IMLEM Meet #4
February, 2025

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery
Meet #4 - February, 2025

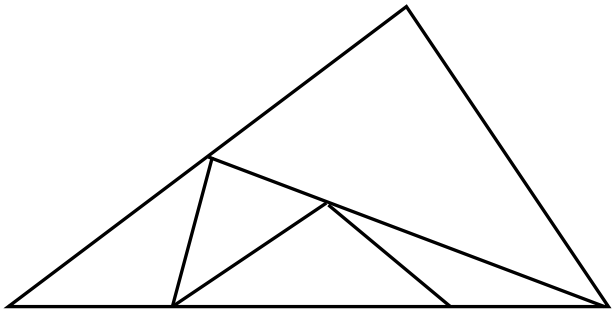
Calculator Meet

1) The value of this sum of cubes is a four-digit number. What is that value?

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$$

2) Plane A has 100 more seats than Plane B. Together, the two planes have 600 seats. How many seats does Plane B have?

3) How many triangles are in this figure? There are more than five triangles.



Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #4 - February, 2025

Answers

1) 2025

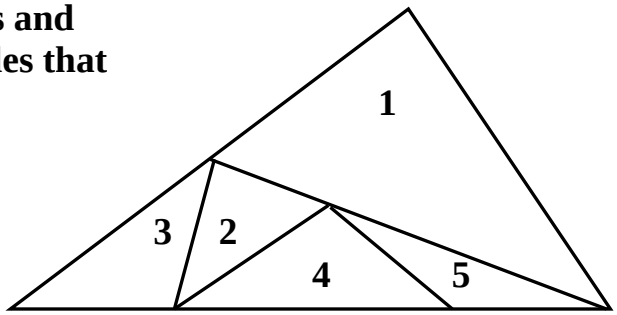
2) 250

3) 9

1) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$
= 1 + 8 + 27 + 64 + 125 + 216 + 343 + 512 + 729
= 2025

2) A non-algebra solution: Two numbers have a sum of 600 and have a difference of 100. Take half of 600, namely 300. Then take half of 100, or 50. Finally, add 50 to 300 and subtract 50 from 300. Plane A has 350 seats while Plane B has 250 seats.

3) Number the individual triangles 1, 2, 3, 4, 5. Then count the individual triangles and combinations of the smaller triangles that make larger triangles:



1
2
3
4
5
4+5
2+4+5
2+3+4+5
1+2+3+4+5

That is a total of 9 triangles.

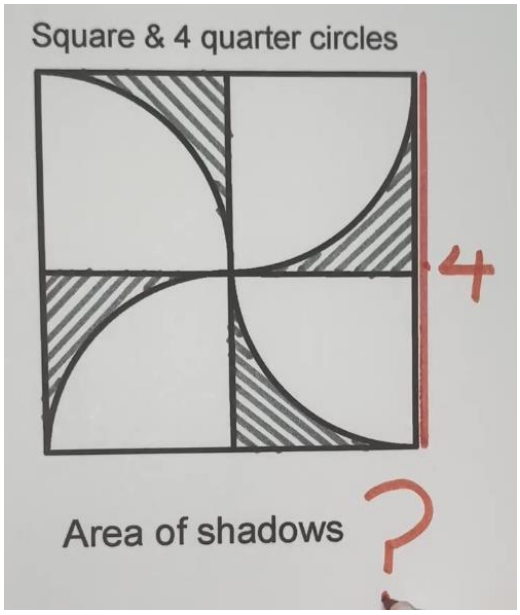
Category 2
Geometry
Meet #4 - February, 2024

Calculator Meet

1) The circumference, or perimeter of a circle is calculated by multiplying the number π by the diameter of the circle. A bug crawls from the two o'clock position on the circumference of a 6-inch radius clock to the seven o'clock position. How many inches did the bug travel? Use $\pi \approx 3.14$. Round your final answer to the nearest whole number of inches. (The bug travels in a clockwise direction.)

2) During the bug's journey along the circumference of the clock in problem #1, in a clockwise direction, both the hour hand and the minute hand rotated, albeit at different rates. The bug began its journey at 2:00 P.M. and concluded the journey at 2:35 P.M. How many degrees did the minute hand rotate during the bug's journey?

