. What is the least possible number of units in its perimeter?
- 6) The sum of four consecutive prime numbers is 168. What is the value of the largest of the four primes?

ANSWERS

- 1) _____
- 2) _____
- 3) _____
- 4) _____
- 5) _____
- 6) _____
- 7) _____
- 8) _____
- 9) _____

- 7) The product of two consecutive odd integers is 323. What is the sum of those two odd integers?
- 8) The yard behind Bobby’s house is shaped like the right trapezoid below. Bobby hired someone to seed the lawn at a rate of 78 cents per ten square feet. How many dollars will Bobby pay? Round your answer to the nearest whole dollar.



- 9) The first four prime numbers are 2, 3, 5, and 7. The first four composite numbers are 4, 6, 8, and 9. If the ninth prime number is multiplied by the ninth composite number, then what is the resulting product?

Solutions to Category 6
Team Round
Meet #2 - November, 2025

- 1) Note the pattern that emerges when the first few powers of 3 are listed:

<u>N</u>	<u>3^N</u>
1	3
2	9
3	27
4	81
5	243
6	729

and so on. The pattern of the units digit falls into groups of four digits that repeat to infinity: 3, 9, 7, 1, 3, 9, 7, 1, 3, ...
 Divide the exponent of 53 by four to get 13 groups of four digits (a total of 52 digits) followed by the 53rd digit of 3.

- 2) The fact that each box holds 90 thumbtacks is irrelevant information.

$$\frac{240 \text{ boxes}}{8 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hour}}$$

$$= 1,800 \text{ boxes per hour.}$$

- 3) One side of the regular octagon is $72 / 8$, or 9 cm.
 One side of the square is $9 + 3$, or 12 cm.
 The area of the square is $(12)(12)$, or 144 square centimeters.
- 4) Use only prime numbers for N:

<u>N</u>	<u>2^N - 1</u>	
7	63	too small
11	2,047	just right
13	8191	too large

- 5) The sum of any two sides of any triangle must be less than the length of the third side. A scalene triangle has all sides of different length. The triangle in question has side lengths that are even integers.)
 The triangle of minimum perimeter, satisfying all these conditions, has lengths 4, 6, and 8 units. The perimeter is $4 + 6 + 8 = 18$ units.

ANSWERS

- 1) 3
 2) 1,800
 3) 144
 4) 2,047
 5) 18
 6) 47
 7) 36
 8) 796
 9) 368

- 6) The average of the four consecutive prime numbers is near $168 / 4$, or 42.
 $37 + 41 + 43 + 47 = 168$. The largest of these four primes is 47.
- 7) The two consecutive odd integers would be in the vicinity of the square root of 323, or about 18.
 $(17)(19) = 323$.
 The sum of the two primes is $17 + 19$, or 36.
- 8) The area of the right trapezoid is $(1/2)(\text{height})(\text{sum of the bases})$
 $= (1/2)(120)(80 + 90)$
 $= (60)(170)$
 $= 10,200$ square feet.
 At 78 cents per TEN square feet, divide 10,200 by 10 to get 1,020 78-cent units.
 Multiply 78 cents by 1,020 square feet to get 79,560 cents.
 Convert to dollars by dividing by 100 to get \$795.60.
 Round to the nearest dollar to get \$796.
- 9) Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23
 23 is the 9th prime number.
 Composite numbers: 4, 6, 8, 9, 10, 12, 14, 15, 16
 16 is the 9th composite number.
 Multiplying 23 by 16 yields 368.