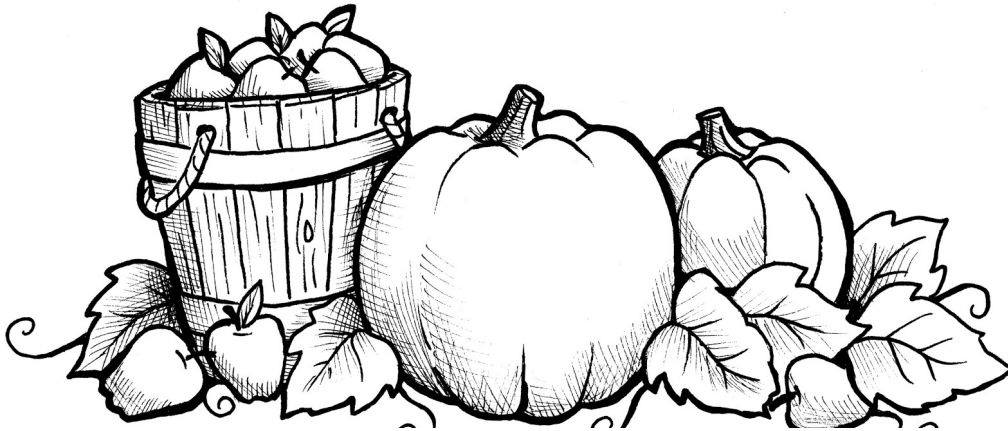


IMLEM Meet #1
November, 2020

Intermediate
Mathematics League
of
Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1

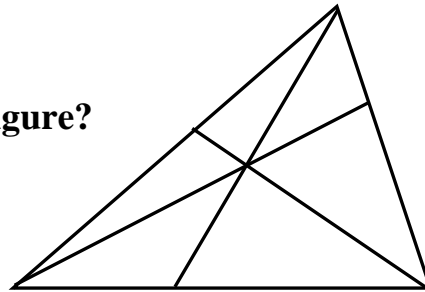
Mystery

Meet #1 - November, 2020

- 1) The letters in the name JASPER are written, in sequence, until the 100th letter is reached. What is the 100th letter in the sequence?

J A S P E R J A S P E R J A S . . .

- 2) How many triangles of any size are in this figure?



- 3) Green aliens have seven fingers while gray aliens have five. An alien spaceship landed in Billerica, Massachusetts. Ms. Ruggieri greeted them and invited them to join her math team. She observed that the aliens were all either green or gray and that they had a total of 162 fingers. How many were gray aliens if there were between 10 and 20, exclusive of 10 and 20?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #1 - November, 2020

1) There are 6 letters in the word JASPER. To identify the 100th letter, divide $100 \div 6 = 16$ with a remainder of 4. There are 16 complete words spelled JASPER. The remainder of 4 has us counting the next five letters as J A S P. The 100th letter is P.

2) There are 6 of the smallest triangles,
6 consisting of three of the smallest triangles, created by the diagonals,
3 consisting of two of the smallest triangles, and
1 of the largest triangle.
 $6 + 6 + 3 + 1 = 16$.

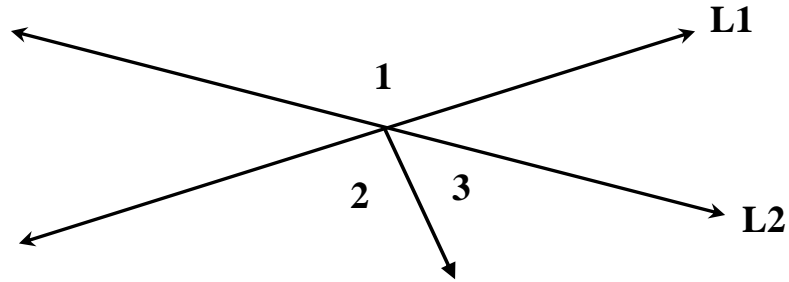
3) At first glance, the equation $7X + 5Y = 162$ represents the data, where X = the number of green aliens and Y = the number of gray aliens. Normally, a second equation involving the same two variables is needed in order to create a solution. However, in this instance, it is only necessary that the numbers be whole numbers and that the number of gray aliens is between 10 and 20. One strategy is to select integers between 10 and 20 and multiply by 7, subtract from 162, and check to see if the remainder is divisible by 5. This ONLY works when $Y = 17$.

