IMLEM Meet #1
October, 2018

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #1 - October, 2018

1) A ping and a pong cost a total of $ 1.10. A ping costs a dollar more than a pong. How many cents does a pong cost? Express your answer as a whole number of cents.

2) Arnold can do 80 push-ups in four minutes. How many push-ups could Arnold and six of his friends do, collectively, in 17 minutes if they all exercise at the same rate?

3) A pot of soup is one-fourth full. When 20 ounces of broth is added to the soup and mixed thoroughly, the pot is two-thirds full. How many ounces of soup would there be in a full pot?

ANSWERS

1) _____ cents

2) _____ push-ups

3) _____ ounces

The first mass-produced car in the U.S.A. was the Model T Ford. The first production run started on Oct. 1, 1908. The Volkswagen Beetle surpassed the Model T's total production in 1972.

Solutions to Category 1
Mystery
Meet #1 - October, 2018

Answers	
1)	5
2)	2,380
3)	48

1) A popular incorrect answer is likely 10. Guessing and checking yields an answer of 5, as 5 cents plus $1.05 = $1.10 and there is a dollar difference between $1.05 and 5 cents.

2) Reducing the ratio of 80 push-ups in four minutes, Arnold can do 20 push-ups in one minute. In 17 minutes, Arnold plus six of his friends could do
(20) (1 + 6) (17)
= (20) (7) (17)
= 2,380 push-ups.

3) The 20 ounces of broth added takes up 2/3 - 1/4, or 8/12 - 3/12, or 5/12 of the pot. One-twelfth of the pot would be 4 ounces. A full pot, therefore, would be (4) (12), or 48 ounces.

Category 2
Geometry
Meet #1 - October, 2018

1)

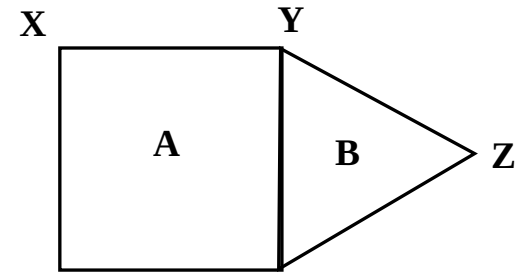
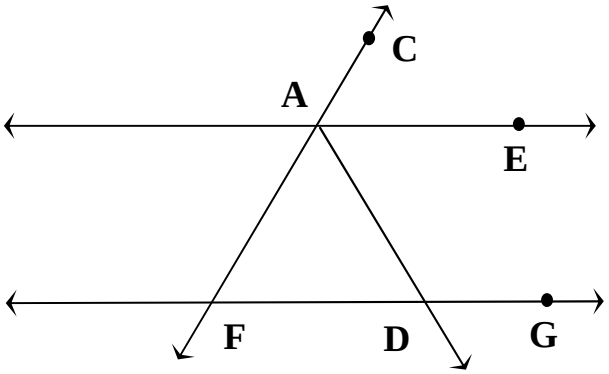
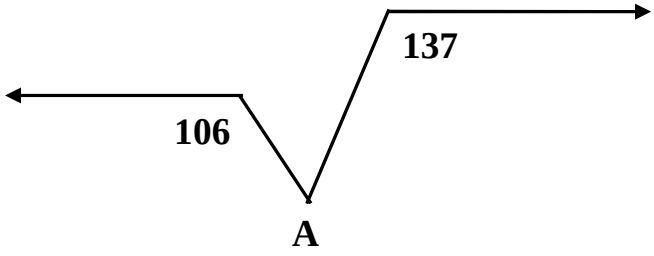


Figure A is a square and figure B is an equilateral triangle. How many degrees are in exterior angle XYZ if its measure is greater than 180 degrees? The square and triangle share a common side.

2) The two horizontal lines are parallel. Segment AD bisects angle EAF. Angle CAE measures 72 degrees. How many degrees are in angle ADG if its measure is less than 180 degrees?



3) The horizontal lines are parallel. How many degrees are in the complement of acute angle A? The measures of two obtuse angles are given in the diagram.



