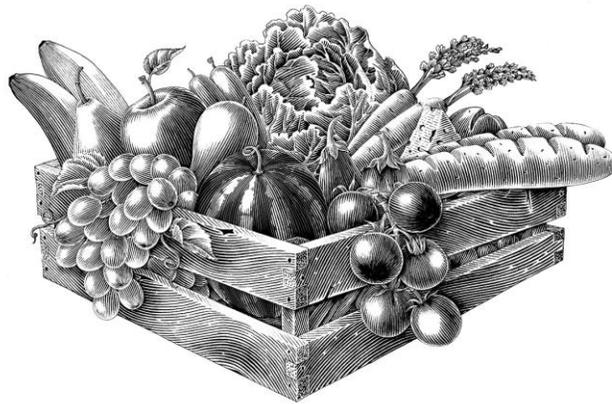


IMLEM Meet #1
October, 2025

Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any “out of control” behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.
- No calculators (or only scientific calculators allowed for meets #4, #5)
- Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

NEW FOR 2025-2026: The October meet, Category 1, will include an extra problem. Questions #1, 2, 3, and 4 will be worth 2, 2, 1, and 1 points, respectively. The November meet will include A fourth question in category 2 as well as in category 1, with point values as just described. Meet #3 in January will include an extra question in categories 1, 2, and 3. Meet #4 will include an extra question in all of the first four categories. Meet #5 will include an extra question in each of the first five categories.

The success of this innovation will be evaluated in the spring, after all five meets have been completed.

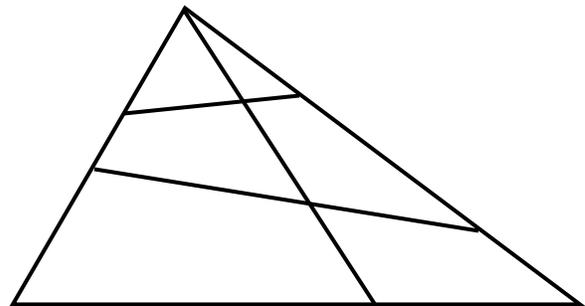
Category 1
Mystery
Meet #1 - October, 2025



1) (worth 2 points) The sum of two consecutive whole numbers is 27. What is the value of the smaller number?

2) (worth 2 points) If the letters of the alphabet correspond to the consecutive integers from 1 – 26, so that A = 1, B = 2, C = 3, and so on, then what is the sum of the values of the letters in the word MATH ?

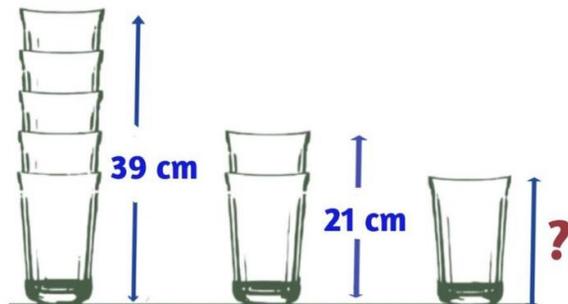
3) (worth 1 point) How many triangles of any size are in this figure?



4) (worth 1 point) Five identical tumblers (glasses) are stacked, as shown below, and have a total height of 39 centimeters. When only two such tumblers are stacked, their total height is 21 centimeters. How many centimeters tall is one tumbler?

Height of one Tumbler ?

<u>ANSWERS</u>
1) _____
2) _____
3) _____
4) _____

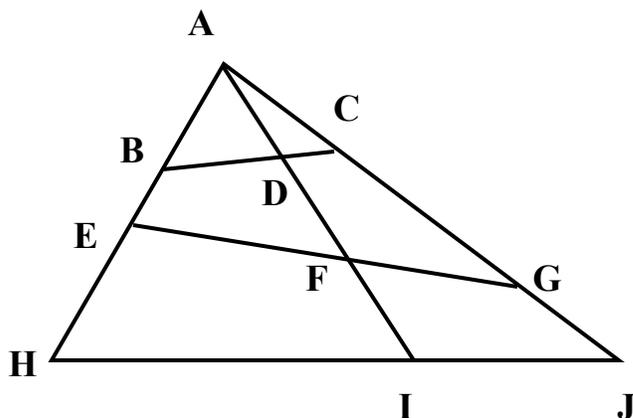


**Solutions to Category 1
Mystery
Meet #1 - October, 2025**

1) Estimate the value of the two integers by dividing 27 by 2. The two numbers whose sum is 27 are 13 and 14. The smaller of the two is 13.

