IMLEM Meet #1
October, 2023

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

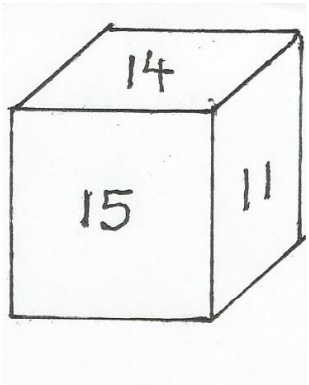
Category 1
Mystery
Meet #1 - October, 2023

1) **It the following pattern repeats continuously, then what is the value of the 83rd letter in the pattern if letters of the alphabet are assigned values, such that A = 1, B = 2, C = 3, etc.?**

A N D O V E R A N D O V E R A . . .

2) **The famous scientist, Mike Roe Scope, noticed while examining a drop of water taken from the Middlesex Fells in Stoneham that two new species of critters that he named Cruds and Plinks. Cruds have four legs while Plinks have seven legs. He counted a total of 59 legs in the water drop. There is more than one of each. How many Plinks were there?**

3) **The numbers on the six faces of this cube are consecutive whole numbers. The sums of the two numbers on each of the three pairs of opposite faces are equal. What is the sum of the six numbers on the cube?**



ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #1 - October, 2023

Answers

1) 5

2) 5

3) 81

1) The word ANDOVER contains seven letters. Divide 83 by 7 to get 11 complete words plus six leftover letters. So, the 83rd letter is E. The value of E is 5, since E is the fifth letter of the alphabet.

2) Since we know the total number of legs but do not know the number of critters, try creating an organized table that keeps track of guesses and checks. Note: the number of critters MUST be a whole number.

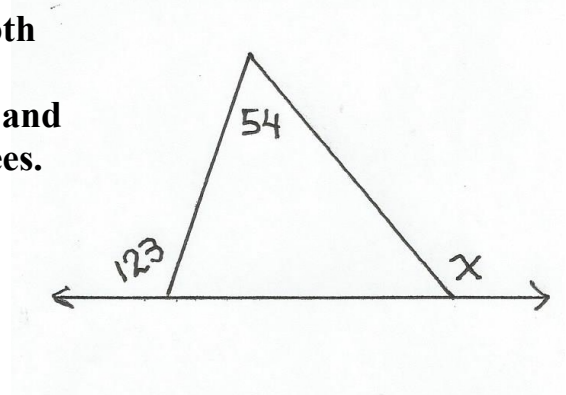
# of Cruds	# of Crud legs	# of Plink legs	Total # of legs
1	4	not a whole #	
2	8	not a whole #	
3	12	not a whole #	
4	16	not a whole #	
5	20	not a whole #	
6	24	35	59

If there are 6 Cruds having 24 legs, then there are 5 Plinks having 35 legs, for a total of 59 legs.

3) The six numbers must be consecutive integers. Opposite faces must have the same sum. If the numbers were 10, 11, 12, 13, 14, 15, then the three equal sums would be 25. However, 14 would have to be opposite 11 and yet they are on adjacent faces and could not be on opposite faces. So, eliminate this possibility. If the numbers were 11, 12, 13, 14, 15, 16, then the three equal sums would be 27, as 11 would be opposite 16, 14 would be opposite 13, and 15 would be opposite 12. This works! So, the sum of the six numbers is 11 + 12 + 13 + 14 + 15 + 16, or 81.

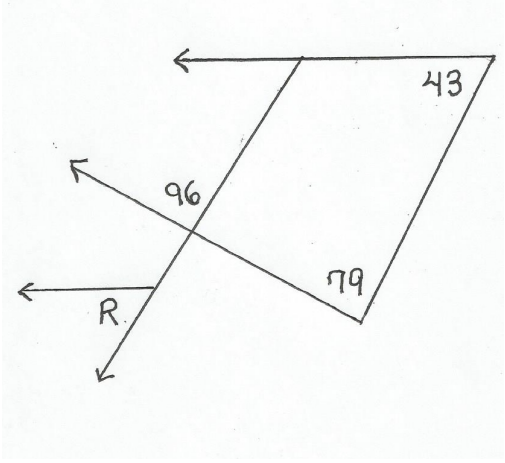
Category 2
Geometry
Meet #1 - October, 2023

1) The base of a triangle is extended in both directions, as shown, with one exterior angle given as measuring 123 degrees and one interior angle measuring 54 degrees. How many degrees are in the exterior angle marked X ?



2) The sum, S, of the interior angles of a convex polygon is given by the formula $S = 180 (N - 2)$ where N = the number of sides of the polygon. A convex nonagon, having nine sides, has two interior angles, each the complement of a 22-degree angle, and three interior angles, each the supplement of a 36-degree angle. The remaining four angles are equal and have the same measure. How many degrees are there in one of these four equal angles?

