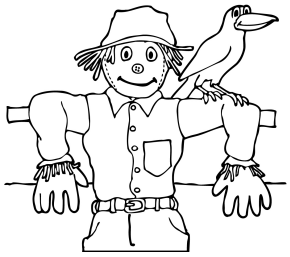
IMLEM Meet #2
November, 2019

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #2 - November, 2019



1) How many hours are there in 9 weeks?

2) The square of an integer is the result of multiplying an integer by itself. For example, the square of 4 is 16 because 4 x 4 = 16. How many positive integers (whole numbers) have squares that are between 60 and 600?

3) Patients at a local hospital used 240 boxes of tissues in 3 days. There are 288 tissues in each box. How many tissues, on average, were used each minute?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #2 - November, 2019

Answers

1) 1512

2) 17

3) 16

1) There are 7 days in a week and 24 hours in a day. Multiply (7)(9)(24) to get 1512.

2) The squares of the following integers fall between 60 and 600:

Integer	8	9	10	11	12	13	14	15	16	17	18	19
Square	64	81	100	121	144	169	196	225	256	289	324	361

Integer	20	21	22	23	24
Square	400	441	484	529	576

Counting the consecutive integers from 8 through 24, inclusive, yields a total of 17 integers.

3) 240 boxes in 3 days reduces to 80 boxes per day. Multiply 80 boxes per day by 288 tissues per box to get 23,040 tissues total. The number of minutes in a day is (60)(24), or 1,440. Divide 23,040 by 1,440 to get the number of tissues per minute, or 16.

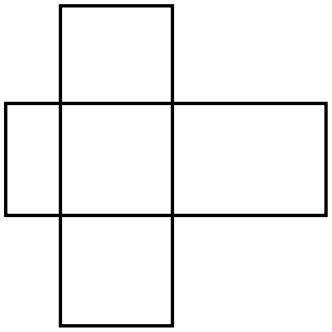
For students who prefer to set up ratios and then reduce / cross cancel to simplify the arithmetic, the following example can provide such a simplified approach:

$$\left(\frac{1 \text{ hour}}{60 \text{ min}}\right)\left(\frac{1 \text{ day}}{24 \text{ hours}}\right)\left(\frac{240 \text{ boxes}}{3 \text{ days}}\right)\left(\frac{288 \text{ tissues}}{1 \text{ box}}\right)$$

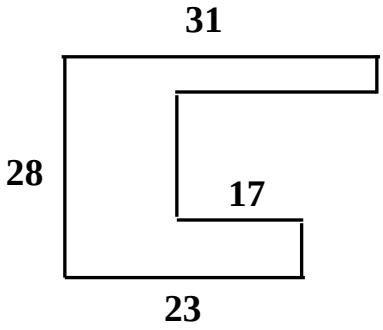
Category 2
Geometry
Meet #2 - November, 2019

1) What is the greatest number of non-overlapping squares of perimeter 12 inches that can be cut from a square that has an area of 225 square inches?

2) Two identical rectangles, each with an area of 51 square feet, intersect as shown. The dimensions of each rectangle are prime numbers. All angles at points of intersection are right angles. How many feet are in the perimeter of the outer 12-gon (12-sided polygon)?



3) How many inches are in the perimeter of this figure? All angles are right angles. All measurements are in feet and are not to scale.



Answers

1) _____

2) _____ feet

3) _____ inches

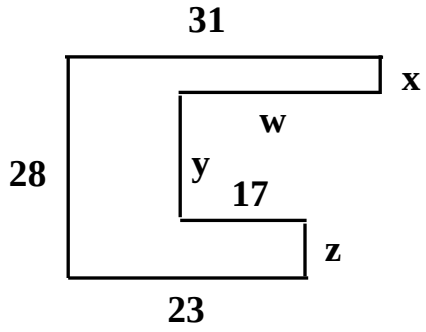
Solutions to Category 2
Geometry
Meet #2 - November, 2019

Answers	
1)	25
2)	68
3)	1824

1) The cut squares of perimeter 12 inches are 3" by 3". The square from which they are cut, having an area of 225 square inches, measures 15" by 15". One can fit five rows of five, or 25 of the smaller squares to tessellate inside the larger square.