Check: $7(11) + 5(17)$
 $= 77 + 85$
 $= 162$

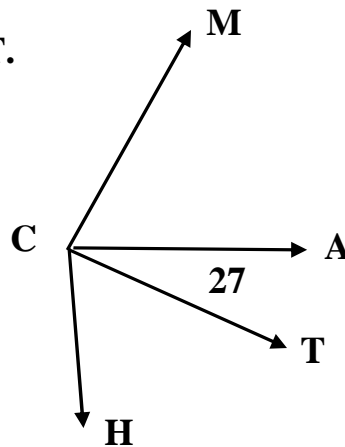
<u>Answers</u>	
1)	P
2)	16
3)	17

Category 2
Geometry
Meet #1 - November, 2020

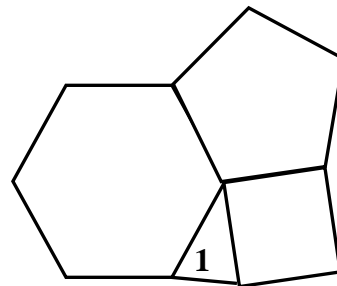
- 1) Lines L1 and L2 intersect. Angle 1 measures 117 degrees. The measure of angle 3 is half the measure of angle 2. How many degrees are in the measure of angle 2 ?



- 2) Angle MCA is complementary to angle ACT.
 Angle ACT is complementary to angle TCH.
 Angle ACT measures 27 degrees.
 How many degrees are in angle MCH if its measure is greater than 180 degrees?



- 3) A regular hexagon and a regular pentagon share a side. The pentagon shares a different side with a square. How many degrees are in the measure of angle 1 ?



<u>Answers</u>	
1)	_____
2)	_____
3)	_____

Solutions to Category 2
Geometry
Meet #1 - November, 2020

- 1) When two lines intersect in a plane, the opposite, or "vertical" angles, are congruent and have the same measure. Since angle 1 measures 117 degrees, then the combined sum of angles 2 and 3 is also 116 degrees. Angle 3 is half of angle 2,

$$\text{so } X + 2X = 117$$

$$3X = 117$$

$$X = 39 \text{ and } 2X = 78.$$

Therefore, the measure of angle 2 is 78 degrees.

- 2) Angle MCH has two measure - one less than 180 degrees and one greater than 180 degrees, such that their sum is 360 degrees. The smaller of the two angles is the sum of right angles MCT and ACH, minus 27 degrees. $90 + 90 - 27 = 153$ degrees. So, the larger angle MCH is $360 - 153$, or 207 degrees.

- 3) Since the triangle is isosceles, as every side length in the diagram is the same, angle 1 is equal to the other base angle. We just need to find the measure of the remaining angle.

Each measure of the regular hexagon is $(4)(180) / 6$, or 120 degrees.

Each measure of the regular pentagon is $(3)(180) / 5$. or 108 degrees.

Each measure of the square is 90 degrees.

At the center point where the polygons meet, let X = the measure of the apex of the isosceles triangle. Then

$$120 + 108 + 90 + X = 360$$

$$318 + X = 360$$

$$X = 42$$

So, the apex angle of the triangle measures 42 degrees and the sum of the base angles is $180 - 42$, or 138 degrees. The two base angles are congruent, so angle 1 measures half of 138, or 69 degrees.

Answers

1) 78

2) 207

3) 69

Category 3
Number Theory
Meet #1 - November, 2020

- 1) What is the smallest positive whole number that is divisible by 2, 3, 4, and 5 ?

- 2) If A = the sum of all prime numbers between 10 and 30 and B = the sum of all composite numbers between 31 and 47 then what is the value of $B - A$?

- 3) There is only one whole number between 100 and 200, not including 100 or 200, that has exactly nine factors, including the number 1 and the number itself. What is that number?

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

Solutions to Category 3
Number Theory
Meet #1 - November, 2020

1) Find the lowest common multiple of 2, 3, 4, and 5.
That number is 60.

2) $A = 11 + 13 + 17 + 19 + 23 + 29$
 $= 112$

$B = 32 + 33 + 34 + 35 + 36 + 38 + 39 + 40 + 42 + 44$
 $+ 45 + 46$
 $= 464$

Then $B - A = 464 - 112 = 352$.

3) Positive whole numbers with an ODD number of factors are perfect squares or, simply, squares. The squares between 100 and 200 are as follows: 121, 144, 169, and 196. With 121 and 169 being squares of primes, those can be quickly eliminated. The number 144 is abundant and has waaaaaay too many factors. The factors of 196 are 1, 2, 4, 7, 14, 28, 49, 98, and 196 - exactly nine factors.

