IMLEM Meet #4
February, 2023

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery
Meet #4 - February, 2023

Calculator Meet

1) What is the smallest positive integer that, when multiplied by the decimal 17.26, will yield a product that is greater than 1000?

2) The five-digit number 2018N is a multiple of 9. What is the remainder when this five-digit number is divided by 8?

3) Felix chose a number on day 1 and multiplied it by some other number on day 2. Every day, he multiplied the result he got on the previous day by the same number he multiplied on day 2. On day 7, the result was 5,832. On day 12, the result was 1,417,176. What number did he choose on day 1?

Answers

1) ______

2) ______

3) ______

Solutions to Category 1
Mystery
Meet #4 - February, 2023

Answers
1) 58
2) 3
3) 8

1) First, divide 1000 by 17.26. That quotient is 57.937 . . . The question asks for the smallest INTEGER that, when multiplied by 17.26, yields a product greater than 1000. The integer 57 is insufficient, yielding a product that is less than 1000. However, 58 is the smallest integer that yields a product greater than 1000.

2) The sum of the digits must be a multiple of 9. The sum of the given four digits is 2 + 0 + 1 + 8 = 11.
If N = 7, then the sum of all five digits is 18, a multiple of 9. To get to the next multiple of 9, or 27, N would have to be 16 which is obviously too large.
Divide the five-digit number 20187 by 8 to get a quotient of 2,532 with a remainder of 3. Therefore, the remainder is 3.

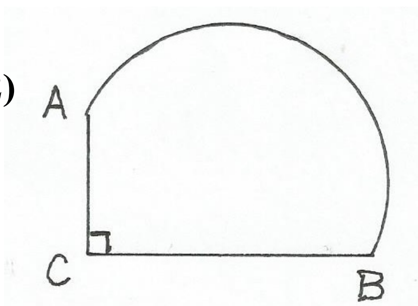
3) Both the 7th and 12th terms are given as 5,832 and 1,417,176 respectively.
Divide 1,417,176 by 5,832 to get 243. This number is the result of five factors of the chosen multiplier that gets from the 7th term to the 12th term. Take the fifth root of 243 to get the number 3. Now check that
the 7th term, 5,832, times 3 = the 8th term, or 17,496 and
the 8th term, 17,496, times 3 = the 9th term, or 52,488 and
the 9th term, 52,488, times 3 = the 10th term, or 157,464 and
the 10th term, 157,464 times 3 = the 11th term, or 472,393 and
the 11th term, 472,393 times 3 = 1,417,176.
Now, go backwards from the 7th term to the first term, first dividing the 7th term by 3, then the sixth term by 3, and so on:
5,832 / 3 = 1,944 (6th term)
1,944 / 3 = 648 (5th term)
648 / 3 = 216 (4th term)
216 / 3 = 72 (3rd term)
72 / 3 = 24 (2nd term)
24 / 3 = 8 (1st term).
Therefore, the number that Felix chose on day 1 is 8.

Category 2
Geometry
Meet #4 - February, 2023

Calculator Meet

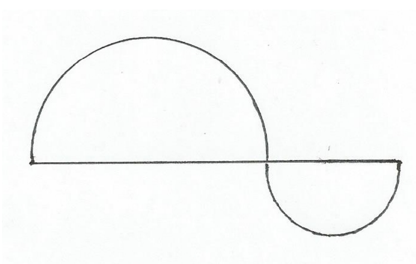
1) The Lexington Field House is a 60-year-old sports facility that includes an indoor circular track with a radius of 46 yards. If Regina runs three laps around the track, then how many yards does she run? Use $\pi \approx 3.14$. Round your final answer to the nearest tenth of a yard.

2)



Arc AB is a semi-circle. AC = 16 cm. BC = 30 cm. How many square centimeters are contained in the figure? Use $\pi \approx 3.1$. Round your final answer to the nearesr whole number.

3)



The diameters of these two semi-circles lie in a straight line and have a sum of 24 inches. Consider the perimeter of the entire figure, including the diameters and the semi-circles. If it is in the form $J + K\pi$, then what is the value of J + K?