2) If the dimensions of each rectangle are prime numbers, then the length and width are 3 and 17 feet, respectively, as 3 and 17 are the only prime factors of 51. The overlapping square measures 3' by 3'. If the two rectangles did not overlap, then each would have a perimeter of 2(3) + 2(17), or 40 feet. Subtract the four lengths of 3' of the overlapping square to get the perimeter of the outer 12-gon: 2(40) - 4(3) = 68 feet.

3) There are four lengths with no measurement labels. The sum of the three vertical segments is equal to the height of the figure, or 28 feet. From the diagram, x + y + z = 28.
The remaining length, the horizontal one, w, can be found as follows: Subtract 17 from 23 to get the width of the left-most vertical space, or 6 feet.
Now subtract 6 from 31 to get the length of the remaining horizontal segment, which is 25 feet.
The perimeter, in feet, is
28 + 31 + 17 + 23 + 28 + 25 = 152 feet.
The question asks for this measurement in inches, so
152(12) = 1824 inches.



Category 3
Number Theory
Meet #2 - November, 2019

1) The prime factorization of 210 is A x B x C x D where A, B, C, and D are different positive prime number divisors of 210 and the symbol "x" is a multiplication sign. What is the value of A + B + C + D ?

2) Willie whistles every 18 minutes, hums every 24 minutes, and howls every 42 minutes. If he whistled, hummed, and howled at 7:12 A.M., then at what time will he next whistle, hum, and howl? You MUST include either A.M. or P.M., whichever is correct, to earn credit for a correct answer.

3) If $\frac{A}{B}$ is the lowest terms form of the fraction $\frac{924}{18{,}018}$, then what is the value of A + B ?

Answers

1) _____

2) _____ ____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - November, 2019

Answers	
1)	**17**
2)	**3:36 P.M.**
3)	**41**

1) 210 = 2 x 105
= 2 x 3 x 35
= 2 x 3 x 5 x 7
The sum 2 + 3 + 5 + 7 = 17.

2) First, find the LCM of 18, 24, and 42:
18 = 2 x 3 x 3
24 = 2 x 2 x 2 x 3
42 = 2 x 3 x 7
The LCM is 2 x 2 x 2 x 3 x 3 x 7 = 504.
Next, add 504 minutes to the time of 7:12 A.M. The number of hours in 504 minutes is 504 / 60, or 8 hours with a remainder of 24 minutes. Adding 8 hours and 24 minutes to 7:12 A.M. yields 3:36 P.M. The student MUST include the P.M. in order to get full credit. There is no partial credit for an answer of just 3:36.

3) Prime factoring the numerator and denominator reveals the factors that are "cancellable" to facilitate the simplification of the original fraction.

$$\frac{924}{18{,}018} = \frac{2x2x3x7x11}{2x3x3x7x11x13} = \frac{2}{3x13} = \frac{2}{39}$$

Therefore, A + B = 2 + 39 = 41.

Category 4
Arithmetic
Meet #2 - November, 2019

1) A televised Patriots football game lasted three hours and 24 minutes, consisting of actual football programming and commercials. The total time of the commercials was 51 minutes. What percent of the televised Patriots football game was actual football programming?

2) In lowest terms, what fraction is the sum of $0.36 + 0.3\overline{6}$ **?**

Reminder: $0.3\overline{6}$ **= 0.36666 . . .**

3) E  H **is defined to be equal to** $\dfrac{\frac{1}{E} - \frac{1}{H}}{\frac{1}{E} + \frac{1}{H}}$**. Find the value of**

6  15. **Express your answer as a fraction in lowest terms.**

Answers

1) _____ **%**

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - November, 2019

Answers

1) 75 (%)

2) $\frac{109}{150}$

3) $\frac{3}{7}$

1) Three hours and 24 minutes = 180 + 24 minutes = 204 minutes. 204 - 51 = 153, the number of minutes of football programming. The percent of football programming is 153 / 204, or 3/4, or 75%.

2) 0.36 + 0.3666666 . . . = 0.7266666 . . .
To convert this sum to a fraction, use this subtraction technique:

Let 10X = 7.2666666 . . .
X = 0.7266666 . . .
Subtract bottom from top: 9X = 6.54
Divide both sides by 9: X = 6.54 / 9
Multiply top & bottom by 100: X = 654 / 900
Then simplify, or reduce to lowest terms: 654 / 900 = 327 / 450 = 109 / 150.

