IMLEM Meet #5
April, 2019

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery *Calculator Meet*
Meet #5 - April, 2019

1) The day after the day after tomorrow is four days before Monday. What day is today?

2) Six horses eat six bales of hay in six hours. At the same rate, how many hours would it take three horses to eat three bales of hay?

3) More than 20 students but fewer than 40 students paid a total of $ 527 for Livingston Taylor concert tickets back in the year 1998. If each ticket cost the same whole number of dollars, then how many students were in the group?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #5 - April, 2019

Answers
1) Monday
2) 6
3) 31

1) Listing the days of the week may help. Then work backwards from Monday to discover that today is . . . Monday!

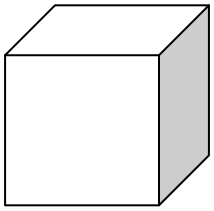
2) The first sentence implies that it takes six hours for each horse to eat one bale of hay. As long as the horses:bales ratio is 1:1, then it takes six hours.

3) The number 527 was purposefully created to be the product of two prime numbers, one of which lies between 20 and 40 and the other does not.
527 = 31 x 17, a result that students can quickly achieve with a calculator.
Therefore, one ticket costs $17 and the number of students is 31.

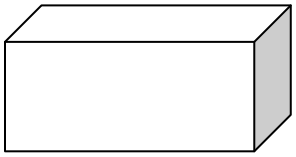
Category 2
Geometry
Meet #5 - April, 2019

Calculator Meet

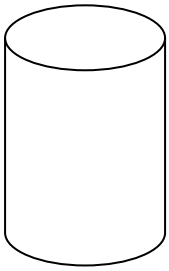
1) The total surface area of a cube is 150 square centimeters. How many cubic centimeters are in its volume?



2) Each edge of a rectangular solid is a prime number of inches long. The volume of the solid is 165 cubic inches. How many square inches are in its total surface area?



3) How many 1-gallon cans of paint must Vincent buy in order to paint the entire inside surface of a closed cylindrical tank, including the floor and ceiling, that is 67 feet high and 23 feet in diameter? One gallon of paint covers 432 square feet. Use $\pi \approx 3.14$.



Answers

1) _____ cu. cm

2) _____ sq. in.

3) _____ cans

Solutions to Category 2
Geometry
Meet #5 - April, 2019

Answers
1) 125
2) 206
3) 14

1) There are six congruent square faces on a cube. Divide the given total surface area (150) by 6 to get the area of one surface (25). Then take the square root of 25 to get the length of one edge (5). Finally, cube 5 to get the volume: 5x5x5 = 125 cubic centimeters.

2) Prime factor the volume of 165: 165 = 3 x 5 x 11.
The total surface area = 2lw + 2lh + 2wh
= 2(3)(5) + 2(3)(11) + 2(5)(11)
= 30 + 66 + 110
= 206 square inches.

3) Find the inside surface area, in square feet, of the cylinder. Then divide by the number of square feet that one can of paint will cover. Any remainder must be rounded *UP*:

Total surface area = the area of two circles plus the wraparound rectangle
= 2(pi)(radius)(radius) + (height)(circumference)
= 2(3.14)(11.5)(11.5) + (67)(2)(3.14)(11.5)
= 830.53 + 4838.74
= 5669.27 square feet

The number of paint cans = 5669.27 divided by 432
= 13.12331

Round up so that the number of cans is 14.

Category 3
Number Theory
Meet #5 - April, 2019

Calculator Meet

1) Forty-three students on the track team run long-distance races while 37 run sprint races. There are 54 students on the team. Every student runs at least one race. How many students run both a long-distance race and a sprint race?

2) Set X = { multiples of 8 between 30 and 90 }
Set Y = {Factors of 240 }
$X \cap Y$ represents the intersection of set X and set Y, that is, the set of all elements (in this case, numbers) that belong to both set X and set Y. How many elements belong to $X \cap Y$?

3) There are 411 students at the Norwood Coakley Middle School.