Answers

1) ________ yards

2) ________ sq. cm

3) ________ inches

Solutions to Category 2
Geometry
Meet #4 - February, 2023

Answers
1) 866.6
2) 688
3) 36

**1) One lap around the track = one circumference = 92 pi.
(If the radius is 46, then the diameter is twice the radius and the circumference is (pi)(diameter).
Regina runs three laps, so her distance is three times a circumference: (3)(pi)(diameter) = (3)(3.14)(92)
= 866.64 yards. Round to the nearest tenth of a yard to get 866.6 yards.**

**2) The diameter of the semi-circle is the hypotenuse of the right triangle ABC. Use the Pythagorean Theorem to calculate the length of the hypotenuse: 16^2 + 30^2 = hypotenuse ^2
Then 256 + 900 = H^2 and 1156 = H^2 so H = 34 and the radius is 17.
The area of the figure is (area of right triangle) + (area of semi-circle)
= (1/2)(base)(altitude) + (1/2)(pi)(square of radius)
= (1/2)(30)(16) + (1/2)(3.1)(17^2)
= 240 + 447.95
= 687.95.
Rounding to the nearest whole number yields an answer of 688 square centimeters.**

**3) The perimeter of the figure is equal to the sum of the lengths of the two diameters plus the lengths of the two semi-circles. Interestingly . . . and the students can discover this through experimenting . . . it doesn't matter what the lengths of the two diameters is, as long as their sum is 24. So, the perimeter is D1 + D2 + C1 + C2
= (D1 + D2) + (C1 + C2)
= (24) + [(1/2)(pi)(D1)] + [(1/2)(pi)(D2)]
= 24 + [(1/2)(pi)] (D1 + D2)
= 24 + [(1/2)(pi)] (24)]
= 24 + 12 pi.
Then J + K = 24 + 12 = 36.**

Category 3
Number Theory *Calculator Meet*
Meet #4 - February, 2023

1) A day on planet Sacco, in a galaxy far far away, lasts 16 hours. Astronaut Albert is performing his daily tasks on Sacco, collecting rock and soil samples and hitting golf balls. At 13 o'clock, he deposits soil samples into a machine that analyzes its contents. Seventy-four hours later, the results are available. At what time are the results available?

2) The following series is the sum of all multiples of 7 from 112 through 1,806, inclusive:

112 + 119 + 126 + . . . + 1,792 + 1,799 + 1,806

What is the value of that sum?

3) A geometric sequence is a string of numbers, such that the ratio of any two consecutive terms is the same as the ratio of any other two other consecutive terms. Find the sum A + B + C + D for the following geometric sequence:

8 24 72 A 648 B C 17,496 D

Answers

1) ________

2) ________

3) ________

Solutions to Category 3
Number Theory
Meet #4 - February, 2023

Answers
1) 7
2) 233,037
3) 60,480

1) Divide 74 by 16 to determine how many days and leftover hours lapsed whole Albert performed his tasks.
74 / 16 = 4 days with 10 hours left over.
Now add 10 hours to the 13 o'clock start = 23, which is 7 o'clock on the 16-clock.

2) Subtract the sum of all multiples of 7 from 7 through 105, inclusive (let's call that A) from the sum of all multiples of 7 from 7 through 1,806, inclusive (let's call that B).
105 is the 15th multiple of 7 while 1,806 is the 258th multiple of 7.
A = (7 + 105)(1/2)(15) = (112)(1/2)(15) = 840.
B = (7 + 1,806)(1/2)(258) = (1,813)(1/2)(258) = 233,877.
Finally, B - A = 233,877 - 840 = 233,037.

3) Dividing any two known consecutive terms gives us the common ratio, for example, 24 / 8 = 3.
(72)(3) = A = 216
Now checking that (216)(3) = 648 . . . and it does.
Then (648)(3) = 1,944 = B
(1,944)(3) = 5,832 = C
Now checking that (5,832)(3) = 17,496 . . . and it does.
Then (17,496)(3) = 52,488 = D.
Finally, A + B + C + D = 216 + 1,944 + 5,832 + 52,488 = 60,480.

Category 4
Arithmetic *Calculator Meet*
Meet #4 - February, 2023

1) When Jolene was in the 2nd grade, she was 45 inches tall. Now in 7th grade, she is 63 inches tall. She had grown X% since 2nd grade. What is the value of X ?

2) Judd earns a monthly paycheck that consists of a fixed salary plus a commission based on how many dollars worth of shoes he sells. He earned an 8% commission this month for selling $13,620 worth of shoes. His total paycheck this month is $1,723.60. How many dollars was his fixed salary?