3) 6  15 $= \dfrac{\frac{1}{6} - \frac{1}{15}}{\frac{1}{6} + \frac{1}{15}} = \dfrac{\frac{5}{30} - \frac{2}{30}}{\frac{5}{30} + \frac{2}{30}} = \dfrac{\frac{3}{30}}{\frac{7}{30}} = \frac{3}{7}.$

Category 5
Algebra
Meet #2 - November, 2019

1) If 3A + 2C = 17, then what is the value of 15A + 10C - 37 ?

2) The first formula converts a temperature in degrees Kelvin, K, to degrees Fahrenheit, F. The second formula converts degrees in Fahrenheit, F, to degrees Celsius, C. How many degrees Celsius is a temperature that is 288.15 degrees Kelvin?

$$\frac{9(K-273.15)}{5}+32=F$$

$$\frac{5(F-32)}{9}=C$$

3) A coin collection consists of twice as many dimes as quarters and seven more nickels than dimes. There are only quarters, nickels, and dimes. If there had been eleven more nickels, the value of the entire collection of coins would have been $ 13.55. How many dimes are there?

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - November, 2019

Answers	
1)	48
2)	15
3)	46

1) If 3A + 2C = 17, then five times that, or 5(3A + 2C), is 15A + 10C, or 5(17), or 85.
Then 15A + 10C - 37 = 85 - 37, or 48.

2) First substitute 288.15 for K in the first formula to find the value of F. Then substitute that value for F into the second formula to solve for C:

$$\frac{9(K-273.15)}{5}+32=F \qquad \frac{5(F-32)}{9}=C$$

$$\frac{9(288.15-273.15)}{5}+32=F \qquad \frac{5(59-32)}{9}=C$$

$$\frac{9(15)}{5}+32=F \qquad \frac{5(27)}{9}=C$$

$$27+32=F \qquad 15=C$$

$$59=F$$

Therefore, the Celsius temperature is 15 degrees.

3) Let X = the number of quarters
2X = the number of dimes
2X + 7 = the number of nickels
Then 25X = the value of the quarters, in cents
10(2X) = the value of the dimes, in cents
5(2X + 7) = the value of the nickels, in cents

Write an equation that reflects the fact that, if there had been eleven more nickels (worth a total of 55 cents), then the total value of the coin collection would have been $ 13.55, and then solve for X:

$$25X + 10(2X) + 5(2X + 7) + 55 = 1355$$
$$25X + 20X + 10X + 35 + 55 = 1355$$
$$55X + 90 = 1355$$
$$55X = 1265$$
$$X = 23, \text{ the number of quarters}$$
$$2X = 46, \text{ the number of dimes}$$

Therefore, there are 46 dimes.

Category 6
Team Round ***Each of the following nine***
Meet #2 - November, 2019 ***problems is worth <u>four</u> points.***

1) X and Y are integers whose product is 7. What is the smallest possible integer value of X^Y ?

2) What is the largest prime factor of the product of all the even numbers from 2 through 200, inclusive?

3) A diagonal of a cube is a line segment connecting any two of its non-consecutive vertices, or corner points. Including the face (surface) and space diagonals, what is the total number of diagonals of a cube?

4) The degree measures of the angles of a certain triangle are in the ratio 2:5:8. The largest angle is how many degrees greater than the smallest?

5) If I divide a number, E, by its reciprocal, the quotient is seven times as large as E. What is the value of E?

6) In a mix of 252 nuts of just walnuts and peanuts, , there are two walnuts for every seven peanuts. How many peanuts are in the mix?

<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

7) Three consecutive odd integers have a sum of 261. What is the largest of the three integers?

8) If 

means "subtract 2 from W,"

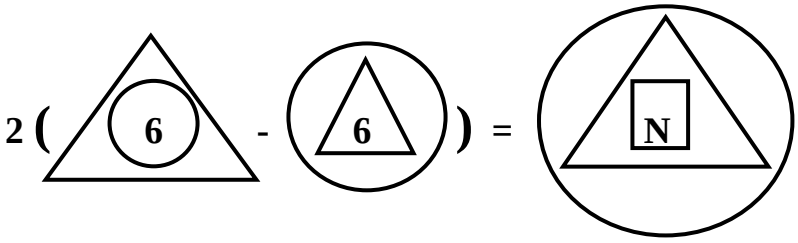
and 

means "multiply W by 5,"

and 

means "square W,"

then find the positive decimal value of N , such that



See the next page for problem #9.