3) The following problem was found on the Internet. A large square is subdivided into four smaller congruent squares. One side of the larger square is 4 inches long. How many square inches are in the total area of the four shaded regions? Use $\pi \approx 3.1416$. Round your final answer to the nearest tenth of a square inch.
Reminder: The area of a circle is calculated by multiplying the number π by the square of the radius.



Answers

1) _____ inches

2) _____ degrees

3) _____ sq. in.

Solutions to Category 2
Geometry
Meet #4 - February, 2025

Answers	
1)	16
2)	210
3)	3.4

1) The bug crawls 5/12 of the circumference (perimeter) of the clock. Its distance covered is
(5/7)(pi)(diameter) = (5/12)(3.14)(12)
= (5)(3.14)
= 15.7
Rounded to the nearest inch, the bug travelled 16 inches.

2) The minute hand rotated from the 12 position to the 7 position, or 7/12 of 360 degrees = 210 degrees.

3) The four smaller squares, if removed and properly rotated, could form a square equivalent to the larger square with a circle "inside" that is tangent to the four sides of the large square. The area of the shadows, or shaded regions, is equal to the area of the larger square, minus the area of the circle.
= (length)(width) - (pi)(square of the radius of the circle)
= (4)(4) - (3.1416)(2^2)
= 16 - (3.1416)(4)
= 16 - 12.5664
= 3.4336
Rounded to the nearest tenth of a square inch, the answer is 3.4.

Category 3
Number Theory ***Calculator Meet***
Meet #4 - February, 2025

1) The first six numbers in an arithmetic sequence are given below. What is the value of the 35^{th} number in this sequence?

7 14 21 28 35 42 . . .

2) Several numbers in a geometric sequence are given below, with three of the numbers represented by the letters T, A, and B. What is the sum of T + A + B ?

3 12 48 192 T A 12,288 B 196,608

3) The first term of an arithmetic sequence is 17 and the 60th term is 194 and the 65th term is 209. The sum of the first 60 terms is 6330. What is the sum of the 20th and 30th terms?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #4 - February, 2025

Answers	
1)	245
2)	52,992
3)	178

1) This sequence consists of multiples of 7.
The value of the 35^{th} number is (7)(35) = 245.

2) The ratio of any two consecutive numbers in this geometric sequence is 4:1; to get from number to the next, multiply by 4.
So, T = (192)(4) = 768 and A = (768)(4) = 3,072.
Verify that (3,072)(4) = 12,288. Check!
Then B = (12,288)(4) = 49,152.
Check that (49,152)(4) = 196,608. Check!
Finally, T + A + B = 768 + 3,072 + 49,152 = 52,992.

3) The words "arithmetic sequence" indicate that there is a constant difference between any two consecutive terms. To find that difference, it is easiest to take the difference between the 60th and 65th terms and divide by the number of differences, five, that it took to reach the 65th term: (209 - 194) / 5 = 15 / 5, or 3. So, there is a constant difference of 3 between any two consecutive terms. For any Nth term, the value of that term is 3N + 14, given that the value of the first term is 17. So, the value of the 20th term is 3(20) + 14, or 74, and the value of the 30th term is 3(30) + 14, or 104. The sum of the 20th and 30th terms is 74 + 104, or 178. Note: The sentence "The sum of the first 60 terms is 6330" is extraneous information, but is inserted into the problem so that students must exercise their critical reading skills to determine what data are relevant and what data are not.

Category 4
Arithmetic
Meet #4 - February, 2025

Calculator Meet

1) At the start of the 2023 school year, there were 180 6th graders enrolled at the Miles River Regional School. By the end of the school year, the number of sixth graders increased by 5%. How many sixth graders were enrolled at the end of the school year?

2) The length of a rectangle was increased by 20% and its width was increased by 30%. This resulted in an increase in the original rectangle's area by A%. What is the value of A ?

3) The equation to the right calculates the value A, that is the result of investing an initial amount of money, M, at an annual (yearly) rate of R for T years when the interest is compounded W times annually. Stan wanted to invest $30,000 at his bank at an annual rate of 4.6%, compounded annually (yearly), for 15 years. Ollie insisted that he would earn more interest at his bank at an annual rate of 4%, compounded five times monthly, for 15 years. How much more interest was earned with the more profitable deal? Round your answer to the nearest hundred dollars.

$$A = M\left(1+\frac{R}{W}\right)^{WT}$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #4 - February, 2025

Answers	
1)	189
2)	56
3)	4,200

1) 105% of 180 = (1.05)(180) = 189.