2) $M = 13$, $A = 1$, $T = 20$, and $H = 8$.
 $13 + 1 + 20 + 8 = 42$.

3)



There are 9 triangles, as follows: ABD, ADC, ABC, AEF, AFG, AEG, AHI, AIJ, and AHJ.

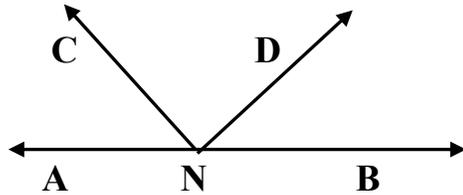
4) One visible tumbler plus four tumbler tops have a total height of 39 cm.
 One visible tumbler plus one tumbler top have a total height of 21 cm.
 The difference between the two stacks is three tumbler tops with a total Height of $39 - 21$, or 18 cm. So, one tumbler top is $18 / 3$, or 6 cm tall.
 Since one tumbler plus one tumbler top = 21 cm, then one tumbler = $21 - 6$, Or 15 cm.

<u>Answers</u>	
1)	13
2)	42
3)	9
4)	15

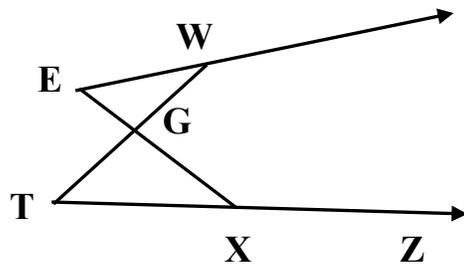
Category 2
Geometry
Meet #1 - October, 2025



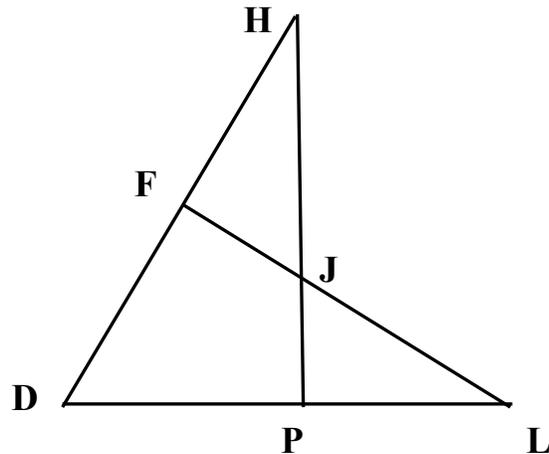
- 1) In the diagram to the right,
 - AB is a straight line,
 - angle CNA measures 72 degrees,
 - angle DNB measures 39 degrees.
 How many degrees are in the measure of angle CND ?



- 2) In the diagram to the right,
 - angle WEG measures 58 degrees,
 - angle EXZ measures 132 degrees,
 - $GT = GX$.
 How many degrees are in the measure of angle EWG ?



- 3) In the diagram below, angle DHP measures 40 degrees, angle DFL measures 105 degrees, and angle DPH is a right angle. How many degrees are in the measure of angle DLF ?



<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 2
Geometry
Meet #1 - October, 2025**

- 1) Since AB is a straight line, angle ANB is a straight angle measuring 180 degrees.

$$72 + (\text{angle CND}) + 39 = 180$$

$$111 + (\text{angle CND}) = 180$$

$$\text{angle CND} = 69$$

- 2) The sum of the angles of a triangle is 180 degrees.

$$132 + \text{angle EXT} = 180, \text{ so angle EXT} = 48 \text{ degrees.}$$

For triangle GTX, if the sides are congruent, then the base angles are congruent, so angle WTX = angle EXT = 48 degrees. Then for triangle GTX, $48 + 48 + \text{angle TGX} = 180$, $96 + \text{angle TGX} = 180$, and angle TGX = 84 degrees.

When lines intersect, the opposite (or vertical) angles are congruent. So, angle TGX = angle EGW = 84 degrees.

Finally, for triangle EGW, $58 + 84 + \text{angle EWG} = 180$, $142 + \text{angle EWG} = 180$, and angle EWG = 38 degrees.

- 3) In triangle DPH, $40 + 90 + \text{angle HDP} = 180$, then $130 + \text{angle HPD} = 180$, and angle HDP = 50 degrees.

In triangle DFL, $50 + 105 + \text{angle DLF} = 180$
Then $155 + \text{angle DLF} = 180$ and angle DLF = 25 degrees.

Answers

1) 69

2) 38

3) 25

Category 3
Number Theory
Meet #1 - October, 2025



1) How many prime numbers are there between 10 and 20 ?