3) The two horizontal lines, the ones parallel to the bottom edge of this paper, are parallel. Three angles are given as 43, 79, and 96 degrees. How many degrees are in the measure of angle R ?



Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #1 - October, 2023

Answers
1) 111
2) 173
3) 26

1) The interior angle adjacent to the 123-degree angle is its supplement, so 180 - 123 = 57 degrees. The third remaining angle of the triangle measures 180 - (57 + 54), or 69 degrees. Its supplement is X, so 180 - 69 = 111 degrees.

2) The total number of degrees among the interior angles of the nonagon is 180 (9 - 2), or 180 (7) = 1260 degrees.
There are five interior angles whose measures can be calculated thusly:
- **the two angles that are complements of a 22-degree angle are each 90 - 28, or 68 degrees.**
- **the three angles that are supplements of a 36-degree angle are each 180 - 36, or 144 degrees.**

The sum of the four remaining angles is the difference between the sum of all nine angles and the sum of the angles we just calculated:
= 1260 - [2(68) + 3(144)]
= 1260 - [136 + 432]
= 1260 - 568
= 692 degrees.
Now divide 692 by 4 to get the degree measure of one of the four remaining equal angles: 692 / 4 = 173 degrees.

3) The angle adjacent to the 96-degree angle is its supplement and has 180 - 96, or 84 degrees. The sum of the measures of the quadrilateral is 360 degrees, so the remaining angle of the quadrilateral is 360 - (84 + 43 + 79), or 360 - 206, or 154 degrees. The angle adjacent to it is its supplement, so 180 - 154 = 26 degrees.
Because the two horizontal lines are parallel, this 26-degree angle has the same measure as the angle marked R, as the corresponding angles are equal. So, R = 26.

Category 3
Number Theory
Meet #1 - October, 2023

1) **There are four prime numbers between 20 and 40. What is the sum of those four prime numbers?**

2) **Two distinct (different) prime numbers are less than 70. What is the greatest possible product of these two primes?**

3) **There are many whole numbers between 43,610 and 47,282 that are divisible by 15 and 4. What is the difference between the smallest and largest of those whole numbers?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2023

1) The sum of the prime numbers between 20 and 40 is 23 + 29 + 31 + 37 = 120.

2) For the two prime numbers less than 70 to produce the maximum product, they must be in the vicinity of the number 70. They are 61 and 67. Their product is (61) (67) = 4087.

3) Since 15 and 4 are relatively prime, thus having a GCF of 1, we are looking for multiples of (15) (4), or 60, that are the first one greater than 43,610 and the last one smaller than 47,282. Those two numbers are divisible by 3, 4, and 5 and are 43,620 and 47,280. Their difference is 47,280 - 43,620 = 3660.

Answers

1) 120

2) 4087

3) 3660

Category 4
Arithmetic
Meet #1 - October, 2023

1) Find the value of the following expression, according to the correct of operations.

$$7 + 9 \times 2 - 5 - 3 \times 4$$

2) A = the mean of the even numbers between 37 and 51.
B = the median of the composite numbers between 53 and 67.
C = the mode of the digits used in the ten-digit number 7,261,218,052.
What is the value of the sum A + B + C ?

3) For the first six weeks of the summer, at his part-time job, Elias earned weekly paychecks of 64, 57, 77, 56, 81, and 70 dollars. What is the least number of dollars that he must average over the final three weeks of the summer so that he can afford to purchase a $513 laptop that he needs when he starts the second quarter of school in November?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2023

Answers
1) 8
2) 106
3) 36

1) $7 + 9 \times 2 - 5 - 3 \times 4$
$= 7 + 18 - 5 - 12$
$= 8.$

2) $A = (38 + 40 + 42 + 44 + 46 + 48 + 50) / 7$
$= (308) / 7$
$= 44.$

$B =$ the middle number in the set
{ 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66 }
$= 60.$

$C =$ the numeral that appears the most among the ten digits
$= 2.$

$A + B + C = 44 + 60 + 2 = 106.$

3) Add the six weekly paychecks: $64 + 57 + 77 + 56 + 81 + 70 = 405.$
Subtract 405 from the \$513 that the laptop will cost: $513 - 405 = 108.$
Finally, divide 108 by 3 to get the average of thos final three weeks' paychecks: $108 / 3 = 36.$
So, Elias must earn an average of \$36 per week for the final three weeks of the summer.

Category 5
Algebra
Meet #1 - October, 2023

1) If $7A = 28$ and $3B + 5 = 26$ and $6C - 9 = 21$, then what is the value of $8B - 4C + 11A$?

