## IMLEM Meet #1 October, 2022

## Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- Many of you are guests in someone else's school please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.
- No calculators (or only scientific calculators allowed for meets #4, #5)
- Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1 Mystery Meet #1 - October, 2022

1) It the following pattern repeats continuously, then what is the value of the 83rd letter in the pattern if letters of the alphabet are assigned values, such that A = 1, B = 2, C = 3, etc.?

NATICKNATICKNAT...

2) Lizzie and her younger sister, Simonne, were born on the same date, seven years apart. Five years after Simonne was born, her younger brother, Scott was born on the same date. Lizzie was born in the year 1963. On their shared birthdays this year in 2022, how many years older will Lizzie be than Scott?

3) The figure below consists of small cubes. How many of the these cubes are there? Assume that, if there is a "hole" on any surface, then that hole goes all the way through the figure.





Solutions to Category 1 Mystery Meet #1 - October, 2022

- 1) The word NATICK contains six letters. Dividing 83 by 6 yields a quotient of 13 with a remainder of 5, so that NATICK appears 13 times in the pattern to the 78th letter. The fifth letter after that, or the 83rd letter in all, is the letter C and has a value of 3.
- 2) When Scott was born, Lizzie was 12 years older than him. On every birthday thereafter, Lizzie will still be 12 years older than Scott, including in 2022.
- 3) If the entire figure were uninterrupted by the holes, the total number of cubes would be (4)(4)(6) = 96 cubes. The holes intersect in the interior of the figure, so we have to be careful about counting the ones missing. The hole in the front (2 cubes high) goes all the way to the back, thus eliminating 8 cubes. We are now down to 88 cubes. The hole on the right side (also 2 cubes high) implies another gap of 8 cubes. However, two of them were eliminated from our first count. So, we subtract six more cubes to bring our new total to 82 cubes. The hole of a single cube on the top implies a gap of 6 cubes, but two of them were already counted. So, we subtract 4 more cubes which brings us to 78 cubes.



Category 2 Geometry Meet #1 - October, 2022

- 1) Two sides of a triangle are each extended. How many degrees are in the measure of external angle A ?
- 2) In a convex hexagon (6-sided polygon), one interior angle is the complement of a 15-degree angle while another is the supplement of a 19-degree angle. The remaining four interior angles have equal measures. How many degrees are in one of the four equal angles?

$$\begin{array}{c|c} \textbf{3} & & \textbf{B} \\ \hline \textbf{A} & & 123 \\ \hline \textbf{D} \\ \hline \textbf{I10} \\ \hline \textbf{C} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{E} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \textbf{C} \\ \textbf{C} \\ \textbf{C} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\ \end{array} \\ \begin{array}{c} \textbf{B} \\ \textbf{C} \\$$

Line segment AB is parallel to line segment DE. Degree measures of angles are as written on the diagram. What is the measure of angle C if it is less than 180 degrees?

<u>Answers</u>		
1)		
2)		
3)		

Solutions to Category 2 Geometry Meet #1 - October, 2022

- 1) The external angle A is equal to the unlabelled interior angle to which it is opposite, or "vertical." The measure of angle A is 180 - (106 + 57), or 180 - 163, or 17 degrees.
- 2) The sum of the measures of the interior angles of the convex hexagon is 180 (6 2), or 720 degrees (the four triangles into which it can be divided, with diagonals emanating from a single vertex.) The complement of a 15-degree angle is 90 15, or 75 degrees. The supplement of a 19-degree angle is 180 19, or 161 degrees. The sum of the four other angles is 720 (75 + 161), or 720 236, or 484. Divide by 4 to get the measure of one of the four equal remaining angles: 484 / 4 = 121. Therefore, the measure of one of the four equal angles is 121 degrees.

Answers

17

121

1)

2)

3) One strategy is to draw a line through C that is parallel to the other two parallel lines, thus creating pairs of angles that are either congruent or supplementary, as shown in this diagram:



The three angles at the bottom of the diagram form a straight angle, so their sum is 180 degrees. 57 + C + 70 = 180, so C = 53.

