IMLEM Meet #2
November, 2023

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**
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Category 1
Mystery
Meet #2 - November, 2023

1) What 3-digit number has the following properties?

- **It is an even number.**
- **The sum of its digits is 12.**
- **The product of its digits is 48.**
- **There are no odd digits.**
- **The tens digit is two more than the ones digit.**
- **The tens digit is two less than the hundreds digit.**

2) The sum of the first 50 positive odd integers is 2500. What is the sum of the first 50 positive even integers?

3) If the average of the whole numbers A, B, C, D, and E is 90 and the average of A, B, and E is 100, then what is the average of C and D ?

Answers
1) _______
2) _______
3) _______

Solutions to Category 1
Mystery
Meet #2 - November, 2023

Answers
1) 642
2) 2,550
3) 75

1) Conclusion: all three digits are different.
There are not many options to try. The three even digits are consecutive and in descending order, from left to right. Use the process of elimination.

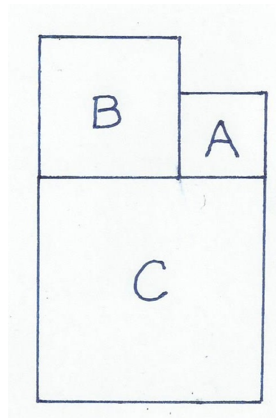
2) Compare the sum of a small string of consecutive odd integers with the sum of another string of the same number of consecutive integers where each number in the even sum is one more than each number in the odd sum.
For example, compare $1 + 3 + 5 = 9$ to $2 + 4 + 6 = 12$. The sums are different by 3.
Now compare, say, $1 + 3 + 5 + 7 + 9 = 25$ to $2 + 4 + 6 + 8 + 10 = 30$.
The sums are different by 5.
So, the sum of the first N positive odd integers is N less than the sum of the first N positive even integers. So, the sum of the first 50 positive odd integers is 50 less than the sum of the first 50 positive even integers. $2500 + 50 = 2,550$.

3) If the average of five numbers is 90, then their sum is (5)(90), or 450.
If the average of three numbers is 100, then their sum is (3)(100), or 300.
So, the sum of C and D is the difference between the other two sums, or 450 - 300, or 150 and their average is 150 / 2, or 75.

Category 2
Geometry
Meet #2 - November, 2023

1) The sides of a square and the sides of an equilateral hexagon (six-sided polygon) have the same length. The perimeter of the hexagon is 84 cm. How many square centimeters are in the area of the square?

2) The area of square A is 36 square inches. The perimeter of square B is 36 inches. How many square inches are in the area of square C ?



3) Quadrilateral AFGH is a rectangle containing five shaded triangles whose bases lie along AF and whose third vertices lie along HG.
AF = 96 feet
FG = 22 feet
B is the midpoint of AF.
C is the midpoint of BF.
D is the midpoint of CF.
E is the midpoint of DF.
How many square feet are in the area of the unshaded region?

Answers		
1)	_______	**sq. cm**
2)	_______	**sq. in.**
3)	_______	**sq. ft.**

Solutions to Category 2
Geometry
Meet #2 - November, 2023

Answers
1) 196
2) 225
3) 1,320

1) If the perimeter of the equilateral hexagon is 84 cm, then divide 84 by 6 to get the length of one side, or 14 cm. Then each side of the square is also 14 cm, so the area of the square is (14)(14), or 196 sq. cm.

2) If the area of square A is 36 square inches, then one side has a length of the square root of 36 , or 6 inches.
If the perimeter of square B is 36 inches, then one side has a length of 36 / 4, or 9 inches.
Then one side of square C measures 6 + 9, or 15 inches. So, the area of square C is (15)(15), or 225 square inches.

3) Regardless ot where, along HG, the third vertex of the shaded triangles lie, the altitude (height) of each shaded triangle os the same as the length of FG, or 22 feet.
The two shaded triangles on the left side of the picture have a combine base of AB, or half of 96, or 48 feet. So, the combined area of those two triangle is (1/2)(48)(22), or 528 square feet.
The three triangle on the right side of the picture have a combined base of 24 feet, as CF is half of BF. So, the combined area of those three triangles is (1/2)(24)(22), or 264 square feet.
The total area of the shaded triangles is 528 + 264, or 792 square feet.
The area of the rectangle is (length)(width), or (96)(22), or 2,112 sq. ft.
The unshaded area can be computed by subtracting the area of the shaded triangles from the area of the rectangle, as follows:
2,112 - 792 = 1,320 square feet.