- **166 like Snickers,**
- **201 like Reese's Peanut Butter Cups**
- **178 like Butterfingers,**
- **80 like Snickers and Reese's Peanut Butter Cups,**
- **90 like Snickers and Butterfingers,**
- **74 like Reese's Peanut Butter Cups and Butterfingers, and**
- **51 like all three candies.**

How many students at the Norwood Coakley Middle School do not like any of the three candies?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #5 - April, 2019

Answers	
1)	26
2)	3
3)	59

1) Since the number of distance runners plus the number of sprint runners exceeds the total number of students on the team, the overlap of those who run both types of races is X:

$43 + 37 - X = 54$
$80 - X = 54$
$X = 26$

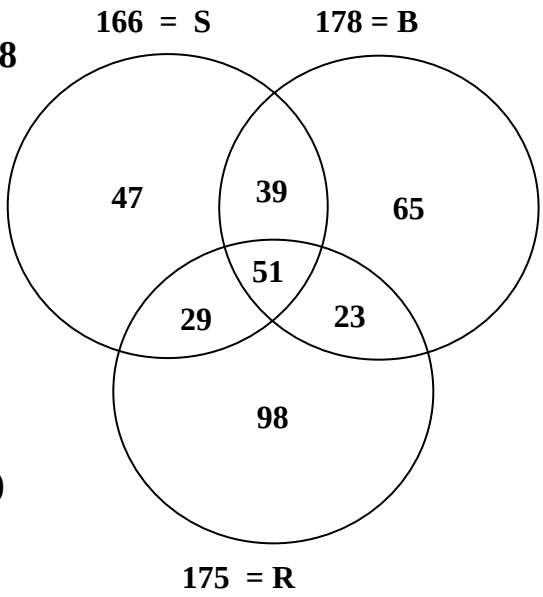
Therefore, the number of students who run both the long-distance races and the sprint races is 26.

2) Set X = { multiples of 8 that are between 30 and 90 }
= { 32, 40, 48, 56, 64, 72, 80, 88 }
Set Y = { 1, 2, 3, 4, 5, 6, 8, 10, 12, 20, 24, 30, 40, 48, 60, 80, 120, 240 }
$X \cap Y$ = { 40, 48, 80 }
Therefore, there are three elements in the intersection of sets X and Y.

3) Since all three activities overlap, the following Venn diagram can help organize the given data. Let S = Snickers, B = Butterfingers, and R = Reese's:

Checking all conditions:

* **Snickers = 47 + 39 + 51 + 29 = 166**
* **Butterfingers = 39 + 51 + 23 + 65 = 178**
* **Reese's = 29 + 51 + 23 + 98 = 175**
* **S and R = 51 + 29 = 80**
* **S and B = 39 + 51 = 90**
* **R and B = 51 + 23 = 74**



The number of students who do not like any of the three candies is the difference between the total number of students and the sum of all the numbers in the Venn Diagram:

$= 411 - (47 + 39 + 65 + 51 + 29 + 23 + 98)$
$= 411 - (352)$
$= 59.$

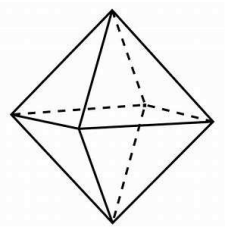
Therefore, only 59 students do not like any of the three candies. On the next page, see one way to actually arrive at the solution.

Starting with the fact that 51 like all three, Subtract 51 from 80 to get 29 - the rest of those who like both snickers and Reese's. Subtract 51 from 74 to get 23 - the rest of those who like Reese's and Butterfingers. Subtract 51 from 90 to get 39 - the rest of those who like Butterfingers and Snickers. Subtract 29 + 51 + 39, or 119, from 166 to get 47 - the number who like just snickers. Subtract 39 + 51 + 23, or 113, from 178 to get 65 - the number who like just Butterfingers. Subtract 29 + 51 + 23, or 103, from 201 to get 98 - the number who like just Reese's. Finally, subtract the total of the numbers in the three intersecting circles, or 352, from 411 to get 59 - the number who do not like any of the three candies.

Category 4
Arithmetic ***Calculator meet***
Meet #5 - April, 2019

1) Frank was carrying the 12 pages of a yearly calendar but accidentally dropped all 12 sheets. He gathered them up off the floor. What is the probability that the sheet on top was from a month starting with the letter "J?" Express your answer as a common (simplified) fraction.

2) Two octahedral dice, with faces numbered 1, 2, 3, 4, 5, 6, 7, and 8 are rolled. What is the probability that the sum of the numbers showing on the bottom surfaces of the two dice is less than ten? Express your answer as a common (simplified) fraction.