3) The formula for compound interest is $A = P(1 + r)^t$ **provided that**
P = the principal, or amount invested,
r = the annual (yearly) rate at which the principal is invested,
t = the amount of time, in years, that the money is invested, and
A = the total amount of money, including the principal and the interest.
Lynda found her great-grandfather's bank book in a box in the attic. His savings account on February 14, 1925 had a balance of $6.17. Assuming an average rate of 5% over the course of time, how many dollars is the savings account worth on February 14, 2023? Round your answer to the nearest dollar.

Answers

1) ________

2) ________

3) ________

Solutions to Category 4
Arithmetic
Meet #4 - February, 2023

Answers	
1)	40
2)	634
3)	736

1) First find the amount of increase, then divide by the original number.
   Amount of increase: 63 - 45 = 18.
   18 / 45 = 0.4 = 40%
   The percent of increase is 40%, so in the expression X%, X = 40.

2) The amount of commission is 8% of $13,620, or (0.08)(13,620) = $1,089.60.
   Fixed salary + the commission = Total paycheck.
   Fixed salary + 1,089.60 = 1,723.60
   Fixed salary = 1,723.60 - 1,089.60
   Fixed salary = $634.

3) P = $6.17.
   r = 5% = 0.05.
   t = 2023 - 1925 = 98 years.
   A = P(1 + r)^t
   = (6.17)(1 + 0.05)^98
   = (6.17)(1.05)^98
   = (6.17)(119.275517 . . .)
   = 735.9299 . . .
   Rounded to the nearest dollar yields $736.

Category 5
Algebra *Calculator Meet*
Meet #4 - February, 2023

1) On a recent walk, Jean burned 72 calories after walking for 18 minutes. If she continued walking at that same pace, how many total calories would she have burned if she walked for a total of 99 minutes?

2) On a 60-question test, four points are awarded for each correct answer while one point is deducted (subtracted) for each incorrect answer. Quizno scored 175 points on his test. How many questions did he answer correctly if he answered every question on the test?

3) My favorite cantaloupe weighs $\frac{5}{6}$ of its own weight plus $\frac{5}{6}$ of a pound. How many pounds does my favorite cantaloupe weigh? (a classic from the very first IMLEM competition in 1963)

ANSWERS

1) _______ calories

2) _______

3) _______ pounds

Solutions to Category 5
Algebra
Meet #4 - February, 2023

Answers
1) 396
2) 47
3) 5

1) Per minute, Jean burned 72 / 18, or 4 calories.
   For the next 81 minutes (99 - 18), she burned
   (81)(4), or 324 calories.
   In all, she burned 72 + 324, or 396 calories.

2) Let C = the number of correct answers
   I = the number of incorrect answers.
   Then this system of equations represents the
   stated facts: $C + I = 60$ and $4C - I = 175$
   Adding the two equations produces $5C = 235$ as the Is "cancel out."
   So, $C = 235 / 5$ or 47 and $I = 13$.
   Quizno answered 47 questions correctly.

3) Let W = the weight, in pounds, of the cantaloupe.
   Then

$$\begin{aligned} (5/6)W + 5/6 &= W \\ 5/6 &= W - (5/6)W \\ 5/6 &= (1/6)W \\ (5/6) / (1/6) &= W \\ 5 &= W \end{aligned}$$

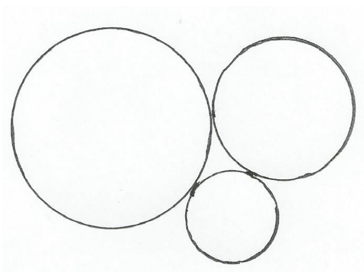
   So, the cantaloupe weighs 5 pounds.

Category 6
Team Round
Meet #4 - February, 2023

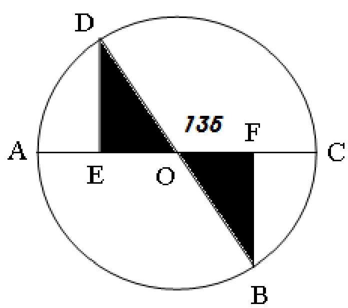
Each of the following six problems is worth six points.

1) The variables X, Y, and Z are positive integers such that XYZ = 1,155. What is the smallest possible value of the sum X + Y + Z ?

2) Brielle is tiling the rectangular floor of a 72 foot by 216 foot recreation center. She uses one-foot by one-foot square tiles to form the border along the edges of the floor (as part of the 72 by 216 foot floor) and 2-foot by 2-foot square tiles to cover the rest of the floor. How many tiles did she use in all?