2) For how many positive integer values is $\frac{144}{W}$ a positive integer?

3) What is the value of N if

- * N is a whole number
- * N is divisible by 6
- * $N > 60$
- * 5 is not a factor of N
- * $N < 100$
- * the sum of the digits of N is 12
- * N is not divisible by 11.

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2025

1) There are four prime numbers between 10 and 20,
Namely 11, 13, 17, and 19.

2) The number 144 is abundant and has a LOT of
Factors – exactly 15.

$$\begin{array}{ll} 144 / 1 = 144 & 144 / 144 = 1 \\ 144 / 2 = 72 & 144 / 72 = 2 \\ 144 / 3 = 48 & 144 / 48 = 3 \\ 144 / 4 = 36 & 144 / 36 = 4 \\ 144 / 6 = 24 & 144 / 24 = 6 \\ 144 / 8 = 18 & 144 / 18 = 8 \\ 144 / 9 = 16 & 144 / 16 = 9 \\ 144 / 12 = 12 & \end{array}$$

3) Since N is between 60 and 100, start with the clue that reduces the possibilities to multiples of 6. Then eliminate the multiples of 6 according to the remaining clues. Doing so leaves only one possibility – the number 84.

Answers

1) 4

2) 15

3) 84

Category 4
Arithmetic
Meet #1 - October, 2025



- 1) Using your knowledge about the order of operations, find the value of the following expression:

$$1 \times 2 + 3 \times 4 + 5 \times 6$$

- 2) The mean (average) of a list of three whole numbers is 6. When one of the numbers is removed from the list, the mean of the remaining two numbers is 5. What is the value of the number that was removed from the list?

- 3) Set A = { 9, 2, 9, 7, 4, 9, 6, 7 }
Set B = { 8, 10, 5, 4, 10, 12, 10 }
Set C = the median of the numbers in Set A
Set D = the mode of the numbers in Set B
What is the mean of Set C and Set D? Express your answer as a decimal.

Answers

1) _____

2) _____

3) _____

**Solutions to Category 4
Arithmetic
Meet #1 - October, 2025**

1) $1 \times 2 + 3 \times 4 + 5 \times 6$
 $= 2 + 12 + 30$
 $= 44$

2) If the mean of three numbers is 6, then their sum is $(3)(6)$, or 18. If removing one number in the set leaves two numbers whose mean is 5, then the sum of those two numbers is $(2)(5)$, or 10. Therefore, the number that was removed is the difference of the two sums, or $18 - 10$, or 8.

3) Step 1: to find the median of the numbers in Set A, first arrange the numbers in order. Then choose the number in the middle of the set.
Set A, reordered = $\{ 2, 4, 6, 7, 7, 9, 9, 9 \}$
Set C = the median of the numbers in Set A = 7 (there are two 7s in the middle of the set).

Step 2: the mode of the numbers in Set B is the number that appears most frequently. That number is 10.

Final step: the mean of Sets C and D is the average of 7 and 10.
 $(7 + 10) / 2 = 17 / 2 = 8.5$, a decimal as required by the question.

Answers

1) 44

2) 8

3) 8.5

Category 5
Algebra
Meet #1 - October, 2025



1) If  = 10, and  = 3, and  = 6, then what is the value of $8 \text{  } - 5 \text{  } + 2 \text{  } ?$

2) A \$\$ B is defined to be the product $(A + B)(A - B)$

A && B is defined to be the sum $A^2 + B^2$

What is the value of $(8 \text{ $$ } 3)(8 \text{ \&\& } 3)$?

3) Find the value of P that makes the following equation true:

$$4P + 6 + 2(5P - 3) = 39 - 3(2P - 7)$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2025

1) To simplify, substituting A, B, and C for the three Halloween symbols: $8A - 5B + 2C$
 $= 8(10) - 5(3) + 2(6)$
 $= 80 - 15 + 12$
 $= 77.$

2) $8 \$\$ 3 = (8 + 3)(8 - 3) = (11)(5) = 55$

$8 \&\& 3 = 8^2 + 3^2 = 64 + 9 = 73$

Finally, $(8 \$\$ 3)(8 \&\& 3) = (55)(73) = 4015$

3) $4P + 6 + 2(5P - 3) = 39 - 3(2P - 7)$
 $4P + 6 + 10P - 6 = 39 - 6P + 21$
 $14P = 60 - 6P$
 $20P = 60$
 $P = 3$

<u>Answers</u>	
1)	77
2)	4015
3)	3

Category 6
Team Round
Meet #1 - October, 2025

Each of the following NINE problems is worth four points.

