IMLEM Meet #2
November, 2021

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**
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Category 1
Mystery
Meet #2 - November, 2021

1) If three apples and one orange cost a total of 81 centavos while three apples and three oranges cost a total of 93 centavos, then how many centavos does one orange cost?

2) In a very tall apartment building, two numbered floors are the same distance from my floor. The two floor numbers have a sum of 126. What is the number of my floor?

3) When their mother turned 40, Benji was four times as old as his brother. When their mother turned 48, Benji was twice as old as his brother. How old will Benji be when their mother turns 60?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #2 - November, 2021

Answers

1) 6

2) 63

3) 36

1) The difference between the two sets of fruit is two oranges. The total price difference is 93 - 81, or 12 centavos. So, one orange costs 12 / 2, or 6 centavos.

2) Let X = the number of my floor
and A = the number of floors above or below
Then X + A = the numbered floor above
and X - A = the numbered floor below.
The sum of the floors above and below is 126, so

$$X + A + X - A = 126$$
$$2X = 126$$
$$X = 63$$

So, My floor is # 63.
For students who lack the necessary algebra skills, they can experiment with selecting random numbers of floors above and below my floor and then observe that those numbers "cancel out," leading them to an answer of 63.

3) As with problem #2, students can experiment with random, yet reasonable choices the brothers' ages.
Here is an algebraic approach:
Let T = the brother's age
and 4T = Benji's age
Eight years later, the brother's age is T + 8 and Benji's age is 4T + 8.
But Benji will only be twice as old as his brother, yielding this equation:

$$4T + 8 = 2(T + 8)$$
$$4T + 8 = 2T + 16$$
$$2T = 8$$
$$T = 4$$

So, when the mother was 40, the brother was 4 and Benji was 16.
Eight years later, when the mother was 48, the brother was 12 and Benji was 24, or twice as old as his brother.
Twenty years after the mother was 40, when the mother will be 60, the brother will be 4 + 20, or 24 while Benji will be 16 + 20, or 36.

Category 2
Geometry
Meet #2 - November, 2021

1) The perimeter of a rectangle is 42 meters. If the length and width are each increased by 7 meters to create a larger rectangle, then how many meters are in the perimeter of the larger rectangle?

2) Five squares of paper all have the same thickness but different side lengths. The smallest square has side lengths of one cm, while the subsequent larger squares have side lengths of 5 cm, 7 cm, 7 cm, and 11 cm, respectively. If all five squares of paper are cut and then reassembled to form five identical squares, then how many cm are in the side length of each square?

3) The lengths of the six sides of a hexagon are consecutive whole numbers of inches. The lengths of the eight sides of an octagon are consecutive odd numbers of inches. The perimeter of the octagon is 183 inches longer than the perimeter of the hexagon. How many inches are in the perimeter of the octagon if the perimeter of the hexagon is less than 100 inches?

Answers

1) _____ meters

2) _____ cm

3) _____ inches

Solutions to Category 2
Geometry
Meet #2 - November, 2021

Answers	
1)	70
2)	7
3)	240

1) If the length and width are each increased by 7 meters, then the perimeter is increased by 4 x 7, or 28 meters. The smaller perimeter is 42 meters, so the larger perimeter is 42 + 28, or 70 meters.

2) The total area of the five squares is (1 x 1) + (5 x 5) + (7 x 7) + (7 x 7) + (11 x 11), or 1 + 25 + 49 + 49 + 121 = 245 square centimeters. If that total area is divided equally to form five identical squares, then each square's area would be 245 / 5, or 49 square centimeters. Each of those squares is, therefore, 7 x 7 cm. So, one side of each square is 7 centimeters.

3) It appears, at first glance, that there is not enough information, so that an algebraic solution may not be possible. However, listing the perimeters of hexagons and octagons in a reasonable range of values and then comparing their differences could provide an expedient solution. The octagon. must have a perimeter of at least 183.

ranges of lengths	5-10	6-11	7-12	8-13	9-14	10-15	11-16
Hexagon perimeter	45	51	57	63	69	75	81
Perimeter + 183	228	234	240	246	254	260	266

range of lengths	15-29	17-31	19-33	21-35	23-37	25-39	27-41	29-43
Octagon perimeter	176	192	208	224	240	256	272	288

The difference between the perimeter of the octagon with shortest side 23 and the hexagon with shortest side 7 is 183. Therefore, the perimeter of the octagon is 23 + 25 + 27 + 29 + 31 + 33 + 35 + 37, or 240 inches.