3) The letters of the word EXCELLENT are arranged to form other "words," with a "word" defined as any 9-letter arrangement of the letters in the word EXCELLENT. How many different words are possible, including the original word?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #5 - April, 2019

Answers	
1)	$\frac{1}{4}$
2)	$\frac{9}{16}$
3)	30,240

1) Of the 12 months, three begin with the letter "J,", namely January, June, and July. Hence, 3/12 of the calendar months begin with the letter "J," or 1/4 in simplified form.

2) The outcomes of the two octahedral dice are arranged along the top row and left-side column, with the sums of the two bottom faces in the interior of the grid:

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Sums less than 10 lie along diagonals that contain the sums 2, 3, 4, 5, 6, 7, 8, and 9. There are 36 such sums of the 64 total sums. The probability of those 36 sums is 36/64, or 9/16 as a common (simplified) fraction.

3) If all nine letters were different, there would be 9! possible arrangements of those letters. But since some of those letters repeat, we must divide by the number of repetitions as follows, accounting for the three Es and two Ls:

$$\frac{9!}{(3!)(2!)}=\frac{(9)(8)(7)(6)(5)(4)(3)(2)(1)}{[(3)(2)(1)][(2)(1)]}=(9)(8)(7)(5)(4)(3)(1)=30,240$$

Therefore, there are 30,240 ways that the letters in the word EXCELLENT can be arranged to form different "words."

Category 5
Algebra ***Calculator Meet***
Meet #5 - April, 2019

1) There are two values of N that make the following quadratic equation true. What is the sum of those two values of N?

$$(N + 4)(N - 9) = 0$$

2) There are two values of W that make the following quadratic equation true. What is the positive difference, or the absolute value of the difference, between those two values of W?

$$10W = W^2 - 24$$

3) A rocket is launched vertically from ground level at an initial velocity (starting speed) of 128 feet per second. For how many seconds is the rocket at least 112 feet above ground level?

Use the quadratic equation $y = gt^2 + vt + h$

where **g = - 16 feet/second/second, the constant of gravity at the surface of the Earth,**
t is the time in seconds that the rocket is in the air,
v is the initial velocity,
h is the initial height of the rocket in feet, and
y is the height in feet of the rocket at any time t seconds.

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #5 - April, 2019

Answers	
1)	5
2)	14
3)	6

1) If (N + 4)(N - 9) = 0, then either N + 4 = 0 or N - 9 = 0, then N = - 4 or N = 9. The sum of these solutions is -4 + 9, or 5.

2)

$$10W = W^2 - 24$$
$$0 = W^2 - 10W - 24$$
$$0 = (W-12)(W+2)$$

So, either W = 12 or W = -2. The positive difference between them is 12 - (-2), or 14.

3) Substitute: G = - 16; V = 128; Y = 112; H = 0.

$112 = (-16)(T^2) + 128T + 0$ **Use the substitutions listed above.**

$0 = -16(T^2) + 128T - 112$ **Subtract 112 from both sides.**

$0 = T^2 - 8T + 7$ **Divide both sides by - 16.**

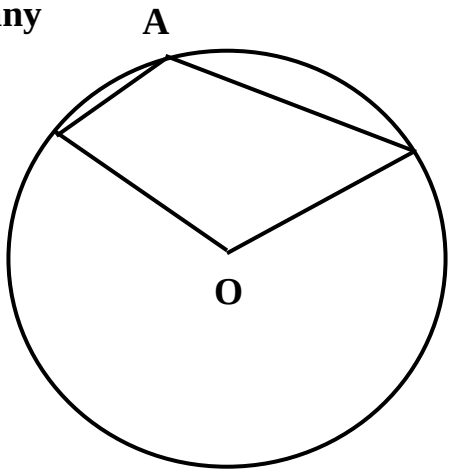
$0 = (T-1)(T-7)$ **Factor.**

T = 1 or T = 7. Therefore, the rocket was at or above 112 feet above the ground from 1 second until 7 seconds into the flight, so the rocket was in flight during the interval $1 \le T \le 7$ or the difference 7 - 1, or 6 seconds.

Category 6
Team Round
Meet #5 - April, 2019

Each of the following nine problems is worth <u>four</u> points.

1) Inscribed angle A = 100 degrees. How many degrees are in central angle O?



2) The five-digit number 40585 has this fascinating property that the sum of the factorials of its digits is equal to the number itself, 40585, as follows:

40585 = 4! + 0! + 5! + 8! + 5!
= 24 + 1 + 120 + 40320 + 120
= 40585.