3) 

Three circles are mutually tangent, as shown. Their diameters are 8, 16, and 24 meters, respectively. How many square meters are in the polygon formed by connecting the centers of the three circles?

4) 

In the circle at the left, O is the center. DE and BF are each perpendicular to AC. AC and DB are diameters, each with length 52 feet. How many square feet are in the shaded area? Angle DOC = 135 degrees.

5) Neptunia captured some sea creatures at the beach and noticed that a) the ratio of sand dollars to periwinkles was 3:8, and b) the ratio of sand dollars to fiddler crabs was 4:7. If there were 60 more periwinkles than sand dollars, then how many fiddler crabs were there?

6) Using the letters A through E to represent the answers to #1-5, respectively, evaluate the following expression:

$$\sqrt{\sqrt[3]{B^2 + AD^2 + (C - E)^3 + 53CD - 24{,}653}}$$

ANSWERS

1) _______ = A

2) _______ = B

3) _______ = C

4) _______ = D

5) _______ = E

6) __________

Solutions to Category 6
Team Round
Meet #4 - February, 2023

ANSWERS	
1)	33
2)	4,317
3)	96
4)	338
5)	63
6)	17

1) First, find the prime factorization of 1,155. There are four prime factors, such that 1,155 = 3 x 5 x 7 x 11. Then try various combinations to discover how to minimize the sum. For example, combining 5x7 with 3 and 11 to make the sum 35 + 3 + 11 = 49 is larger than other sums. The one that works is to multiply the smallest factors to create the smallest product to add to 7 and 11. So, the sum 15 + 7 + 11 = 33 is the smallest possible sum.

2) Since the four corners of the floor share a 1x1 tile, we cant just add the two lengths and two widths. We must subtract four tiles for having "double counted" the corners. The remaining space is 70 by 214 and will accommodate many 2x2 tiles.
So, 1x1 tiles: 72 + 72 + 216 + 216 - 4 = 572 tiles.
2x2 tiles: (half of 70)(half of 214) = (35)(107) = 3,745 tiles.
Total number of tiles = 572 + 3,745 = 4,317 tiles.

3) If the three diameters are 8, 16, and 24, respectively, then their radii are 4, 8, and 12, respectively. Connecting the three centers produces a triangle of side lengths 12, 16, and 20 which make three sides of a right triangle, verifiable via the Pythagorean Theorem. So, the area of that right triangle is half the product of the two legs that form the right angle: (1/2)(12)(16) = 96 square meters.

4) Each base angle of the two right triangles is the supplement of the 135-degree angle, or 45 degrees, making the two triangles isosceles right triangles. Half the diameter of DB (52) is 26, or the length of the hypotenuse of each triangle. Let X = the length of each leg of the triangles. The area of each triangle is (1/2)(X)(X), or (1/2)X^2. Therefore, the total area of the two triangles is (2)(1/2)X^2, or X^2.

SEE NEXT PAGE FOR THE REMAINDER OF THE SOLUTIONS.

For each triangle, using the Pythagorean Theorem, X^2 + X^2 = 26^2.
So, 26^2 is twice the total area. Take half of 26^2, or half of 676, or 338 square feet.

5) Let P = the number of periwinkles
S = the number of sand dollars
F = the number of fiddler crabs

Then S:P = 3:8 and S:F = 4:7 and S + 60 = P

Cross products are equal: 3P = 8S

Substitute S + 60 for P:
$$3(S + 60) = 8S$$
$$3S + 180 = 8S$$
$$180 = 5S$$
$$36 = S$$

Again, cross products are equal: 4F = 7S

Substitute 36 for S:
$$4F = 7(36)$$
$$4F = 252$$
$$F = 63$$

Therefore, there are 63 fiddler crabs.

6)
$$\sqrt{\sqrt[3]{B^2 + AD^2 + (C - E)^3 + 53CD - 24,653}}$$

$$= \sqrt{\sqrt[3]{(4,317)^2 + (33)(338)^2 + (96 - 63)^3 + 53(96)(338) - 24,653}}$$

$$= \sqrt{\sqrt[3]{18,636,489 + (33)(114,244) + (33)^3 + 1,719,744 - 24,653}}$$

$$= \sqrt{\sqrt[3]{18,636,489 + 3,770,052 + 35,937 + 1,719,744 - 24,653}}$$

$$= \sqrt{\sqrt[3]{24,137,569}}$$

$$= \sqrt{289}$$

$$= 17$$