2) If L = the original length and W = the original width, then the adjusted length and width, reflecting the increases, become 1.2L and 1.3W and the resulting area becomes (1.2L)(1.3W) = 1.56LW. So, the increase in area is 56%. So, A = 56.

3)

$$A = M\left(1+\frac{R}{W}\right)^{WT}$$

$$= 30,000\left(1+\frac{0.04}{60}\right)^{(60)(15)}$$

$$= 30,000(1+0.000666667)^{900}$$

$$= 30,000(1.000666667)^{900}$$

$$= 30,000(1.821754575)$$

$$= 54,652.63725$$

= 54,652.64 This is how much money Ollie would have after 15 years.

Now for Stan:

$$A = M(1+R)^{T}$$

$$= 30,000(1+0.046)^{15}$$

$$= 30,0$$

$$= 30,0$$

$$A = M\left(1+\frac{R}{W}\right)^{WT}$$

$$= 58,897.44$$

This is how much money Stan would have after 15 years.

The difference would be 58,897.44 - 54,652.64, or $4,244.80, with Stan having earned more. Rounding the final answer to the nearest hundred yields $4,200.

Category 5
Algebra
Meet #4 - February, 2025

Calculator Meet

1) Seventy-two pounds of food will feed an average of 252 chickens. How many chickens, on average, can be fed with 54 pounds of food?

2) Riley and Bree leave their respective homes at the same time and ride their bikes to a coffee shop somewhere between their homes. Riley rides at 12 miles per hour while Bree rides at 16 miles per hour. If their homes are 42 miles apart, and they arrive at the coffee shop at the same time, then how many miles did Bree travel?

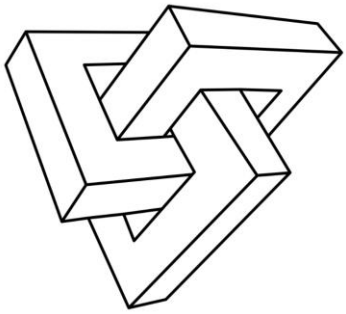
3) Three genetically modified zebras and five elephants weigh a total of 1307 stones. Four such zebras and two elephants weigh a total of 758 stones. If one stone = 20 pounds, then one elephant weighs how many pounds more than one zebra?

ANSWERS

1) _____ chickens

2) _____ miles

3) _____ pounds



Solutions to Category 5
Algebra
Meet #4 - February, 2025

Answers
1) 189
2) 24
3) 2,540

1) Set up ratios into a proportion:
food : chickens 72 : 252 = 54 : F
cross products are equal.
72 F = (252)(54)
72 F = 13,608
F = 13,608 / 72
F = 189
So, 189 chickens could be fed.

2) (rate)(time) = distance
Riley and Bree take the same amount of time. Let T = the number of hours.
Riley's distance: 12 T
Bree's distance: 16 T
The sum of their distances is 42 miles.
So, 12 T + 16 T = 42
28 T = 42
T = 42 / 28
T = 1.5
Finally, Bree travelled (16)(1.5), or 24 miles.

3) Let Z = the weight of one zebra, in stones
E = the weight of one elephant, in stones.
Now write a system of two equations in two variables, then solve for each variable. Finally, convert the number of stones to pounds by multiplying the number of stones by 20.

3Z + 5E = 1307
4Z + 2E = 758

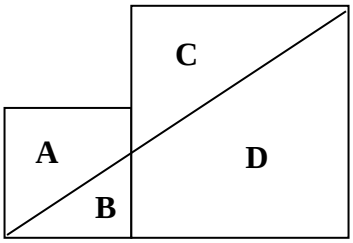
12Z + 20E = 5,228 **multiplied each side of the first equation by 4**
12Z + 6E = 2,274 **multiplied each side of the second equation by 3**

14E = 2,954 **subtracting the two equations**
E = 211 **divided each side by 14**
Z = 84 **substituting 14 for E, then solving for Z.**

In pounds, one elephant weighs more than one zebra by
(20)(211) - (20)(84) = 4,220 - 1,680 = 2,540 pounds.