Category 3 Number Theory Meet #1 - October, 2022

1) There are five prime numbers between 30 and 50. What is the sum of those five prime numbers?

2) A six-digit number 2WW,095 is divisible by 15 where W is the value of both the ten-thousands and thousands digits. There is more than one possible value of W that makes this possible. What is the positive difference between the largest and smallest possible values of W?

3) What is the value of T if T is

- \* a whole number
- \* divisible by 24
- \* greater than 300
- \* not divisible by 17
- \* less than 400
- \* divisible by 21

and the sum of its digits is 12?



Solutions to Category 3 Number Theory Meet #1 - October, 2022

- 1) The sum of the prime numbers between 30 and 50 is 31 + 37 + 41 + 43 + 47 = 199.
- 2) For the 6-digit number to be divisible by 15, it must be divisible by 5 and 3. Since the units digit is 5, the entire number is divisible by 5. For the number to be divisible by 3, the sum of its digits must be a multiple of 3. The sum of the given digits is 2+0+9+5 = 16. so, 2W + 16 must be a multiple of 3. This works when W = 1 or W = 4 or W = 7. The difference between the largest and smallest possible values of W is 7 - 1, or 6.



3) The most helpful clues are that T is divisible by both
21 and 24. The only number between 300 and 400 that
is divisible by both 21 and 24 is 336. Simple division will show that 336
is not divisible by 17 and the sum of the digits is 3+3+6 = 12.

Category 4 Arithmetic Meet #1 - October, 2022

1) Find the value of the following expression, according to the correct order of operations.

 $13 + 7 \times 2 - 3 \times 5 - 8 + 6$ 

2) The arithmetic mean (average) of the four whole numbers 114, 231, 426, and 589 is equal to the arithmetic mean of the three whole numbers 362, 649, and P. What is the value of P?

3) Set M = { 26, 15, 35, 26, 53, 71, 53, 75, 26 } Set A = { 62, 87, 46, 17, 87, 52, 62, 87, 29 } T = the median of Set M H = the mode of Set A What is the value of the mean of T and H?



**Solutions to Category 4** Arithmetic **Meet #1 - October, 2022** 

1) $13 + 7 \times 2 - 3 \times 5 - 8 + 6$ - 13 + 14 15 8 + 6	first multiply	<u>An</u>	swers
- 13 + 14 - 13 - 8 + 0	subtract, from	1)	10
= 10	icit to right	2)	9

2) In order for the two sets of numbers to have the same average, then (114 + 231 + 426 + 589) / 4 = (362 + 649 + P) / 3340 = (1011 + P) / 31020 = 1011 + PP = 9

3) 61

3) To locate the median of Set M, first arrange the numbers in order. Then select the number in the middle of the set. For set M, that number is 35. The mode of set A is the number that appears the most frequently. For set A, that number is 87. So, T = 35 and H = 87. The mean of sets T and H is the average of 35 and 87, or (35+87)/2 = 122/2 = 61.

Category 5 Algebra Meet #1 - October, 2022

1) If 6C + 7 = 85 and 5D - 25 = -260, then what is the value of -5CD ?

2) The letters of the alphabet, { A, B, C, ..., Z }, correspond to the consecutive integers { -9, -8, -7, -6, ..., 16 }. For example, A = -9, B = -8, M = 3, N = 4, and so on. The number, zero, is included among the integers. Find the sum of the numbers corresponding to the letters in the following word:

## **OCTOBER**

3) Find the value of E if 2(E+3) + 7(E-9) - 4(E-6) = 21 - 3(E+2)



Solutions to Category 5 Algebra Meet #1 - October, 2022

- 1) 6C + 7 = 85, so 6C = 78 and C = 13. 5D - 25 = -260, so 5D = -235 and D = -47Finally, -5CD = (-5)(13)(-47) = 3055.
- 2) O = 5, C = -7, T = 10, O = 5, B = -8, E = -5, and R = 8. Then the value of OCTOBER is 5 + (-7) + 10 + 5 + (-8) + (-5) + 8 = 8.