Category 3
Number Theory
Meet #2 - November, 2023

1) The GCF of A and B is the greatest common factor of A and B.

A = the GCF of 12 and 18
B = the GCF of 9 and 36
C = the GCF of 7 and 10

What is the value of the product ABC ?

2) Peter takes a sip of water every 12 minutes. He takes a bite of pretzel every 18 minutes. He makes a phone call every 30 minutes. At 2:47 P.M., Peter took a sip of water, a bite of pretzel, and made a phone call. The next time he did so was at A:BC P.M. What is the value of the expression 2A + 3B + 7C ?

3) The LCM of A and B is the lowest common multiple of A and B.
Let X = the sum of the different prime factors in the prime factorization of 2,310.
Let Y = the sum of the different prime factors in the prime factorization of 396
What is the LCM of X and Y ?

Answers
1) _______
2) _______
3) _______

Solutions to Category 3
Number Theory
Meet #2 - November, 2023

Answers	
1)	54
2)	71
3)	112

1) A = GCF (12, 18) = 6
   B = GCF (9, 36) = 9
   C = GCF (7, 10) = 1
   The product ABC = (6)(9)(1) = 54.

2) 12 = (2^2)(3)
   18 = (2)(3^2)
   30 = (2)(3)(5)
   The LCM of 12, 18, and 30 is (2^2)(3^2)(5) = (4)(9)(5) = 180.
   180 minutes is exactly three hours. So, Peter performed his three tasks three hours after 2:47 P.M., or at 5:47 P.M.
   A = 5, B = 4, and C = 7.
   Finally, 2A + 3B + 7C = (2)(5) + (3)(4) + (7)(7) = 10 + 12 + 49 = 71.

3) X: 2,310 = (2)(3)(5)(7)(11), so X = 2 + 3 + 5 + 7 + 11 = 28
   Y: 396 = (2)(2)(3^2)(11), so Y = 2 + 3 + 11 = 16
   The LCM of X and Y is (2^4)(7) = (16)(7) = 112

Category 4
Arithmetic
Meet #2 - November, 2023

1) Express the sum of $\frac{3}{4} + 47\% + 5.8$ **as a decimal.**

2) Evaluate. Express your answer as a whole number.

$$\frac{3}{0.3} + \frac{4}{0.4} + \frac{5}{0.5} + \frac{6}{0.6}$$

3) When the repeating decimal 0.3888888 . . . . is expressed as a fraction in lowest terms, then what is the sum of the numerator and denominator?

Answers
1) ______
2) ______
3) ______

Solutions to Category 4
Arithmetic
Meet #2 - November, 2023

1) $3/4 + 47\% + 5.8 = 0.75 + 0.47 + 5.8 = 7.02.$

2) $\frac{3}{0.3}+\frac{4}{0.4}+\frac{5}{0.5}+\frac{6}{0.6} = \frac{30}{3}+\frac{40}{4}+\frac{50}{5}+\frac{60}{6}$

$= 10 + 10 + 10 + 10 = 40.$

3) Let $10X = 3.88888888\ldots$
and $X = 0.38888888\ldots$

Then $9X = 3.5$ and $X = 3.5 / 9 = 35 / 90$
$= 7 / 18$
Finally, the sum of the numerator and denominator is $7 + 18$, or 25.

Answers
1) 7.02
2) 40
3) 25

Category 5
Algebra
Meet #2 - November, 2023

1) At the local Hott Nutz shop, a pound of peanuts costs \$6.50 if a minimum of 20 pounds of peanuts is purchased. Julie spent \$201.50 on peanuts. By how many pounds did Julie's purchase exceed the minimum?

2) The formula for computing D, the number of degrees in one interior angle of an N-sided regular polygon, is $D = \frac{180(N-2)}{N}$.

One interior angle of a particular regular polygon measures 156 degrees. How many sides does the polygon have?

3) If $2E + 5F = 13$
and $3G - 6H = 27$
and $0.5J + 2K = 9$
then what is the value of $J + G + 4E + 10F + 4K - 2H$ **?**

Answers

1) ______

2) ______

3) ______

Solutions to Category 5
Meet #2 - November, 2023

Answers
1) 11
2) 15
3) 53

1) Divide 201.50 by 6.50 to get the number of pounds of peanuts: 201.50 / 6.50 = 31 pounds, thus exceeding 20 pounds by 11 pounds.