Category 3
Number Theory
Meet #2 - November, 2021

1) **The prime factorization of 350 is 350 = 2 x 5 x A x B. What is the sum of A + B ?**

2) **What is the smallest positive composite number that does not have any positive prime factors less than 50 ?**

3) **Tim receives a message every 18 minutes, a tweet every 30 minutes, and an email every 42 minutes. If he received a message, a tweet, and an email at 10:38 A.M., then at what time, P.M., will he receive all three? Your answer to this question will be a whole number that is equal to the sum of the digits of that time.**

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - November, 2021

Answers	
1)	12
2)	2809
3)	17

1) 350 = 2 x 5 x 5 x 7. A = 5 and B = 7.
A + B = 5 + 7 = 12.

2) The smallest such number is the square of the smallest prime number that is greater than 50.
53 x 53 = 2809.

3) First, find the LCM of 18, 30, and 42. That will be the product of all the different prime factors of the three numbers, to their highest powers in any factorization.

18 = 2 x 3 x 3
30 = 2 x 3 x 5
42 = 2 x 3 x 7

The LCM (least common multiple) is 2 x 3 x 3 x 5 x 7, or 630.
Now, find the time that is 630 minutes after 2:38. More easily computed, 630 minutes is 10 hours and 30 minutes.
10:38 + 10:30 = 20:68 . . . translates to 9:08 P.M.

Finally, add the digits: 9 + 0 + 8 = 17.

Category 4
Arithmetic
Meet #2 - November, 2021

1) When the fraction $\frac{12}{30}$ is simplified, or reduced to lowest terms, what is the sum of its numerator and denominator?

2) After the repeating decimal 0.26666666 . . . was converted to its fraction form and then reduced to lowest terms, what was the denominator of that lowest-terms fraction?

3) Evan spent 2/9 of his year's allowance on gifts for his family and friends and 20% on items for himself. He put the rest into his savings account at the bank. If he saved 468 dollars, then how many dollars was his yearly allowance?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - November, 2021

Answers	
1)	7
2)	15
3)	810

1) The fraction 12 / 30, when simplified, is 2 / 5.
The sum of the numerator and denominator is
2 + 5, or 7.

2) Let X = 0.266666 . . .
then 10X = 2.666666 . . .
Subtracting gives us 9X = 2.4
Then X = 2.4 / 9 = 24 / 90 = 8 / 30 = 4 / 15.
The denominator is 15.

3) Evan saved 1 - (2/9 + 1/5)
= 1 - (10/45 + 9/45)
= 1 - (19/45)
= 26/45.
Let X = Evan's yearly allowance. He saved 26/45 of it, or $ 468.
So, 26/45 X = 468
X = 468 (45/26)
X = 810
Therefore, Evan's yearly allowance is 810 dollars.

Category 5
Algebra
Meet #2 - November, 2021

1) **Rashad has a collection of coins. The total value is $ 2.82. He has seven quarters, five dimes, and twelve pennies. How many nickels does he have?**

2) **The formula for the total surface area of a cone is** $\pi r^2 + \pi rl$ **.**
Find the number of inches in the slant height, l, of a cone whose total surface area is 77π **square inches and whose radius is seven inches.**

3) **Together, Anna and Elsa have 80 goldfish. If Elsa gives 20 of her goldfish to Anna, then Anna will have 16 more goldfish than Elsa. How many goldfish did Elsa have originally?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Meet #2 - November, 2021

1) The total cent value of the given coins is
7(25) + 5(10) + 12(1), or 175 + 50 + 12, or 237.
Subtract 237 from 282 to get 45 cents, the value of NINE nickels.
For an algebraic solution:
Let X = the number of nickels and 5X = the cent value
Then 12(1) + 5X + 5(10) + 7(25) = 282
12 + 5X + 50 + 175 = 282
5X + 237 = 282
5X = 45
X = 9
So, there are NINE nickels.