3) 
$$2(E+3) + 7(E-9) - 4(E-6) = 21 - 3(E+2)$$
  
 $2E + 6 + 7E - 63 - 4E + 24 = 21 - 3E - 6$   
 $5E - 33 = 15 - 3E$   
 $8E = 48$   
 $E = 6$ 

Answers		
1)	3055	
2)	8	
3)	6	

Category 6 Team Round Meet #1 - October, 2022

Each of the following six problems is worth <u>six</u> points.

- 1) What is the value of Y that makes the following equation an identity? 9(5x + 3) - 4(7x + 6) - 10 = 2(8x - 11) + Y + x
- 2) The numbers in the pattern 1, 3, 6, 10, 15, etc., are known as "triangular numbers." If the sum of three consecutive triangular numbers is 514, then what is the largest of these three consecutive triangular numbers?
- 3) What is the sum of all the prime numbers between 160 and 180?
- 4) A test contains 41 questions. Some are worth 2 points and the rest

are worth 3 points. If a perfect score is 100 points, then how many questions are worth three points?

5) In the image to the right, there are seven bowling pins. A perfect score is 100 points to be achieved by knocking down just some of the pins. This can be done in only one way. What is the positive difference between the highest and lowest values of the winning combination? (from the Internet site "Mind Stretchers")



6) The only way to purchase Dover Donuts is to buy one or more boxes that



each contains six donuts. If a box of donuts costs \$4.79, then what is the least amount of money that Duncan must spend so that each member of his seventh grade class receives exactly two donuts? There are 172 students in his class, including Duncan. Express your answer as a decimal number of dollars. Solutions to Category 6 Team Round Meet #1 - October, 2022

<u>ANSWERS</u>	1) $9(5x + 3) - 4(7x + 6) - 10 = 2(8x - 11) + Y + x$ 45x + 27 - 28x - 24 - 10 = 16x - 22 + Y + x
1) 15	45x + 27 - 26x - 24 - 10 = 10x - 22 + 1 + x $17x - 7 = 17x - 22 + Y$
2) 190	-7 = -22 + Y Y = 15
3) 682	2) Extend the list of triangular numbers until
4) 18	numbers is 514: 1 3 6 10 15 21 28 2(-45-55)((-78-01-105-120-12)(-152))
5) 35	56 45 55 66 78 91 105 120 156 155 171 190. 153+171+190=514. The largest is 190.
6) 277.82	3) One need test the odd numbers between 160 and 180 for divisibility to eliminate the

Divisible by 5: 165, 170, 175. Divisible by 3 (not also by 5 or 2): 171, 177. Divisible by 7 (not also by previous): 161 Divisible by 11: none Divisible by 13: 169 There is no need to investigate larger possible prime factors, as their squares are larger than 180. So, the sum of the remaining odd numbers, all prime, is 163 + 167 + 173 + 179 = 682.

composite numbers. Those left are prime.

- 4) An algebraic solution is possible. For the younger or less-experienced students who have not yet studied systems of equations, alternative solutions include making an organized list and looking for patterns. Either way, there are 23 two-point questions and 18 three-point questions. Check: 23(2) + 18(3) = 46 + 54 = 100 points. Therefore, there are 18 three-point questions.
- 5) This one is tricky because it is not stated HOW MANY of the pins are to be used. Some guessing and checking options may be reduced by seeking some combination of units digits whose sum is a multiple of ten. The winning combination is 13 + 39 + 48 = 100. The difference between the highest and lowest values is 48 13 = 35.
- 6) The number of donuts needed is  $172 \times 2$ , or 344 donuts. Divide 344 by 6 to get the minimum number of boxes needed. 344/6 = 57 with remainder 2. Thus, it is necessary to buy 58 boxes. The total cost is \$4.79 x 58, or \$277.82.