Answers

1) _____

2) _____

3) _____

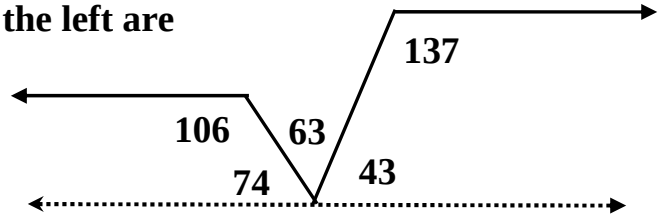
Solutions to Category 2
Geometry
Meet #1 - October, 2018

Answers
1) 210
2) 126
3) 27

1) One interior angle of the square measures 90 degrees. One interior angle of the triangle measures 60 degrees. The combined interior angle XYZ measures 90 + 60, or 150 degrees. So, the exterior angle XYZ measures 360 - 150, 210 degrees.

2) Since angle CAE measures 72 degrees, then so does angle AFD, as alternate interior angles are congruent. Angle FAE is the supplement of each of those angles and measures 180 - 72, or 108 degrees. Since AD bisects angle FAE, then each of the half-angles measures half of 108 degrees, or 54 degrees each. Angle FDA is the remaining as-yet unmeasured angle of triangle FDA. Since the sum of the measures of a triangle is 180 degrees, then angle FDA measures 180 - (72 + 54), or 54 degrees. Its supplement, angle ADG - the angle requested in the question - measures 180 - 54, or 126 degrees.

3) Drawing an auxiliary line through the lowest point in the diagram, parallel to the two horizontal lines, can be helpful. The two interior angles on the left are supplementary, as are the two on the right, hence the measures of 74 and 43 degrees in the diagram. Angle A, combined with the 74 and 43 degree angles, form a straight line. Angle A = 180 - (74 + 43), or 180 - 117, or 63 degrees. The question asks for the complement of angle A, so 90 - 63 = 27 degrees.



Category 3
Number Theory
Meet #1 - October, 2018

1) The number 18 has six factors. Three of these are 1, 2, and 6. What is the sum of the other three factors?

2) The four digit number $3NN1$ is divisible by 9. What is the value of N?

3) The sum of the composite numbers between 140 and 160 is how much greater than the sum of the prime numbers between 140 and 160? "Between" 140 and 160" does not include the numbers 140 and 160.

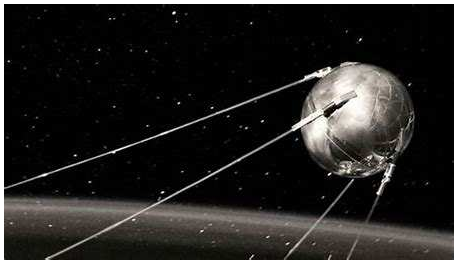
Answers

1) _____

2) _____

3) _____

The Space Age began on October 4, 1957 when Russia launched the first satellite, Sputnik, into orbit, thus propelling the USA to promise to be the first to land a man on the moon.



Solutions to Category 3
Number Theory
Meet #1 - October, 2018

1) The factors of 18 are: 1, 2, 3, 6, 9, and 18. The sum of the missing factors is 3 + 9 + 18 = 30.

2) The sum of the digits must be a multiple of nine in order for the 4-digit number to be divisible by nine. If that sum were 9, then 2N = 5 and N = 2.5 (not a whole number). If that sum were 18, then 2N = 14 and N = 7 (a whole number). If that sum were 27, then 2N = 23 and N = 11.5 (not a whole number and not a single digit). So, the answer is 7.

3) The composite numbers between 140 and 160 are 141, 142, 143, 144, 145, 146, 147, 148, 150, 152, 153, 154, 155, 156, 158, and 159 and their sum is 2393. The prime numbers between 140 and 160 are 149, 151, and 157 and their sum is 457.
The difference between these two sums is 2393 - 457, or 1936.

Answers

1) 30

2) 7

3) 1936

Category 4
Arithmetic
Meet #1 - October, 2018

1) What is the value of $142 - 3(18 + 5)$ **?**

2) As of July 9, the following Boston Red Sox baseball players had the given number of base hits (BH), doubles (D), and runs (R):

	BH	D	R
Mookie Betts	94	24	71
J.D. Martinez	108	21	64
Brock Holt	48	12	18
Andrew Benintendi	98	20	65
Xander Bogaerts	79	25	44
Rafael Devers	82	24	42
Mitch Moreland	68	15	41

Let A = the arithmetic mean (average) of the base hits
B = the mode of the doubles
C = the median of the runs

What is the sum of A + B + C ? Round your answer to the nearest whole number.

