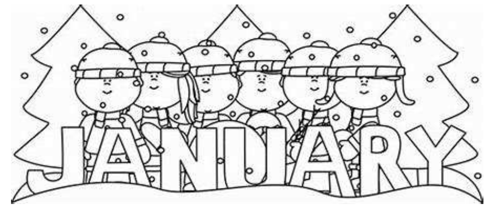
IMLEM Meet #3
January, 2020

# Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • No calculators (or only scientific calculators allowed for meets #4, #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

**Category 1**
**Mystery**
**Meet #3 - January, 2020**



**1) What is the sum of all the different prime factors of 360 ?**

**2) A pair of two-digit numbers, 6A and A2, have a product of 5576. What is the value of the digit A ?**

**3) Arrange the digits 0, 1, 2, and 3 to form a four-digit number that is divisible by the smallest four prime numbers.**

**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 1**
**Mystery**
**Meet #3 - January, 2020**

| **Answers** |
| --- |
| 1) 10 |
| 2) 8 |
| 3) 2310 |

**1) First write the prime factorization of 360:**

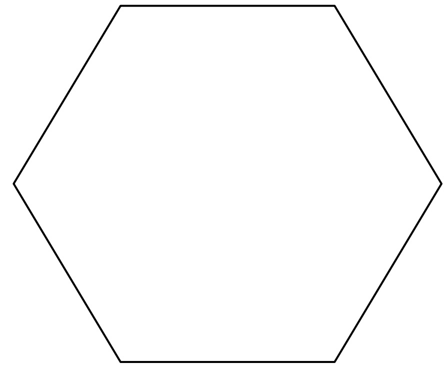
**360 = 2 x 180**
**= 2 x 2 x 90**
**= 2 x 2 x 2 x 45**
**= 2 x 2 x 2 x 3 x 15**
**= 2 x 2 x 2 x 3 x 3 x 5**

**The sum of all the DIFFERENT prime factors is**
**2 + 3 + 5 = 10.**

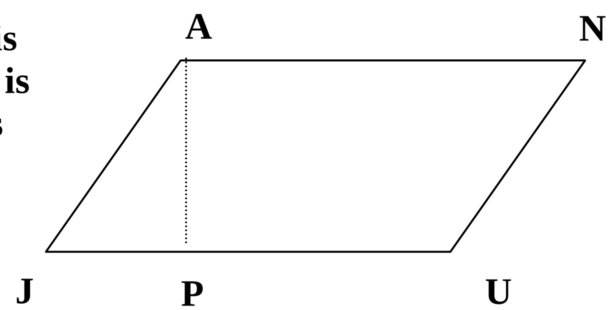
**2) For 5576 to be divisible by a number between 60 and 70, the other factor should be somewhere in the 80s. Guessing and checking and employing some number sense yields 68 x 82 = 5576. Therefore, the value of A is 8.**

**3) The four smallest prime numbers are 2, 3, 5, and 7. The four-digit number must have a units digit of 0 if it is to be divisible by 2 and 5. The sum of the digits, 0 + 1 + 2 + 3, is 6, so the four-digit number is automatically divisible by 3. Now it is a matter of arranging the digits 1, 2, and 3 so that the four-digit number is divisible by 7. The four-digit number is 2310.**

**Category 2**
**Geometry**
**Meet #3 - January, 2020**



**1) A diagonal of a convex polygon, like the one pictured, is a line segment connecting two non-consecutive vertices, or corners. How many diagonals does a regular hexagon have?**




**2) The area of parallelogram JANU is 1104 square meters. Segment AP is perpendicular to JU and measures 24 meters. The length of UP is 28 meters. How many meters are in the perimeter of JANU ?**

**3) Bud challenged Lou to a race at a small local playground. Both are pre-schoolers. Bud is able to run at an average rate of 4 feet per second while Lou can run at 5 feet per second. They start at one corner of the rectangular playground that is 60 yards wide and 80 yards long and finish at the diagonally-opposite corner. Bud ran in a diagonal line across the playground while Lou ran along its length and width. Who won the race? by how many seconds? (You must get both answers correct in order to receive credit.) Note: 3 feet = 1 yard.**

**Answers**

1) _______

2) _______ meters

3) _______ who?