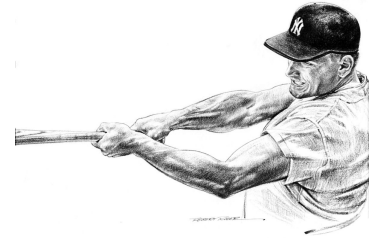
Answers

1) 60

2) 352

3) 196

Category 4
Arithmetic
Meet #1 - November, 2020



- 1) What is the value of $17 + 5(25 - 3 \times 4)$ where the symbol \times is a multiplication sign?
- 2) Babe Ruth broke the all-time Major League Baseball homerun record in 1927, hitting 60 homeruns. The record stood for 34 years until a virtually unknown player with a sub-par career hit "the shot heard 'round the world" on the final day of the 1961 season when Roger Maris, then with the New York Yankees, hit homerun #61. The stem and leaf plot below indicates how many homeruns were hit by Roger Maris in each of his 13 major league seasons. The number in the left-hand column is the tens digit. The number in the right-hand column is the units, or ones digit. What is the mean (average) number of homeruns that Maris hit per season? Round your answer to the nearest whole number.

0	5, 8, 9, 9
1	3, 4, 6, 9
2	3, 6
3	3, 9
6	1

- 3) The mean (average) of nine different positive whole numbers is 17. Each number is greater than 5. Six of the numbers are 7, 10, 16, 21, 28, and 36. What is the largest possible value that one of the remaining three numbers could be?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - November, 2020

1) $17 + 5(25 - 3 \times 4)$
 $= 17 + 5(25 - 12)$
 $= 17 + 5(13)$
 $= 17 + 65$
 $= 82$

2) To compute the average, add the 13 numbers, then divide by 13. Finally, round to the nearest whole number:

$$(5 + 8 + 9 + 9 + 13 + 14 + 16 + 19 + 23 + 26 + 33 + 39 + 61) / 13$$
$$= (275) / 13$$
$$= 21.15 \dots$$

Rounded to the nearest whole number produces an answer of 21.

3) If the average of nine numbers is 17, then their sum is $(9)(17)$, or 153. The sum of the given six numbers is $7 + 10 + 16 + 21 + 28 + 35$, or 117. Compute the difference between 153 and 117 to get the sum of the remaining three numbers, or 36. To maximize the value of one of these three numbers, and to make them all different AND satisfy that they be greater than 5, make two of the numbers as small as possible while satisfying those conditions. Those two smallest numbers are 6 and 8 (not 7, because 7 was already given). Subtract the sum $6 + 8$, or 14, from 36 to get the largest possible remaining number, or 22.

Answers

1) 82

2) 21

3) 22

Category 5

Algebra

Meet #1 - November, 2020

- 1) If $A = 7$ and $B = 3$ and $C = 6$, then evaluate $AB - BC + ABC$.

- 2) What value of  makes the following equation true?

$$3(2 \text{  - 4) + 4 \text{ } = 33 - 5(\text{ } - 3)$$

- 3) If $7 - M = -2$ and $0.75N = 15$ and $-5(7 - E) = -50$ then find the value of H such that the following equation is true:

$$2(M - E) - H = 6N$$

Answers

1) _____

2) _____

3) _____

**Solutions to Category 5
Algebra
Meet #1 - November, 2020**

1) $AB - BC + ABC$
 $= (7)(3) - (3)(6) + (7)(3)(6)$
 $= 21 - 18 + 126$
 $= 3 + 126$
 $= 129$

2) Replacing the turkey with the letter W,
for simplicity:

$$3(2W - 4) + 4W = 33 - 5(W - 3)$$

$$6W - 12 + 4W = 33 - 5W + 15$$

$$10W - 12 = 48 - 5W$$

$$15W = 60$$

$$W = 4$$

<p>3) $7 - M = -2$ $-M = -9$ $M = 9$</p>	<p>$0.75N = 15$ $N = 15 / 0.75$ $N = 20$</p>	<p>$-5(7 - E) = -50$ $-35 + 5E = -50$ $5E = -15$ $E = -3$</p>
---	---	--

<u>Answers</u>	
1)	129
2)	4
3)	-96

Now substitute the values of M, N, and E, from above, into this equation:

$$2(M - E) - H = 6(20)$$

$$2(9 - (-3)) - H = 120$$

$$2(12) - H = 120$$

$$24 - H = 120$$

$$-H = 96$$

$$H = -96$$

Category 6
Team Round
Meet #2 - November, 2020

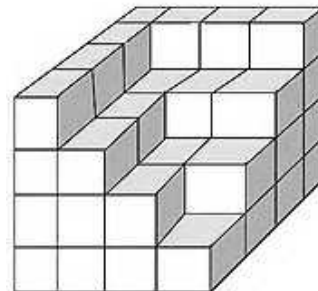
Each of the following nine problems is worth four points.