2) Situation #1: $10A + 1 = 7(2A - 5)$

Situation #2: $3C^2 = 75$

Situation #3: The following equation is an identity in N:

$$4N - 7 + W = -2(3N + 4) + 10N$$

Finally, find the value of X if

X = (the smaller value of C)(the larger value of C) - AW

3) What value of N makes the following equation a true statement?

$$3(2N + 1) + 4(7N - 5) = 10(6N - 5) - 7(8N - 50) + 43$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2023

Answers	
1)	80
2)	- 16
3)	12

1) $7A = 28$, so $A = 4$
$3B + 5 = 26$, so $3B = 21$ and $B = 7$.
$6C - 9 = 21$ so $6C = 30$ and $C = 5$.
Finally, $8B - 4C + 11A$
$= 8(7) - 4(5) + 11(4)$
$= 56 - 20 + 44$
$= 80$.

2) $10A + 1 = 7(2A - 5)$
$10A + 1 = 14A - 35$
$36 = 4A$
$9 = A$

$3C^2 = 75$
$C^2 = 25$
$C = 5$ or $- 5$

$4N - 7 + W = -2(3N + 4) + 10N$
$4N - 7 + W = - 6N - 8 + 10N$
$10N + W = - 1 + 10N$
$W = - 1$

$X = (- 5)(5) - (9)(- 1)$
$= - 25 + 9$
$= - 16$

3) $3 (2N + 1) + 4 (7N - 5) = 10 (6N - 5) - 7 (8N - 50) + 43$
$6N + 3 + 28N - 20 = 60N - 50 - 56N + 350 + 43$
$34N - 17 = 4N + 343$
$30N = 360$
$N = 12$

Category 6
Team Round
Meet #3 - October, 2023

Each of the following NINE problems is worth <u>four</u> points.

1) For how many integers between 10 and 100 is the tens digit equal to 3, 7, or 8 and the units digit (ones digit) is equal to 1, 2, 5, or 9 ?

2) A + B + C = 4 and D + E = 5.
What is the value of AE + CE + BD + BE + CD + AD ?

3) The sum of five consecutive odd integers is 365. What is the product of the smallest and largest of these five numbers?

4) Seven non-collinear lines intersect on the same plane. What is the maximum possible number of intersection points?

5) In a certain quadrathalon, athletes compete in four events. Regina runs three times farther than she swims, bicycles four times farther than she runs, and hops four miles fewer than she swims. How many miles does Regina run? (The total distance for the entire race is 81 miles.)

6) What is the largest prime factor of the sum of the two largest two-digit prime numbers?

7) How many different ways can 73 be written as the sum of two prime numbers?

8) What is the only integer less than 100 whose three prime factors have a sum of 14 ?

9) If three different numbers from the set { - 3, - 2, - 1, 4, 5 } are multiplied, then what is the largest possible product?

<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

Solutions to Category 6
Team Round
Meet #3 - October, 2023

ANSWERS	
1)	12
2)	20
3)	5313
4)	21
5)	15
6)	31
7)	1
8)	70
9)	30

1) Each of the three tens digits is paired with each of the units digits to create 3 x 4, or 12 possible two-digit numbers.

2) $AE + CE + BD + BE + CD + AD$
$= (AE + BE + CE) + (AD + BD + CD)$
$= E(A + B + C) + D(A + B + C)$
$= (E + D)(A + B + C)$
$= (5)(4)$
$= 20$

3) $X + (X + 2) + (X + 4) + (X + 6) + (X + 8) = 365$
$5X + 20 = 365$
$5X = 345$
$X = 69$

Then the smallest, 69, and the largest, 77, have a product of (69)(77), or 5313.

4) Look for a pattern as increasing numbers of lines intersect:

lines	points of intersection
2	1
3	3
4	6
5	10
6	15
7	21

Students may recognize these as "triangular" numbers.

5) Let the following variable expressions represent the various distances, in miles, covered: X = swim, X - 4 = hop, 3X = run, 4(3X) = bicycle.

$X + (X - 4) + 3X + 4(3X) = 81$
$17X - 4 = 81$
$17X = 85$
$X = 5$

Regina runs 3X, or 3(5), or 15 miles.

SOLUTIONS TO PROBLEMS #6-9 are on the next page.

6) The sum of the two largest two-digit prime numbers is 89 + 97, or 186.
The prime factorization of 186 is 186 = 2 x 3 x 31.
So, the largest prime factor of 186 is 31.

7) 73 = 2 + 71. That's it!! 73 must be the sum of one even and one odd prime number. Since 2 is the only even prime, all other primes are odd. The sum of two odd numbers is even.

8) If the sum of three primes is 14, then one of them must be even. Two is the only even prime. So, the other two primes must have a sum of 12. The only possibility is that 5 + 7 = 12.
Therefore 2 x 5 x 7 = 70.

9) Two of the integers must be negative. So, (- 3)(- 2)(5) produces the largest possible product of three of the integers. The answer is 30.