- 1) The difference between the squares of two consecutive odd integers is 64. Which of the two odd integers is prime?
- 2) What is the sum of all prime numbers between 120 and 140 ?
- 3) Evan has test scores of 89, 72, 93, and 86. What must be the sum of Evan's next two test scores so that his average for all six tests is 90 ?
- 4) The supplement of an angle V is 14 degrees more than three times its complement. How many degrees are in angle V ?
- 5) (a classic from the 1960s) Moe DeLawn cut $\frac{1}{3}$ of the grass in the backyard while a cousin, Sprey, cut $\frac{2}{5}$ of the grass. It then took Moe an additional one hour and 40 minutes to complete the job. If Moe had cut all of the grass alone, without the aid of Sprey, then how many hours would it have taken Moe to do the job? Express your answer as a decimal.
- 6) Seven non-collinear lines intersect on the same plane. What is the maximum possible number of intersection points?

ANSWERS

- 1) _____
- 2) _____
- 3) _____
- 4) _____
- 5) _____
- 6) _____
- 7) _____
- 8) _____
- 9) _____

- 7) Craig earned the following quiz scores for the first quarter marking period: 95, 83, 92, 89, 83, 92, 87, 83, 91, 90, and 95. He gets to choose the mean, the median, or the mode of these scores for his grade. What is the value of the greatest of these three measures of central tendency?
- 8) Three consecutive integers have a sum of 84. What is the value of the largest of the three integers?
- 9) The ratio of two numbers is 3 : 5. Their sum is 56. What is the value of the smaller of the two numbers?

**Solutions to Category 6
Team Round
Meet #1 - October, 2025**

ANSWERS

- 1) 17
- 2) 534
- 3) 200
- 4) 52
- 5) 6.25
- 6) 21
- 7) 90
- 8) 29
- 9) 21

1) Experimenting should lead students to considering odd integers greater than 10. $11^2 = 121$, $13^2 = 169$. the difference of their squares is 48. $15^2 = 225$. The difference of the squares of 13 and 15 is 56. $17^2 = 289$. The difference of the squares of 15 and 17 is 64. Voila! The prime number is 17.

2) The primes between 120 and 140 are 127, 131, 137, and 139. Their sum is $127 + 131 + 137 + 139 = 534$.

3) For the average of the six scores to be 90, their sum must be $(6)(90)$, or 540. The sum of the four given scores is $89 + 72 + 93 + 86$, or 340. The sum of the remaining two scores is $540 - 340$, or 200.

4) Let the measure of the requested angle = X
Then its complement = $90 - X$ and its supplement is $180 - X$.
 $180 - X = 3(90 - X) + 14$
 $180 - X = 270 - 3X + 14$
 $2X = 104$
 $X = 52$.

5) Adding the portions of the grass that the two cousins mowed, $1/3 + 2/5 = 5/15 + 6/15 = 11/15$. The remaining portion, $15/15 - 11/15$, or $4/15$, took one hour and 40 minutes (or $1 \frac{2}{3}$ hours) to complete.

Let H = the number of hours required for one person to do the job. Then $1 \frac{2}{3} = \frac{4}{15} X$. Multiply both sides by $15/4$ to give $(5/3)(15/4) = X$ and $X = 75/12$, or 6.25 hours.

6) Drawing the diagram for the first few intersecting lines can help create a numerical sequence to help predict the number of intersection points for an increasing number of lines. Keep track.

<u># of lines</u>	<u># of intersection points</u>
1	0
2	1
3	3
4	6
5	10
6	15
7	21

Solutions to Category 6 – continued

7) Rearrange the numbers into ascending order:

{ 95, 83, 92, 89, 83, 92, 87, 83, 91, 90, 95 } becomes the set

{ 83, 83, 83, 87, 89, 90, 91, 91, 92, 95, 95 }.

Median = middle number = 90.

Mode = most frequent number = 83.

Mean = average = the sum of the numbers divided by the number of numbers = $979 / 11 = 89$.

So, the median is the greatest value, or 90.

8) $X + (X + 1) + (X + 2) = 84$

$$3X + 3 = 84$$

$$3X = 81$$

$$X = 27$$

$$X + 1 = 28$$

$$X + 2 = 29$$

So, the largest of the three consecutive integers is 29.

9) Let $U =$ the scaling factor

$$3U + 5U = 56$$

$$8U = 56$$

$$U = 7$$

The scaling factor is 7.

Then the two original numbers are $7(3)$ and $7(5)$, or 21 and 35.

The smaller of the two numbers is 21.