2) Substitute 156 for D into the formula:

$$D = \frac{180\,(N-2)}{N}$$

$$156 = \frac{180(N-2)}{N}$$

$$\begin{aligned}
156N &= 180\,(N-2) \\
156N &= 180N - 360 \\
360 &= 180N - 156N \\
360 &= 24N \\
N &= 360 / 24 \\
N &= 15
\end{aligned}$$

So, the polygon has 15 sides.

3) Arrange the six terms in the question into an order that makes consecutive terms comparable to variables of each of the three equations.

$J + G + 4E + 10F + 4K - 2H$	original
$= 4E + 10F \quad + \quad G - 2H \quad + \quad J + 4K$	terms rearranged
$= 2(2E + 5F) + (1/3)(3G - 6H) + 2(0.5J + 2K)$	equivalences
$= (2)(13) + (1/3)(27) + (2)(9)$	
$= 26 + 9 + 18$	
$= 53.$	

Category 6
Team Round
Meet #1 - November, 2023

Each of the following six problems is worth six points.

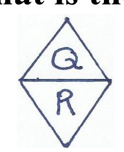
1) Evaluate: $\dfrac{(86 + 2 \times 7)^5 - 91}{3^4}$

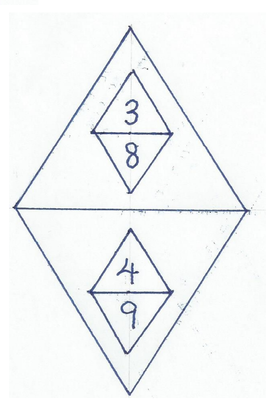
2) The average weight of five capybaras is 110.4 pounds. If no capybara weighs less than 90 pounds and if the difference in the weights of any two capybaras is at least seven pounds, then what is the greatest number of pounds that any one of the capybaras could weigh?

3) A vintage circular phonograph record makes 33 revolutions per minute. How many degrees does it turn in one second?

4) How many of the different divisors (factors) of 144 are composite?

5) The author G.H. Hardy once wrote, "I remember once going to see the mathematician Ramanujan when he was lying ill at Putney. I had ridden in a taxi cab number 1729 and remarked that the number seemed to me a rather dull one and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."
If A and B are two of the numbers whose cubes add to 1729 while C and D are two other such numbers, then what is the sum A + B + C + D?

6) If (Q)(R) + (Q + R) can be represented by 

then what is the value of 

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ANSWERS

1) _________

2) _________

3) _________

4) _________

5) _________

6) _________

Solutions to Category 6
Team Round
Meet #2 - November, 2023

1) $\dfrac{(86+2\times 7)^5-91}{81} = \dfrac{(86+14)^5-91}{81}$

$= \dfrac{(86+14)^5-91}{81} = \dfrac{(100)^5-91}{81}$

$= \dfrac{10{,}000{,}000{,}000\ -\ 91}{81}$

$= \dfrac{9{,}999{,}999{,}909}{81}$

= 123,456,789

2) The total weight of all the capybaras is (110.4)(5), or 552 pounds. If four of weighed the minimum possible weights of 90, 97, 104, and 111 pounds for a total of 402 pounds, then the fifth capybara would weigh 552 - 402, or 150 pounds.

3) At 33 revolutions per minute, divide by 60 to get the number of revolutions per second, or 33 / 60 of a revolution. Then multiply 33 / 60 by 360 to get the number of degrees, or 198 degrees.

4) The divisors (factors) of 144 are as follows:
1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144.
The composite divisors are 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144.
That's 12 composite divisors.

5) 1729 = 1 + 1728 = 1^3 + 12^3 A = 1 and B = 12.
1729 = 729 + 1000 = 9^3 + 10^3 C = 9 and D = 10.
Finally, A + B + C + D = 1 + 12 + 9 + 10 = 32.

6) (3)(8) + (3 + 8) = 24 + 11 = 35
(4)(9) + (4 + 9) = 36 + 13 = 49
(35)(49) + (35 + 49) = 1,715 + 84 = 1,799.

ANSWERS

1) 123,456,789

2) 150

3) 198

4) 12

5) 32

6) 1,799