The only three other numbers with this property are 1, 2, and N, where N is a whole number between 120 and 150. What is the value of N?

3) The median of nine consecutive odd integers is 73. What is the sum of the smallest and largest of these nine integers?

4) Express the base 2 numeral 1011010 as a base 5 numeral.

<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

5) If C is a whole number so that C^3 is a 4-digit number whose thousands digit is 9, then what is the value of C^2?

6) Ten lines lie in a plane. What is the maximum number of points where they can intersect?

7) The area of a square is 13 square yards. Its length is doubled and its width is quadrupled. How many square feet are in the area of the resulting rectangle?

8) In a basketball game, Jean shot 80% from the foul line while Gail shot 75%, They each scored 12 foul shots, How many total foul shots did the two girls take?

9) Johnny has ten fruits from which to choose. He wants three pieces of fruit. How many 3-fruit possibilities does he have?

Solutions to Category 6
Team Round
Meet #5 - April, 2019

ANSWERS	
1)	160
2)	145
3)	146
4)	330
5)	441
6)	45
7)	936
8)	31
9)	120

1) The intercepted arc of inscribed angle A is twice the measure, in degrees, of angle A. So, the major arc measures 200 degrees and the minor arc 360 - 200, or 160 degrees. Then, central angle O measures 160 degrees.

2) It is a quick matter of eliminating certain numbers as candidates for N for either producing obviously too small or too large results, thus narrowing the choices to a few. Too small: 121, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 142, 143, and 144. Too large: 126, 127, 128, 129, 136, 137, 138, 139, 146, 147, 148, and 149. That leaves as potential successes 125, 135, and 145. For 125: 1! + 2! + 5! = 1 + 2 + 120, or 123, so no. For 135: 1! + 3! + 5! = 1 + 6 + 120, or 127, so no again. For 145: 1! + 4! + 5! = 1 + 24 + 120 = 145. Yay!!!

3) If necessary, students could list the nine numbers to determine the smallest and largest of the set: 65, 67, 69, 71, 73, 75, 77, 79, and 81. The sum of the smallest and largest is 65 + 81, or 146 . . . the same as twice 73.

4) The base 2 numeral 1011010 has a base 10 value of, from right to left:

$$1(0) + 1(2) + 0(4) + 1(8) + 1(16) + 0(32) + 1(64)$$
$$= 0 + 2 + 0 + 8 + 16 + 0 + 64$$
$$= 90.$$

In base 5, this is $3(25) + 3(5) + 0(1) = 330$ (base 5).

5) The only cube in the 9000s is the cube of 21, or 9,261. The value of the square of C is (21)(21), or 441.

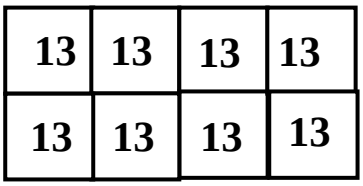
THE SOLUTIONS TO PROBLEMS #6-9 ARE ON THE NEXT PAGE.

6) Ten lines is too many to actually draw on a sheet of paper. Looking for a pattern as the number of intersecting lines increases is more efficient and, likely, more accurate:

number of lines:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
number of intersection points:	**0**	**1**	**3**	**6**	**10**	**15**	**21**	**28**	**36**	**45**

This sequence is known as Triangular Numbers.
So, the max number of points where ten lines can intersect is 45.

7) Increasing the dimensions as listed would create six squares of area 13. So, 8(13) = 104. The question asks for SQUARE FEET. So, multiply the 104 square yards by 9 to get 936 square feet.



8) Gail: 75% of G = 12, so G = 12 / 0.75,
or 16 shots.
Jean: 80% of J = 12, so J = 12 / 0.8, or 15 shots.
In all, Jean and Gail took 16 + 15, or 31 shots.

9) Since the order of Johnny choosing his three fruits is irrelevant, the solution reduces to 10 C 3 = "10 choose 3" = 10! / [(10 - 3)!](3!)

= (10x9x8x7x6x5x4x3x2x1) / (7x6x5x4x3x2x1)(3x2x1)
= (10x9x8) / (3x2x1)
= 10x3x4
= 120.

Using a scientific calculator, a student who identifies the solution as "10 choose 3" can quickly access such a function on the calculator.