_______ seconds

**Solutions to Category 2**
**Geometry**
**Meet #3 - January, 2020**

| Answers | |
|---|---|
| 1) | 9 |
| 2) | 152 |
| 3) | Bud |
| | 9 |

1) Three diagonals can be drawn from each vertex. That's 18 diagonals but each is counted twice. Therefore, there are just 9 diagonals.

2) Since base x height = area, then base x 24 = 1104, so the base = 1104 / 24, or 46 meters. Subtract 28 from the 46-meter base so that JP = 18 meters. use the Pythagorean Theorem to find the length of JA = 30 meters. So, the parallelogram has two lengths of 30 meters and two lengths of 46 meters. The perimeter of JANU = 2(30) + 2(46) = 60 + 92 = 152 meters.

3) The Pythagorean Theorem yields a diagonal of 100 yards. Lou ran 60 + 80 yards, or a total of 140 yards, or 420 feet. Bud ran along the diagonal of 100 yards, or 300 feet. The time it took for each to run is calculated by dividing the distance by the rate:

Bud: 300 / 4 = 75 seconds
Lou: 420 / 5 = 84 seconds.

Bud finished in less time by 84 - 75, or 9 seconds, and won the race. To receive credit, a student must get both answers correct ( Bud won and by 9 seconds).

**Category 3**
**Number Theory**
**Meet #3 - January, 2020**

**1) The base 10 numeral 310,000 written in scientific notation is $3.1 \times 10^{A}$. What is the value of A ?**

**2) What is the base three numeral that is equivalent to the base two numeral 1100111 ?**

**3) The numerals 3031 and 32 are base 5 numerals. In base 5, what is the value of the quotient when 3031 (in base 5) is divided by 32 (in base 5) ?**

| **Answers** |
| --- |
| 1) ________ |
| 2) ________ |
| 3) ________ |

**Solutions to Category 3**
**Number Theory**
**Meet #3 - January, 2020**

| **Answers** | |
|---|---|
| 1) | 5 |
| 2) | 10211 |
| 3) | 43 |

**1) 310,000 = $3.1 \times 10^5$, so A = 5.**

**2) The base two numeral 1100111, from right to left, has a base 10 value of**
**1(1) + 1(2) + 1(4) + 0(8) + 0(16) + 1(32) + 1(64)**
**= 1 + 2 + 4 + 0 + 0 + 32 + 64**
**= 103.**
**Its base 3 value is**
**= 1(81) + 0(27) + 2(9) + 1(3) + 1(1)**
**= 81 + 0 + 18 + 3 + 1**
**= 103.**
**So, the base 3 value is 10211.**

**3) Converting first to base 10:**
**3031 (in base 5) = 1(1) + 3(5) + 0(25) + 3(125)**
**= 1 + 15 + 0 + 375**
**= 391 (in base 10)**
**32 (in base 5) = 2(1) + 3(5)**
**= 2 + 15**
**= 17 (in base 10)**
**In base 10, 391 divided by 17 = 23.**
**Converting back to base 5:**
**23 (in base 10) = 4(5) + 3(1) (in base 5) = 43 (in base 5)**

**Category 4**
**Arithmetic**
**Meet #3 - January, 2020**

**1) Evaluate:** $\sqrt{25} + \sqrt[3]{216} - \sqrt[4]{10,000} + \sqrt[5]{243} - \sqrt[6]{64}$

**2) How many whole numbers are between** $\sqrt[3]{72}$ **and** $\sqrt[4]{7200}$ **?**

**3) What is the value of F if** $\frac{15}{27} = \sqrt{\frac{25}{F}}$ **?**

**ANSWERS**

1) ________

2) ________

3) ________

**Solutions to Category 4**
**Arithmetic**
**Meet #3 - January, 2020**

| **Answers** |
|---|
| 1) 2 |
| 2) 5 |
| 3) 81 |

1) $\sqrt{25} + \sqrt[3]{216} - \sqrt[4]{10{,}000} + \sqrt[5]{243} - \sqrt[6]{64}$

$= 5 + 6 - 10 + 3 - 2$
$= 2$

2) $\sqrt[3]{72}$ is between 4 and 5, as 4 cubed is 64 and 5 cubed is 125.