Category 6
Team Round ***Each of the following six***
Meet #4 - February, 2025 ***problems is worth <u>six</u> points.***

1) A circular wheel makes 15 revolutions (complete turns) in one minute. How many degrees will it rotate in 12 seconds?



2) In the figure to the right, imagine that sections A and D are colored red. A small square consists of sections A and B while a larger square consists of sections C and D. One side of the smaller square is 6 cm while one side of the larger square is 8 cm. To the nearest whole number, how many square cm are in the combined areas of the red sections?

3) What is the whole number value of X if $3^X = X^9$ **?**

4) If the product of (W)(D) = 144 and the quotient of W / D = 4, then what is the value of the sum W + D ? (Both W and D are positive integers.)

5) Phineas starts adding consecutive natural numbers, starting with the number 1. After some time, Phineas attains a sum of 1000 and realizes that one number was mistakenly used twice. What is that number?

6) What is the value of H if $2^{H+2} + 2^{H+7} = 33$ **?**

<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

Solutions to Category 6
Team Round
Meet #4 - February, 2025

1) (15)(360) = 5,400 degrees in one minute.
Divide by 5, as there are five 12-seconds in one minute. So, 5,400 / 5 = 1,080 degrees.

2) Triangles B and C are similar (by AAA).
so the ratio of their altitudes is the same as the ratio of their bases: 6:8.
Let X = the altitude of the larger triangle
so that ¾ X = the altitude of the smaller triangle. Then 3/4 X + X = 8

$$7/4\ X = 8$$
$$X = (8)(4/7)$$
$$X = 32/7$$

and 3/4 X = 24/7

Area of triangles: (1/2)(8)(32/7) + (1/2)(6)(24/7)

$$= 128/7 + 72/7$$
$$= 200/7$$

shaded area = area of squares – area of triangles.

$$= (6)(6) + (8)(8) – 200/7$$
$$= 36 + 64 – 200/7$$
$$= 100 – 200/7$$
$$= 500/7$$

or 71.428 . . . or 71 when rounded to the nearest whole number.

3) The key is to see the relationship between the 3 and the 9.
With a bit of trial and error, a student can be led to the conclusion that X = 27. An algebraic solution still relies on the student knowing the relationship between the 3 and the 9.

$$3^X = X^9$$
$$(3^X)^3 = (X^9)^3$$
$$(3^3)^X = (X^27)$$
$$27^X = X^27$$

The only whole number value for which this could be true is when X = 27.

4) If A / D = 4, then 4D = A. Substitute 4D for A into the first equation.
(4D)(D) = 144 . . . then 4(D^2) = 144 . . . then D^2 = 36 . . . and D = 6 and W = 4D = (4)(6) = 24. Finally, W + D = 24 + 6 = 30.

THE SOLUTIONS FOR PROBLEMS #5 & 6 ARE ON THE NEXT PAGE.

ANSWERS

1)	1,080
2)	71
3)	27
4)	30
5)	10
6)	- 2

5) Students unfamiliar with the summation formula for consecutive natural numbers can use their calculators to quickly add consecutive natural numbers until they reach near 1000. The formula Sum = [N(N + 1)]/2 where N is the largest addend in the sum can be employed, assuming the sum is exactly 1000:

1000 = [N(N + 1)]/2

2000 = N(N + 1)

Look for two consecutive numbers, N and N + 1, whose product is about 2000. Those two consecutive numbers are approximately the square root of 2000, or about 44. So, try N = 44 to see how close to 1000 is that sum of the natural numbers from 1 through 44, inclusive: (44)(45)/2 = 990.
In order for the sum to equal exactly 1000, then another 10 would have had to be added. Therefore, the number that was included twice is 10.

6) $2^{H+2} + 2^{H+7} = 33$

The two powers of 2 that have a sum of 33 are 2^0 and 2^5, or 1 and 32. Note that the exponents of 0 and 5 have a difference of 5, just like the exponents of the original equation.
So, H + 2 = 0 and H + 7 = 5.
Therefore H = - 2.