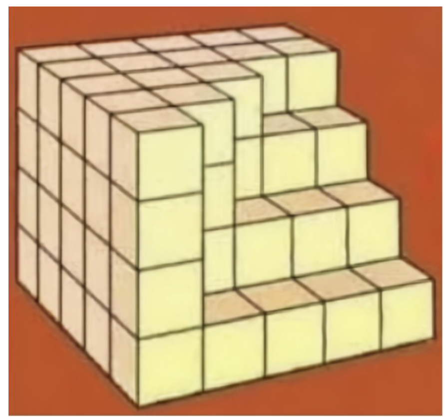
IMLEM Meet #3
January, 2022

Intermediate Mathematics League of Eastern Massachusetts

Category 1
Mystery
Meet #3 - January, 2022

1) Lisa worked for 30 hours and earned \$225. Amy earned \$2 more per hour than Lisa. How many dollars did Amy earn if she worked for 24 hours? Express your answer as a whole number.

2) How many of the smallest cubes are in this figure?



3) Mickey has twice as many sea monkeys as Peter. Mike has three fewer than Mickey. Together, Mickey, Mike, and Peter have as many sea monkeys as Davey. Davey has 217 sea monkeys. How many sea monkeys does Mike have?

Answers

1) ______

2) ______

3) ______

Solutions to Category 1
Mystery
Meet #3 - January, 2022

Answers
1) 228
2) 80
3) 85

1) Lisa's hourly wage was 225 / 30, or \$7.50 per hour. Amy's hourly wage is given as \$2 more than Lisa's, or \$9.50 per hour. So, if Amy works 24 hours, she earns (\$9.50 per hour)(24 hours), or \$228.

2) Counting the cubes by layers:
bottom layer: 5 x 5 = 25
next layer up: 5 x 4 + 1 = 21
next layer up: 5 x 3 + 3 = 18
top layer: 5 x 2 + 6 = 16
Total = 25 + 21 + 18 + 16 = 80 cubes.

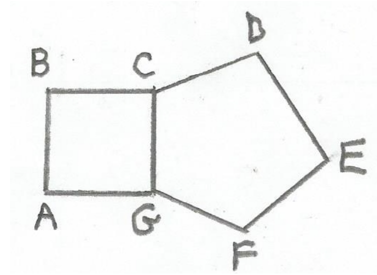
3) Guessing and checking could be a successful strategy but is time consuming . . perhaps too time consuming, given the time limit. Here is an algebraic approach:
Let X = the number of Peter's sea monkeys
then 2X = the number of Mickey's sea monkeys
and 2X - 3 = the number of Mike's sea monkeys
Davey has 217 sea monkeys.

$$
\begin{aligned}
X + 2X + 2X - 3 &= 217 \\
5X - 3 &= 217 \\
5X &= 220 \\
X &= 44
\end{aligned}
$$

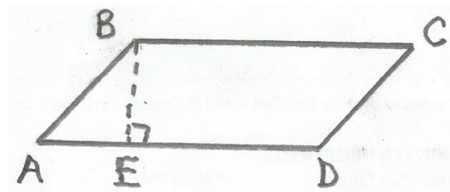
So, Mike has 2X - 3, or 2(44) - 3, or 85 sea monkeys.

Category 2
Geometry
Meet #3 - January, 2022

1) A square and a regular pentagon share a common side CG. How many degrees are in the measure of exterior angle BCD ?



2) If the area of parallelogram ABCD is 464 square centimeters, then how many centimeters are in its perimeter?
AE = 12 cm. BE = 16 cm. Angle BEA is a right angle.



3) Jack and Jill agree to meet at a local 60 foot by 80 foot rectangular field and run a race. Jill averages 5 feet per second while Jack averages 4 feet per second. They start at one corner of the field. Jack runs diagonally across the field to the opposite corner while Jill runs along the length and width of the field to meet Jack at the opposite corner. By how many seconds did Jack win the race?

Answers

1) _______ degrees

2) _______ cm

3) _______ seconds

Solutions to Category 2
Geometry
Meet #3 - January, 2022

Answers
1) 162
2) 98
3) 3

1) One interior angle of a square measures 90 degrees while one interior angle of a regular pentagon measures 3(180) / 5, or 108 degrees. Combining the interior angles of both, angle BCD, measures 90 + 108, or 198 degrees. The exterior angle at that vertex measures 360 - 198, or 162 degrees.

