IMLEM Meet #4
March, 2021

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery
Meet #4 - March, 2021

Calculator Meet

1) Six carpenters can frame a house in four days. However, the boss hired more carpenters so that the job could be completed in three days. How many more carpenters were hired?

2) Evan turns E years old in the year E^2. If E^2 occurred during the 20th century, then in what year was Evan born?

3) Sydney had a truckload of sand delivered to her driveway. The volume of the sand measured 8 cubic yards. She can fill a wheelbarrow that holds 4 cubic feet in 45 seconds. She wants dump the sand into a large sandbox in her backyard with the sand. It takes an average of 22 seconds for her to take a filled wheelbarrow to the sandbox, an average of 5 seconds to dump the sand into the sandbox, and an average of 17 seconds to return the empty wheelbarrow back to the driveway. If she starts at her driveway and finishes the job after dumping her final load of sand into the sandbox, then how many seconds would it take for her, in total, to complete the job?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #4 - March, 2021

Answers

1) 2

2) 1892

3) 4789

1) If 6 carpenters can frame a house in 4 days, then one could do so in (6)(4), or 24 days. It would require 24 / 3, or 8 carpenters to do the job in 3 days. So, an additional 2 carpenters were hired.

2) The only square number occurring during the 20th century, or from the years 1901-2000, is (44)(44), or 1936. So, Evan is 44 years old in the year 1936. He was born in the year 1936 - 44, or 1892.

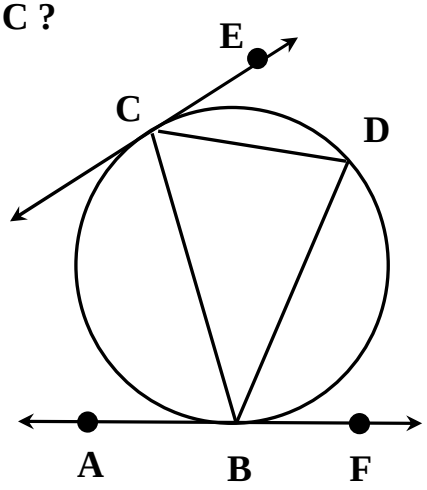
3) Convert 8 cubic yards to cubic feet:
(27)(8) = 216 cubic feet.
The number of full wheelbarrows of sand is 216 / 4, or 54.
Amount of time required to fill the 54 wheelbarrows: (54)(45) = 2430 sec.
Number of trips to the sandbox: 54
Amount of time for those 54 trips to the sandbox: (54)(22) = 1188 sec.
Number of dumps: 54
Amount of time to do the dumping: (54)(5) = 270 sec.
Number of return trips to the driveway: 53
Amount of time making the return trips: (53)(17) = 901 sec.
Total amount of time: 2430 + 1188 + 270 + 901 = 4789 seconds.

Category 2
Geometry
Meet #4 - March, 2021

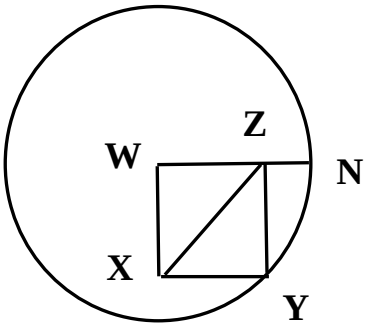
Calculator Meet

1) Using $\pi \approx 3.14$, how many inches greater is the circumference of a 30-inch-diameter circle than a 20-inch-diameter circle?

2) Lines AF and CE are tangent to the circle and are not parallel. Angle FBC measures 124 degrees. Points B, C, and D lie on the circle. How many degrees are in the measure of angle BDC ?



3) In the circle below, WN is a radius, quadrilateral WXYZ is a rectangle, and the diagonal XZ of the rectangle = 7.2 centimeters. How many centimeters are in the circumference of circle W ? Use $\pi \approx 3.14$. Round your answer to the nearest whole number.