$\sqrt[4]{7200}$ is between 6561 and 10,000, as 9 to the 4th power is 6561 and 10 to the 4th power is 10,000.

So, the whole numbers between $\sqrt[3]{72}$ and $\sqrt[4]{7200}$ are 5, 6, 7, 8, 9.
That makes 5 such whole numbers.

3) $\frac{15}{27} = \sqrt{\frac{25}{F}}$

Squaring both sides of the equation produces $\frac{225}{729} = \frac{25}{F}$.

Cross products are equal:

$225\text{ F} = (25)(729)$
$225\text{ F} = 18{,}225$
$\text{F} = 81$

Another, perhaps simpler approach, is to simplify 15 / 27 to 5 / 9.
Then the numerator, 5, is the square root of 25 and the denominator, 9, is the square root of 81.

**Category 5**
**Algebra**
**Meet #3 - January, 2020**

1) **Evaluate this absolute value expression:**

$$|16| + |0| - |-9|$$

2) **Find the sum of all possible integers, F, that make this absolute value inequality true:**

$$|3F - 2| \leq 17$$

3) **If $10 + 4(3Y - 5) < 98$ and $10 - 4(3Y - 5) < -6$, then the set of all possible values of Y is represented in the graph below. What is the value of L + M ?**


**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 5**
**Algebra**
**Meet #3 - January, 2020**

| Answers |
|---|
| 1) 7 |
| 2) 6 |
| 3) 12 |

1) $|16| + |0| - |-9| = 16 + 0 - 9 = 7.$

2) $|3F - 2| \leq 17$

$$3F - 2 \leq 17 \quad \text{and} \quad 3F - 2 \geq -17$$
$$3F \leq 19 \quad \text{and} \quad 3F \geq -15$$
$$F \leq 19/3 \quad \text{and} \quad F \geq -5$$

The integers that satisfy this condition are
{ -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 }
The sum of these integers is 6.

3) $10 + 4(3Y - 5) < 98 \quad \text{and} \quad 10 - 4(3Y - 5) < -6$

$$10 + 12Y - 20 < 98 \quad \text{and} \quad 10 - 12Y + 20 < -6$$
$$12Y < 108 \quad \text{and} \quad -12Y < -36$$
$$Y < 9 \quad \text{and} \quad Y > 3$$

(Note: Watch for the reversal of the sense of inequality when dividing both sides by a negative number!)

On the graph, therefore, L = 3 and M = 9. The sum L + M = 3 + 9 = 12.

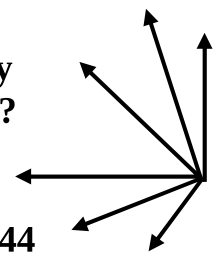
**Category 6**
**Team Round**
**Meet #3 - January, 2020**

***Each of the following nine problems is worth <u>four</u> points.***

1) The three colors of candies in the dish - red, blue, and yellow, respectively - are in the ratio 3 : 8 : 9. What fraction of the candies, in lowest terms, are blue?

2) What fraction, in lowest terms, of the times on a 12-hour digital clock have the digit 7 at least once? All times include just hours & minutes.

3) Six rays intersect at a common point, as shown. How many angles measuring between 0 and 180 degrees are formed? (Note: All angles have different measures.)

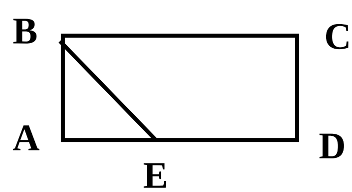


4) If you square all of the positive whole-number factors of 144 and then multiply the resulting factors, the result is equal to $144^N$. What is the value of N?

5) Xavier's age is 2/3 of Yolanda's age. Yolanda's age is 2/3 of Zorba's age. The sum of Xavier's age and Yolanda's age is 20 years. How many years old will Zorba be 10 years from now?

6) What is the remainder when $3^{333}$ is divided by 10?