2) To find the length of one of the short sides, say BA, Use the Pythagorean Theorem so that
AE^2 + BE^2 = AB^2. Then 12^2 + 16^2 = AB^2.
144 + 256 = AB^2, then 400 = AB^2 and AB = 20.
To find the length of the longer side, say BC, divide the area by the height:
464 / 16 = 29. So, the perimeter of parallelogram ABCD = 2(20) + 2(29), or 40 + 58, or 98 centimeters.

3) Since the length, width, and diagonal are all divisible by both 4 and 5, the times for Jack and Jill can be calculated rather quickly. First, though, find the distance that Jack runs by employing the Pythagorean Theorem or by scaling the 60:80:D triple down to 3:4:X so that X = 5 and D = 100. Then calculate the times it takes Jack and Jill to reach the point diagonally opposite their starting point:

Jack: 100 feet / 4 feet per second = 25 seconds.
Jill: 140 feet / 5 feet per second = 28 seconds.

Therefore, Jack wins the race by 28 - 25, or 3 seconds.

Category 3
Number Theory
Meet #3 - January, 2022

1) What is the base 10 value of the base 4 numeral 2031 ?

2) The product $\left(5.8 \times 10^{8}\right)\left(6.2 \times 10^{-2}\right)$ is equal to $M \times 10^{N}$ that is written in scientific notation. If M is rounded to the nearest whole number, then what is the value of M + N ?

3) A plane (a flat surface that extends infinitely) can be divided into two distinct spaces, or half-planes, with a single line. Two intersecting lines can divide a plane into as many as four distinct spaces. What is the maximum (most) number of spaces into which a plane can be divided by six lines?

Answers

1) ________

2) ________

3) ________

Solutions to Category 3
Number Theory
Meet #3 - January, 2022

Answers	
1)	141
2)	11
3)	22

1) 2031 in base 4 is, from right to left,
$1(1) + 3(4) + 0(16) + 2(64)$
$= 1 + 12 + 0 + 128$
$=$ 141 in base 10.

2) $\left(5.8 \times 10^{8}\right)\left(6.2 \times 10^{-2}\right) = \left(35.96 \times 10^{6}\right)$
$= \left(3.596 \times 10^{7}\right)$. So, M = 4 and N = 7
and the sum M + N = 4 + 7 = 11.
Remember that the value of M was required to be rounded to the nearest whole number.

3) Count the number of distinct spaces into which a small number of lines can divide the plane. Then look for a pattern.

# of lines	# of spaces	observation
1	2	
2	4	2 more than previous
3	7	3 more than previous
4	11	4 more than previous
5	16	5 more than previous
6	22	6 more than previous

Therefore, six lines can divide the plane into as many as 22 spaces.

Category 4
Arithmetic
Meet #3 - January, 2022

1) What is the value of $5^0 + 5^1 + 5^2 + 5^3$ **?**

2) How many whole numbers are between $\sqrt[3]{47}$ **and** $\sqrt[4]{2638}$ **?**

3) Compute: $\sqrt[3]{\sqrt{\sqrt{12^2} \times \sqrt[4]{81}} \times \sqrt[3]{64} \times \sqrt[5]{243} \times \sqrt{9}}$

ANSWERS
1) _________
2) _________
3) _________

Solutions to Category 4
Arithmetic
Meet #3 - January, 2022

Answers
1) 156
2) 4
3) 6

1) $5^0 + 5^1 + 5^2 + 5^3 = 1 + 5 + 25 + 125 = 156.$

2) $\sqrt[3]{47}$ **lies between 3 and 4.**

$\sqrt[4]{2638}$ **lies between 7 and 8.**

So, the whole numbers in between the given expressions are 4, 5, 6, and 7. Therefore, there are four whole numbers in between.

3)
$$\sqrt[3]{\sqrt{\sqrt{12^2} \times \sqrt[4]{81}} \times \sqrt[3]{64} \times \sqrt[5]{243} \times \sqrt{9}}$$

$$= \sqrt[3]{\sqrt{12 \times 3} \times 4 \times 3 \times 3}$$

$$= \sqrt[3]{\sqrt{36} \times 36}$$

$$= \sqrt[3]{6 \times 36}$$

$$= \sqrt[3]{216}$$

$$= 6$$

Category 5
Algebra
Meet #3 - January, 2022

1) **Evaluate this absolute value expression:** $|0| + |16| + |-23|$

2) **What is the sum of all possible positive integer values of M that make the following inequality true?**

$$3M - 5 < 21$$

3) **The graph below is the set of all real values of W that make the following absolute value inequality true:**

$$|W - A| \leq C$$

What is the value of A + C ?