Answers

1) _____ inches

2) _____ degrees

3) _____ cm

Solutions to Category 2
Geometry
Meet #4 - February, 2021

Answers	
1)	31.4
2)	56
3)	45

1) Circumference of larger circle: (2)(pi)r) = (2)(3.14)(15)
= 94.2
Circumference of smaller circle: (2)(3.14)(10)
= 62.8
Difference: 94.2 - 62.8 = 31.4 inches.

2) Angle FBC = 124 degrees, so arc CDB = (2)(124), or 248 degrees. Angle BDC intercepts minor arc BC whose measure is 360 - 248, or 112 degrees. So, angle BDC measures half of the intercepted arc, or 56 degrees.

3) Since XZ is a diagonal of the rectangle, and diagonals of a rectangle are congruent, then XZ = WY = the radius of the circle = 7.2 cm. Then the circumference of the circle is (2)(pi)(radius) = (2)(3.14)(7.2) = 45.216 cm. Rounding to the nearest whole number yields an answer of 45 cm.

Category 3
Number Theory
Meet #4 - March, 2021

Calculator Meet

1) What is the sum of the first 150 numbers in the arithmetic sequence below?

11 15 19 23 27 . . .

2) The pattern below is formed according to the following two rules:

a) If the number is even, then the next number is half of it.

b) If the number is odd, then the next number is one more than three times that number.

6 3 10 5 16 . . .

What is the value of the 400th number in the pattern?

3) On Planet Earth, one day is about 24 hours but is only about 16 hours on Planet Neptune. Ephraim took a cross-country trip on Planet Earth that lasted 9 days and 4 hours, starting at 14 o'clock on a 24-hour clock. Ignoring the difference in time among time zones, and using a 16-hour clock for Planet Neptune, at what time would a clock on Neptune register when Ephraim completed his trip if the trip started when the Neptune clock read 10 o'clock?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #4 - February, 2021

Answers	
1)	46,350
2)	4
3)	6

1) Consecutive numbers in this sequence have a difference of 4. Each number is 7 more than the corresponding number in the sequence of positive multiples of 4, or
4 8 12 16 20 . . .
The 150th multiple of 4 is (150)(4), or 600.
Seven more than 600 is 607.
The sum of the first 150 terms is
(607 + 11)(150 / 2), or (618)(75), or 46,350.

2) The sequence, continued, looks like this:
6 3 10 5 16 8 4 2 1 4 2 1 4 2 1 . . .
Eliminating the first six numbers leaves us 394 terms consisting of blocks of three numbers, in sequence, 4 2 1 4 2 1 4 2 1 . . .
Dividing 394 by 3 yields 131 with remainder 1. So, the final number in the 400-number sequence is 4.

3) The Earthly trip took (9)(24) + 4, or 220 hours. On Neptune, to calculate the number of days, divide 220 by 16 = 13 with remainder 12. It is the 12 that is significant in helping to answer the question: 12 hours after 10 o'clock is 6 o'clock.

Category 4
Arithmetic
Meet #4 - March, 2021

Calculator Meet

1) Fonz purchased a 1959 Corvette for $ 17,350. When he registered the car, he paid a sales tax of 6 %. How much sales tax did he pay?

2) Forrest walked 26,700 steps on Wednesday, 10% more steps on Thursday, and 10% more steps on Friday than on Thursday. How many more steps did Forrest take on Friday than on Wednesday?

3) The equation to the right calculates the value A, that is the result of investing an initial amount of money, M, at an annual (yearly) rate of R for T years when the interest is compounded W times annually. Kelly will be able to retire once her investment grows to at least $800,000. How much money must Kelly invest at an annual rate of 3.6% that compounds interest twice a month for 23 years? Round your answer up to the nearest hundred dollars to assure that Kelly will have at least $800,000 when she retires.

$$A = M\left(1+\frac{R}{W}\right)^{WT}$$

ANSWERS

1) $ _____

2) _____ steps

3) $ _____

Solutions to Category 4
Arithmetic
Meet #4 - March, 2021

Answers	
1)	1,041
2)	5,607
3)	349,800

1) Multiply (17,350)(0.06) = 1,041.