3) The arithmetic mean, or average, of six numbers in a set containing fifteen numbers is 28. The arithmetic mean of five other numbers in this set is 32. What is the arithmetic mean of the remaining numbers in the set if the sum of all fifteen numbers is 407? Round your answer to the nearest tenth.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2018

Answers
1) 73
2) 150
3) 19.8

1) $142 - 3(18 + 5)$
$= 142 - 3(23)$
$= 142 - 69$
$= 73$

2) **A = (the sum of the number of hits) divided by the total number of players**
= 577 / 7, or about 82.4 . . .
= 82 when rounded to the nearest whole number.

B = 24, the only repeated number of doubles

C = 44 , as 44 is the middle number when the seven numbers of runs are arranged in order: 18, 41, 42, 44, 64, 65, 71.

The sum of A + B + C = 82 + 24 + 44 = 150.

3) If the mean of six numbers in the set is 28, then their sum is (6)(28), or 168. If the mean of five other numbers is 32, then their sum is (5)(32), or 160. If the total of all fifteen numbers is 407, Then the total of the remaining four numbers is 407 - 168 - 160, or 79. The mean of those four numbers is 79 / 4, or 19.75. Rounding to the nearest tenth yields 19.8.

Category 5
Algebra
Meet #1 - October, 2018

1) What value of W makes the following equation an identity?

$$2 (5x + 3W) = 7x + 3 (x + 8)$$

2) If  **= 10, and**  **= 2, and**  **= 6, then what is the value of** 5  $+ 3$  $- 4$  **?**

3) What value of  **makes the following equation true?**

$3 (2$  $- 5) - 2 (4$  $- 7) = 3$  $- 25 - 11$ 

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2018

1) $2(5x + 3W) = 7x + 3(x + 8)$

$10x + 6W = 7x + 3x + 24$

$6W = 24$

$W = 4$

2) $5(10) + 3(2) - 4(6)$

$= 50 + 6 - 24$

$= 56 - 24$

$= 32$

3) To simplify, substitute a letter, say T, for the ghost:

$3(2T - 5) - 2(4T - 7) = 3T - 25 - 11T$

$6T - 15 - 8T + 14 = -8T - 25$

$-2T - 1 = -8T - 25$

$24 = -6T$

$-4 = T$

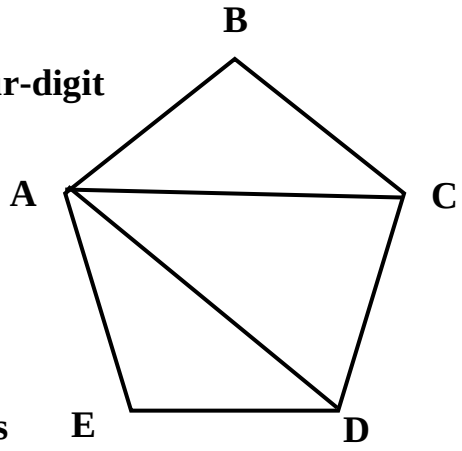
Answers
1) 4
2) 32
3) - 4

Category 6
Team Round
Meet #2 - October, 2018

Each of the following nine problems is worth <u>four</u> points.

1) If the following pattern of letters continues, then what is the 93rd letter?
A M E S B U R Y A M E S B U R Y A M E S . . .

2) Polygon ABCDE is an equilateral pentagon. How many degrees are in the measure of angle ACD?



3) Arrange the digits 0, 1, 2, and 3 to form a four-digit number that is divisible by 2, 3, 5, and 7.