Answers	
1)	9
2)	4
3)	52

2) $\pi r^2 + \pi rl = 77\pi$
Divide both sides by pi to get
r squared + rl = 77
r(r + l) = 77
Substitute 7 for r, as given in the problem:
7(7 + l) = 77
7 + l = 11
l = 4

3) Let A = the number of Anna's goldfish
and 80 - A = the number of Elsa's goldfish
and 80 - A - 20 = the number of goldfish that Elsa has after giving 20 to Anna
and A + 20 = the number of goldfish that Anna has after Elsa gave her 20

Now Anna will have 16 more than Elsa:

80 - A - 20 + 16 = A + 20
76 - A = A + 20
56 = 2A
28 = A
So, Anna originally had 28 goldfish and Elsa originally had 80 - 28, or 52 goldfish.

Category 6
Team Round
Meet #2 - November, 2021

Each of the following twelve problems is worth <u>three</u> points.

1) **If the average of A, B, and C is 24 and the average of A and C is 16, then what is the value of B ?**

2) **The ages, in years, of the five oldest members of the Yanco family are consecutive odd integers whose total is 445 years. How many years old is the oldest of the Yanco family?**

3) **How many whole numbers have squares that are between 2 and 450 ?**

4) **If I square all the positive factors of 36 and multiply the resulting numbers, the product is equal to 36^N. What is the value of N ?**

5) **How many hours are there in four weeks?**

<u>ANSWERS</u>

1) _____
2) _____
3) _____
4) _____
5) _____
6) _____
7) _____
8) _____
9) _____
10) _____
11) _____
12) _____

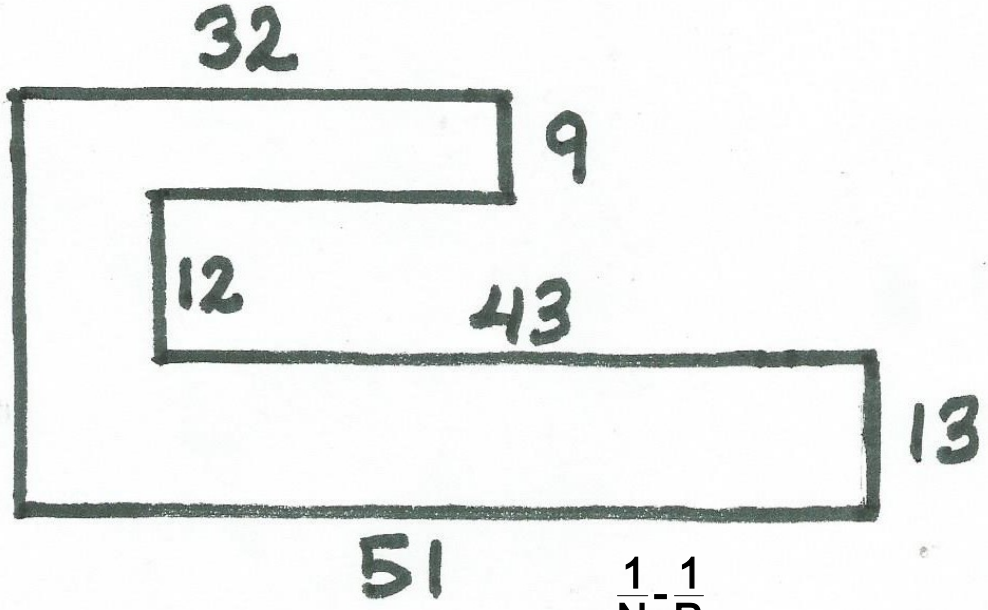
6) **0.02% of 20% of what number is 200% of 2000 ?**

7) **Six students sit in a circle arrangement. Their names are Abigail, Brandon, Connor, Donna, Evagrio, and Frank. They start counting. as follows: Abigail says 1, then Brandon says 2, and so on. When the number contains a digit of 7, such as 37 or 77, or is a multiple of 7, such as 28 or 63, then the person leaves the circle and the counting continues. After five students have left the circle, then who is the last student remaining?**

8) **If @ = the sum of all composite numbers greater than 80 but less than 90, and # = the sum of all prime numbers greater than 50 but less than 60, then what is the value of @ - # ?**

9) **The average person can speak 120 words in 1.5 minutes and can write 96 words in 2 2/3 (two and two-thirds) minutes. In a ten minute period, how many more words can be spoken than written?**

10) How many units are in the perimeter of this figure? All angles are right angles.