- 1) The supplement of an angle is 14 degrees more than three times its complement. How many degrees are in that angle?
- 2) The smallest whole number of a 17-member set is 8, the largest is 39, and the median is 22. What is the smallest possible arithmetic mean, or average, of the entire set? Round your answer to the nearest whole number. All 17 numbers are different.
- 3) For the first six days, Paolo ate the following numbers of pumpkin seeds: increasing the numbers of seeds each day according to this pattern: 2, 3, 5, 8, 12, and 17. On which day will Paolo first eat at least 100 seeds?
- 4) Two distinct (different) prime numbers are each less than 90. What is the greatest possible product of those two prime numbers?
- 5) The median of five consecutive prime numbers is 53. What is the mean of the smallest and largest of those five prime numbers?

ANSWERS

- 1) _____
- 2) _____
- 3) _____
- 4) _____
- 5) _____
- 6) _____
- 7) _____
- 8) _____
- 9) _____

- 6) How many consecutive zeroes are at the end of the product $27 \times 28 \times 29 \times 30 \times \dots \times 200$?
- 7) Eight non-collinear lines intersect on the same plane. What is the maximum possible number of intersection points?
- 8) $4W25W$ is a five-digit whole number that is divisible by 18. What is the value of W ?
- 9) This figure consists of congruent cubes. How many of these cubes are in the figure?
There are no cubes missing from below or behind the ones that are visible.



Solutions to Category 6
Team Round
Meet #1 - November, 2020

ANSWERS

- 1) 52
- 2) 20
- 3) 15
- 4) 7387
- 5) 52
- 6) 43
- 7) 28
- 8) 8
- 9) 50

1) Let X = the angle, $90 - X$ its complement, and $180 - X$ its supplement.

$$\text{Then } 180 - X = 3(90 - X) + 14$$

$$180 - X = 270 - 3X + 14$$

$$180 - X = 284 - 3X$$

$$2X = 104$$

$$X = 52$$

2) Half the numbers, or eight numbers, are less than 22 while half are more than 22 but less than 39. To create the smallest possible mean of 17 different numbers, that set must have these numbers: 8, 9, 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 29, and 39. Add those numbers, divide by 17, and round to the nearest whole number. $335 / 17 = 19.705 \dots$ or 20 when rounded to the nearest whole number.

3) 2, 3, 5, 8, 12, 17, 23, 30, 38, 47, 57, 68, 80, 93, 107
 Paolo eats at least 100 seeds on the 15th day.

4) The two prime numbers must be as high as possible but less than 90, so 83 and 89.
 $(83)(89) = 7387$.

5) The five consecutive prime numbers are 43, 47, 53, 59, and 61,
 The mean of the smallest and largest is $(43 + 61) / 2$, or 52.

6) The multiples of 10 produce 18 zeroes. The numbers 100 and 200 each produce an extra zero, for 2 more zeroes. Between multiples of 10 are two numbers whose product ends in a zero, such as 2×5 or any even number $\times 5$, for an additional 17 zeroes. Now for the surprise - there are six more zeroes, due to there being numbers that have additional factors of 5 - one each extra from 50, 75, 150, and 175 and two extra from 125. The total number of zeroes is $18 + 2 + 17 + 6$, or 43 zeroes.

SEE NEXT PAGE FOR SOLUTIONS TO #7 - 9.

7) # of lines # of intersection points

1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28

8) For $4W25W$ to be divisible by 18, it must be divisible by both 2 and by 9. So, W must be even and the sum of all digits must be a multiple of 9. This only works when W is 8. Then $4W25W$ is the 5-digit number 48,258 that is both even and the sum of the digits is 27, a multiple of 9.

9) If the figure were $4 \times 4 \times 4$ with no cubes missing, there would be 64 cubes. The bottom tier is missing none. The tier above it is missing 3, the tier above that is missing 4, and the tier on top is missing 9. The total number missing is $1 + 4 + 9$, or 14. Then $64 - 14 = 50$ cubes. Check: The bottom tier has 16 cubes. The tier above it has 15. The next one has 12 and the top has 7 for a total of $16 + 15 + 12 + 7$, or 50.