Answers
1) ______
2) ______
3) ______

Solutions to Category 5
Arithmetic
Meet #3 - January, 2022

Answers

1) 39

2) 36

3) 36

1) $|0| + |16| + |-23| = 0 + 16 + 23 = 39$

2) $3M - 5 < 21$
$3M < 26$
$M < 8.6666\ldots$

The positive integers that satisfy $M < 8.66666\ldots$ are 1, 2, 3, 4, 5, 6, 7, and 8. Their sum is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.

3) Students may translate this inequality as, "The distance between W and A is less than or equal to C." If the endpoints, - 10 and 36, are to be equidistant from A, then A is their midpoint, or 13. So, A = 13. The distance between A and either endpoint is 23 units. So, C = 23.
So, $A + C = 13 + (23)$, or 36.

Category 6
Team Round
Meet #3 - January, 2022

Each of the following NINE problems is worth four points.

1) The animals in the barn are either owls or cows. There are 12 more owls than cows. Each owl has two feet while each cow has four feet. There is a total of 126 feet. How many animals are in the barn?

2) How many prime numbers less than 100 have the digit 7 in the units place (the "ones" place)?

3) What is the largest three-digit number that is divisible by 5, 9, and 12?

4) How many degrees are in the measure of one exterior angle of a regular pentadecagon (a 15-sided polygon)?

5) Catherine walked 17 miles north, then 4 miles west, 6 miles north, 10 miles west, and 25 miles north. How many miles, in a straight line, is Catherine from her starting point?

ANSWERS

1) ________

2) ________

3) ________

4) ________

5) ________

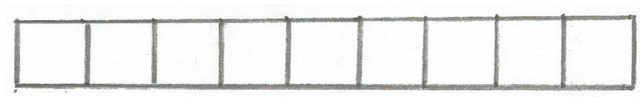
6) ________

7) ________

8) ________

9) ________

6) How many rectangles, including the nine attached squares, are in this figure?



7) Points A, B, C, and D, in that order, lie in a straight line. AB = 8. AC = 11. BD = 17. How many units long is AD?

8) The maximum recommended pulse rate, R, while exercising, for a person who is X years old, is given by the equation $R = 176 - 0.8X$. How many years old is a person whose maximum recommended pulse rate, when exercising, is 140?

9) A grocer has 400 pounds of coffee, 20% of which is decaffeinated. If the grocer buys another 100 pounds of coffee, 60% of which is decaffeinated, then W% of the grocer's total stock of coffee is decaffeinated. What is the value of W ?

Solutions to Category 6
Team Round
Meet #3 - January, 2022

ANSWERS

1)	46
2)	6
3)	900
4)	24
5)	50
6)	45
7)	25
8)	45
9)	28

1) Let X = the number of cows and X + 12 the number of owls. Then $4X + 2(X + 12) = 126$, $6X + 24 = 126$, then $6X = 102$, and $X = 17$. So, there are 17 cows and 29 owls, or 46 animals total.

2) The prime numbers are 7, 17, 37, 47, 67, and 97, or SIX primes in all.

3) The LCM of 5, 9, and 12 is 180. The largest three-digit multiple of 180 is (180)(5), or 900.

4) Divide 360 by the number of 15 equal exterior angles to get 24 degrees.

5) Add the wests to get one leg of a right triangle then add the norths to get the other leg. Then employ the Pythagorean Theorem:
14^2 + 48^2 = D^2, then 196 + 2304 = D^2 and 2500 = D^2, so D = 50.

6) There are 9 squares, 8 1x2s, 7 1x3s, 6 1x4s, 5 1x5s, 4 1x6s, 3 1x7s, 2 1x8s, and 1 1x9, for a total of $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, or 45 rectangles.

7) Subtract AB from AC to get BC = 11 - 8, or 3. Subtract BC from BD to get CD = 17 - 3, or 14. Then add the individual pieces to get AB + BC + CD = 8 + 3 + 14 = 25.

8) $176 - 0.8X = 140$, so $0.8X = 36$, and $8X = 360$, so $X = 45$.

9) 20% of 400 is 80 pounds of decaf; 60% of 100 is 60 pounds of decaf. So, 140 of the 500 pounds of coffee is decaf. Convert 140 / 500 to a percent, which is 28%. Therefore W = 28.