7) ABCD is a rectangle. AE = 12. BC = 23. The area of triangle ABE = 30 square units. How many units are in the perimeter of trapezoid BCDE?




8) What 2-digit positive whole number is three times the sum of its digits?

9) A, B, and the sum A + B are prime numbers.

$$A^3B^2 + A^2B^3 = 700$$

What is the value of the product AB ?

**<u>ANSWERS</u>**

1) ________

2) ________

3) ________

4) ________

5) ________

6) ________

7) ________

8) ________

9) ________

**Solutions to Category 6**
**Team Round**
**Meet #3 - January, 2020**

| ANSWERS | |
|---|---|
| 1) | $\frac{2}{5}$ |
| 2) | $\frac{7}{40}$ |
| 3) | 15 |
| 4) | 15 |
| 5) | 28 |
| 6) | 3 |
| 7) | 52 |
| 8) | 27 |
| 9) | 10 |

1) The total of the parts is 3 + 8 + 9 = 20. The blue part is 8 / 20, or 2 / 5.

2) Each of the twelve hours has six different times that contain the digit 7 at least once, at :07, :17, :27, :37, :47, and :57. There is one exception - from 7:00 through 7:59, the digit 7 appears a total of 60 times. 11(6) + 60 = 126.

$$\frac{126}{(60)(12)} = \frac{126}{720} = \frac{63}{360} = \frac{21}{120} = \frac{7}{40}$$

3) There are 5 individual angles formed by adjacent rays, 4 angles formed by pairs of consecutive angles, 3 angles formed by trios of consecutive angles, 2 angles formed by quads of consecutive angles, and 1 angle formed by all five consecutive angles. That is a total of 5 + 4 + 3 + 2 + 1 = 15 angles.

4) The positive factors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, and 144. They pair up to produce products of 144. There are 7 such pairs plus the lone unpaired factor of 12. Squaring each factor produces another set of 7 pairs that make products of 144, plus another lone factor of 12 which can pair up with the another 12 to make another product of 144. That now makes a total of 15 products of 144 so that N = 15.

5) Guessing and checking can quickly reveal that Xavier = 8, Yolanda = 12, and Zorba = 18. In 10 years, Zorba will by 28.

6) Consecutive positive integral powers of 3, when divided by 10, yield the following remainders: 0, 0, 7, 1, 3, 9, 7, 1, 3, 9, 7, 1 and so on, with the digits 7, 1, 3, and 9 occurring in a repeating pattern. The problem now is to determine which of these digits will be in the units place for the 333rd power of 3. How many groups of these four digits

**SEE NEXT PAGE THE REMAINDER OF THE SOLUTIONS.**

**occur after the initial pair of zeros? Divide 333 by 4 to get how many groups of four digits there are. 333 / 4 = 83 with remainder 1 or 82 with remainder 5. Let's go with the latter. Think of the 333rd number in this sequence as it follows three digits after the sequence of 0, 0, and 82 groups of four digits in the given order of 7, 1, 3, and 9. Therefore, the 333rd units digit is 3.**

**7) The area of triangle ABE = 30 square units = 0.5 B H = 0.5 (12) (H). Therefore, the height, AB, is 30 / 6, or 5 units. The diagonal length, BE, is the hypotenuse of triangle ABE whose legs are 12 and 5, so BE = 13 units. The perimeter of trapezoid BCDE is 23 + 5 + 11 + 13, or 52 units.**

**8) Let T = the ten's digit and U = the units digit.**
**Then 10T + U = 3 (T + U)**
**10T + U = 3T + 3U**
**7T = 2U**
**Since T and U are digits whose values can only be selected from 0 - 9, the only values that make 7T = 2U are when T = 2 and U = 7. Then the two-digit number 27 is three times the sum of its digits.**

**9)** $A^3B^2 + A^2B^3 = 700$ **Express the left member in factored form:** $A^2B^2\,(A + B) = 700$.

**Now prime factor 700 as 2 x 2 x 5 x 5 x 7, or** $2^2 \times 5^2 \times 7$.

**Then A and B are, interchangeably, 2 and 5. Then 2 + 5 = 7 and 2 x 5 = 10. The product of AB is, therefore, 10.**