9) The total number of spheres needed to create a triangular pyramid of spheres is given by the formula

$$N = \frac{X^3}{6} + \frac{X^2}{2} + \frac{X}{3}$$

where X is the number of layers in the pyramid and N is the total number of spheres. How many spheres are in the bottom, or base, of an 8-layer pyramid?

Solutions to Category 6
Team Round
Meet #2 - November, 2019

ANSWERS	
1)	- 1
2)	97
3)	16
4)	72
5)	7
6)	196
7)	89
8)	1.6
9)	36

1) There are four possible values for X and Y, such that their product is 7, as given by these four ordered pairs: (1, 7), (7, 1), (-1, -7), and (-7, -1). Replacing X and Y with these values produces the following answers, in this order: 1, 7, -1, and -1/7. The smallest of these is -1.

2) The largest prime factor of these even numbers is 97, as the even number 194 has 97 as one of its prime factors. The product of the even numbers from 2 through 200, inclusive, has 97 as its largest prime factor.

3) Each of the six faces has two diagonals, plus there are four space diagonals, with each connecting two opposing vertices. So, there are 6(2) + 4, or 16 diagonals.

4) Let X be the scaling factor: 2X + 5X + 8X = 180, then 15X = 180 and X = 12. The largest angle is 8(12), or 96 degrees, and the smallest angle is 2(12), or 24 degrees, and their difference is 96 - 24, or 72 degrees.

5) E / 1/E = 7E . . . then E squared = 7E, so E = 0 or E = 7. However, if E = 0, then E has no reciprocal. So, E = 7.

6) Let N = the scaling factor. Then 2X + 7X = 252, 9X = 252, and X = 28. The number of peanuts in the mix is 28(7), or 196.

7) Let the three consecutive odd integers be represented by T, T + 2, and T + 4. Then T + (T + 2) + (T + 4) = 261 . . . 3T + 6 = 261, 3T = 255, and T = 85. The largest of the integers is 85 + 4, or 89.

The solutions to problems #8 and #9 are on the next page.

8) The symbols in this equation can be translated into this simpler version:

$$2[(6)(6) - 2 - (6 - 2)(6 - 2)] = (5N - 2)(5N - 2)$$
$$2[36 - 2 - (4)(4)] = (5N - 2)(5N - 2)$$
$$2[18] = (5N - 2)(5N - 2)$$
$$36 = (5N - 2)(5N - 2)$$

Then square root both sides:

$$6 = 5N - 2 \quad \text{or} \quad -6 = 5N - 2$$
$$5N = 8 \quad \text{or} \quad -4 = 5N$$
$$N = 8/5 \quad \text{or} \quad -4/5 = N$$

The question required that the answer be a *positive* decimal, so 8/5 = 1.6.

9) To find the number of spheres in the 8th layer, one may perform the following steps:

(a) Find the number of spheres in an 8-layer pyramid,
(b) find the number of spheres in a 7-layer pyramid, and finally
(c) subtract the answer to part (b) from the answer to part (a).

(a) $N = \frac{X^3}{6} + \frac{X^2}{2} + \frac{X}{3}$

$N = \frac{8^3}{6} + \frac{8^2}{2} + \frac{8}{3}$

$N = \frac{512}{6} + \frac{64}{2} + \frac{8}{3}$

$N = \frac{512}{6} + \frac{192}{6} + \frac{16}{6}$

$N = \frac{720}{6}$

$N = 120$

(b) $N = \frac{X^3}{6} + \frac{X^2}{2} + \frac{X}{3}$

$N = \frac{7^3}{6} + \frac{7^2}{2} + \frac{7}{3}$

$N = \frac{343}{6} + \frac{49}{2} + \frac{7}{3}$

$N = \frac{343}{6} + \frac{147}{6} + \frac{14}{6}$

$N = \frac{504}{6}$

$N = 84$

(c) 120 - 84 = 36.

Therefore, there are 36 spheres in the 8th row of the pyramid.