11) $N \bigcirc P$ **is defined by the expression** $\dfrac{\frac{1}{N}-\frac{1}{P}}{\frac{1}{N}+\frac{1}{P}}$.

Find the value of $8 \bigcirc 12$, **then reduce your fractional answer to lowest terms. What is the product of the numerator and denominator of that lowest-terms fraction?**

12) M, A, T, and H are consecutive multiples of 7. $M < A < T < H$.
What is the value of $(M - T)(H - A)$ **?**

Solutions to Category 6
Team Round
Meet #2 - November, 2021

1) If the average of A and C is 16, then their sum is 32.
The average of A, B, and C is 24, so their sum is (3)(24), or 72.
Substituting 32 for A + C gives

$$A + B + C = 72$$
$$B + 32 = 72$$
$$B = 40.$$

2) $Y + (Y+2) + (Y+4) + (Y+6) + (Y+8) = 445$

$$5Y + 20 = 445$$
$$5Y = 425$$
$$Y = 85$$

So, the oldest member of the Yanco family is Y + 8, or 93 years old.

3) The squares between 2 and 350 are 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, and 441. These are the squares of the whole numbers from 2 through 21, inclusive, or 20 whole numbers.

4) The positive factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. The product of their squares is (1)(4)(9)(16)(36)(81)(144)(324)(1296). To find which power of 36 is this product, it may be most expedient to represent the product in factored form and then group the factors into product that equal 36:
(1)(2x2)(3x3)(2x2x2x2)(2x2x3x3)(3x3x3x3)(2x2x3x2x2x3)(2x3x3x2x3x3) (2x2x3x3x2x2x3x3)
= (2x2x3x3)(2x2x3x3)(2x2x3x3)(2x2x3x3)(2x2x3x3)(2x2x3x3)(2x2x3x3) (2x2x3x3)(2x2x3x3)
= 36 ^ 9. So, N = 9.

5) (24 hours / day) (7 days / week) (4 weeks) = 672 hours

ANSWERS

1)	40
2)	93
3)	20
4)	9
5)	672
6)	100,000,000
7)	Donna
8)	481
9)	440
10)	218
11)	5
12)	- 196

6) $(0.0002)(0.2)(X) = (2)(2000)$

$0.00004)(X) = 4000$

$4X = 400{,}000{,}000$

$X = 100{,}000{,}000$

7)

A1 B2 C3 D4 E5 F6 A7	Abigail leaves the circle.
B8 C9 D10 E11 F12 B13 C14	Connor leaves the circle.
D15 E16 F17	Frank leaves the circle.
B18 D19 E20 B21	Brandon leaves the circle.
D22 E23 D24 E25 D26 E27	Evan leaves the circle.

Therefore, only Donna remains.

8) $@ = 81 + 82 + 84 + 85 + 86 + 87 + 88 = 593$

$\# = 53 + 59 = 112$

$@ - \# = 593 - 112 = 481$

9) Spoken words: $[(120) / (1.5)](10) = 800$

Written words: $[(96) / 2\ 2/3)](10) = 360$

The difference is $800 - 360$, or 440 words.

10) There are two unlabelled lengths. The long vertical length on the left side is the sum of the shorter verticals, or $9 + 12 + 13$, or 34.

The missing horizontal length is $32 - (51 - 43)$, or 24.

The perimeter is the sum of all lengths:

$= 32 + 9 + 24 + 12 + 43 + 13 + 51 + 34$

$= 218.$

11) $$\frac{\frac{1}{N}-\frac{1}{P}}{\frac{1}{N}+\frac{1}{P}} = \frac{\frac{1}{8}-\frac{1}{12}}{\frac{1}{8}+\frac{1}{12}} = \frac{\frac{3}{24}-\frac{2}{24}}{\frac{3}{24}+\frac{2}{24}} = \frac{\frac{1}{24}}{\frac{5}{24}} = \frac{1}{5}$$

The product of the numerator and denominator is $(1)(5)$, or 5.

12) The difference between two consecutive multiples of 7 is 7.

The difference between two consecutive multiples of 7 is 14.

$(M - T)(H - A) = (-14)(14) = -196.$