4) If $W=\frac{1}{W}$ **, then evaluate** W^2+6**.**

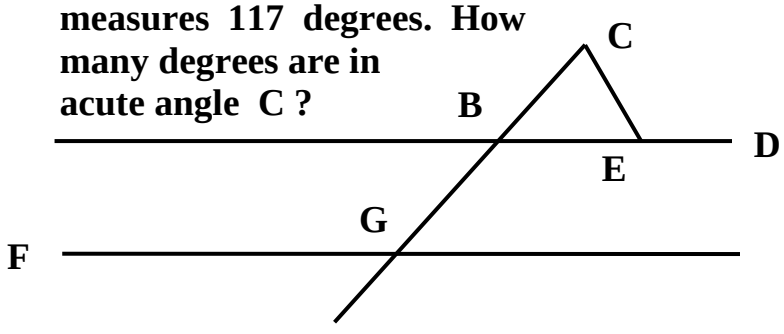
5) Count de Monet has a large bucket that contains pennies. Each day that he deposits more pennies into the bucket, the number of pennies in the bucket doubles. It takes 16 days days to fill the bucket. After how many days was the bucket half full?

6) If the following pattern continues, what is the value of the 300th digit?
2 3 4 5 2 2 3 3 4 4 5 5 2 2 2 3 3 3 4 . . .

7) How many degrees are in one exterior angle of a convex regular polygon that has 12 sides?

8) If a hen and a half can lay an egg and a half in a day and a half, then how many days should it take 15 hens to lay 15 eggs? Express your answer as a decimal.

9) The two horizontal lines are parallel. Angle CED measures 132 degrees. Angle FGB measures 117 degrees. How many degrees are in acute angle C ?



<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

Solutions to Category 6
Team Round
Meet #1 - October, 2018

ANSWERS	
1)	B
2)	72
3)	2310
4)	7
5)	15
6)	4
7)	30
8)	1.5
9)	69

1) The word AMESBURY contains eight letters. Divide 93 by 8 to get 11 with a remainder of 5. The entire word AMESBURY occurs 11 times. The five letters that occur beyond the end of the 11th time are A, M, E, S, B, with the letter B being the 93rd letter.

2) The sum of the measures of the pentagon is (180)(3), or 540 degrees. Each interior angle measures 540 / 5, or 108 degrees. Regarding triangle ABC, angle B measures 108 degrees, so each of the remaining angles measures (180 - 108) / 2, or 36 degrees. Angle ACD = BCD - BCA, or 108 - 36, or 72 degrees.

3) A number divisible by 2 and 5 is divisible by 10. So, the units digit is 0. The sum of the digits 0 + 1 + 2 + 3 = 6 is already divisible by 3, so we only need to arrange the digits 1, 2, and 3 so that the 4-digit number is divisible by 7. Guessing and checking yields the answer 2310.

4) A quick inspection of the first equation gives W = 1, -1. Then $1^2+6=7$.

5) The bucket was half full on the day prior to the 16th day . . . day #15.

6) Counting the repetitions of the four digits helps speed things along. At the start, the four different digits yields a total of four digits. When each digit is repeated, that set has eight digits. For three of each, that set has 12 digits. Look to see when the sum 4 + 8 + 12 + 16 + . . . falls just shy of 300 digits: 4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40 + 44 = 264. The 36th digit of the next set is the answer we seek. In that set, there are 12 twos, 12 threes, and 12 fours, for a total of 36 digits. The 36th digit is, therefore, 4.

THE SOLUTIONS TO QUESTIONS #7-9 ARE ON THE NEXT PAGE.

7) The quickest way to this answer is to know that the sum of the exterior angles of any convex polygon is 360 degrees. 360 / 12 = 30 degrees.

8) The ratio of hens to eggs, 1.5:1.5, simplifies to 1:1. So, one hen could lay one egg in a day and a half. 100 hens could lay 100 eggs in a day and a half. As long as the ratio of hens to eggs remains constant, any number of hens could lay that same number of eggs in a day and a half. The answer is required to be a decimal, so a day and a half = 1.5 days. (This is an oldie but goodie from the very first season of math league in 1963.)

9) The supplemental angles along line BED are critical here. Since corresponding angles are congruent, angle FGB = obtuse angle B = 117 degrees. Its supplement is angle CBE = 180 - 117 = 63 degrees. Angle CEB is the supplement of angle CED, or 180 - 132 degrees = 48 degrees. The three angles of triangle BCE have a sum of 180 degrees. So, angle C = 180 - (63 + 48) = 180 - 111 = 69 degrees.