2) Wednesday: 26,700 steps
Thursday: (26,700)(1.1) = 29,370 steps
Friday: (29,370)(1.1) = 32,307 steps
Subtract Wednesday's # of steps from
Friday's # of steps: 32,307 - 29,370 = 5,607.

3) $A = M\left(1+\frac{R}{W}\right)^{WT}$ **original formula**

$800,000 = M\left(1+\frac{0.036}{24}\right)^{(24)(23)}$ **substituting values, including the number 24 for the number of half-month periods in a year**

$800,000 = M\left(1+0.0015\right)^{(552)}$ **evaluate**

$800,000 = M\left(1.0015\right)^{(552)}$ **evaluate**

$800,000 = M(2.287317)$ **evaluate**

$M = \frac{800,000}{2.287317}$ **solve for M**

$M = 349,754.76$ **divide**

This amount, $349,754.76, must be rounded up to $349,800 in order to guarantee at least $800,000 in retirement money.

Category 5
Algebra
Meet #4 - March, 2021

Calculator Meet

1) What value of C makes the following proportion true?

$$\frac{3C}{9C - 15} = \frac{4}{7}$$

2) A jar of coins contains pennies, nickels, dimes, and quarters. There are 5 more nickels than pennies, twice as many dimes as nickels, and two more quarters than dimes. The total value of all the coins is $32.37. How many quarters are in the jar?

3) Laurel and Hardy live 390 miles apart. They each leave their homes at the same time and head toward each other on the same road. Laurel's average speed is 30 miles per hour less than Hardy's average speed. After three hours on the road, they pass each other! How many miles per hour was Hardy travelling?

ANSWERS

1) C = _____

2) _____ quarters

3) _____ mph

Solutions to Category 5
Algebra
Meet #4 - March, 2021

Answers	
1)	4
2)	86
3)	80

1) $\dfrac{3C}{9C - 15} = \dfrac{4}{7}$

Cross products are equal whenever two ratios are equal:

$$(7)(3C) = (4)(9C - 15)$$
$$21C = 36C - 60$$
$$60 = 15C$$
$$C = 4$$

2) Let X = the number of pennies
then $X + 5$ = the number of nickels
$2(X + 5)$ = the number of dimes
and $2(X + 5) + 2$ = the number of quarters

The total value of the coins is $ 32.37. Expressing this equation in terms of cents, rather than in dollars:

$$1(X) + 5(X + 5) + 10[(2)(X + 5)] + 25([(2)(X + 5) + 2] = 3237$$
$$X + 5X + 25 + 20X + 100 + 50X + 250 + 50 = 3237$$
$$76X + 425 = 3237$$
$$76X = 2812$$
$$X = 37$$

So, the number of quarters is $(2)(37 + 5) + 2 = 86$.

3) (Rate of speed) x (Time) = (Distance)

	Rate	Time	Distance
Laurel	X - 30	3	3(X - 30)
Hardy	X	3	3X

The total distance covered is 390 miles:

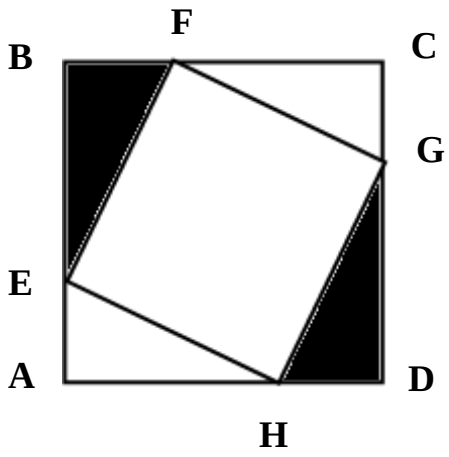
$$3X + 3(X - 30) = 390$$
$$3X + 3X - 90 = 390$$
$$6X - 90 = 390$$
$$6X = 480$$
$$X = 80$$

Hardy was travelling an average speed of 80 miles per hour.

Category 6
Team Round
Meet #4 - March, 2021

Each of the following six problems is worth <u>six</u> points.

1) Square EFGH has a perimeter of 132 centimeters. Square ABCD has a perimeter of 156 centimeters. How many square centimeters are in the shaded area?



2) A Mersenne Prime is a prime number that can be expressed in the form

$2^P - 1$ **where P is a prime number. What is the value of the largest Mersenne Prime that is less than 1000?**

3) The average of ten numbers is 296. If each of the ten numbers is increased by 12.5%, then what will be the new average?

4) The terms of a geometric sequence are such that the ratio of any two consecutive terms is the same as the ratio of any other two consecutive terms. What is the value of the denominator of the eighth term of the following geometric sequence if that fraction is in lowest terms?

$\frac{3}{4} \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{2}{9} \quad \frac{4}{27} \quad . \; . \; .$

ANSWERS

1) _____

2) _____

3) _____

4) _____

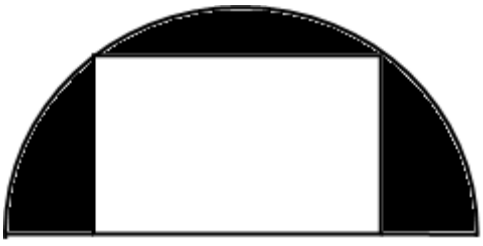
5) _____

6) _____

5) What is the value of the 234th term of the following arithmetic sequence?

39 51 63 75 87 . . .

6) A rectangle is inscribed in a semi-circle with segment lengths as marked. How many square units are in the shaded area? Use $\pi \approx 3.14$.



Solutions to Category 6
Team Round
Meet #4 - March, 2021

ANSWERS	
1)	216
2)	127
3)	333
4)	729
5)	2,835
6)	381.25

1) Divide each perimeter by 4 to get the side lengths of each square: 132 / 4 = 33 and 156 / 4 = 39. The areas of the two squares are (33)(33) and (39)(39), or 1,089 and 1521. The shaded area represents half of the difference, or (0.5)(1,521 - 1,089), or 216.

2) The largest Mersenne less than 1000 occurs when P = 7, so that 2^7 - 1 = 127.

3) If the average of the ten numbers is 296, then their sum is (10)(296), or 2,960. Increasing each number by 12.5%, or by 1/8, increases the sum by 12.5%. The new sum is (9/8)(2,960) or 3,330. The new average is 3,330 / 10, or 333.

4) Divide any term by the previous term to get the common ratio. For example, 1/2 / 3/4 = (1/2)(4/3), or 2/3. So, to compute terms #6-8: term #6 = (term #5)(2/3), or (4/27)(2/3), or 8/81. Term #7 = (term #6)(2/3), or (8/81)(2/3), or 16/243. Term #8 = (term #7)(2/3), or (16/243)(2/3), or 32/729. The denominator of the 8th term is 729.

5) The difference between any two consecutive terms is 12. Each number in this sequence is 27 more than a multiple of 12 when comparing to the sequence 12 24 36 48 60 . . .
The 234th multiple of 12 is (12)(234), or 2,808. Adding 27 to 2,808 yields 2,835.

6) The diameter of the semi-circle is 10 + 30 + 10, or 50 units. The radius is half of 50, or 25 units. To find the width of the rectangle, draw a radius from the center of the diameter to the upper-right vertex of the rectangle, forming a right triangle whose hypotenuse is the radius of the semi-circle, or 25 units, and one of whose legs is 15 units. The Pythagorean Theorem yields a rectangular width of 20 units.
Shaded area = (area of semi-circle) - (area of rectangle)
= (1/2)(3.14)(25)(25) - (20)(30) = 981.25 - 600 = 381.25 square units.