Intermediate Mathematics League of Eastern Massachusetts

1) If Bob's birthday is today, a Thursday, and his sister's birthday was 115 days ago, then on what day of the week was Bob's sister born?

2) The product of two numbers is the same as their sum. One of the numbers is five. What is the other number? Express your answer as a decimal.

3) How many triangles of any size can be traced by connecting any three vertices (corner points) along the lines drawn in this figure? The three line segments that pass through the interior of the largest triangle are concurrent, so they pass through the same point in the interior (inside) of the largest triangle.
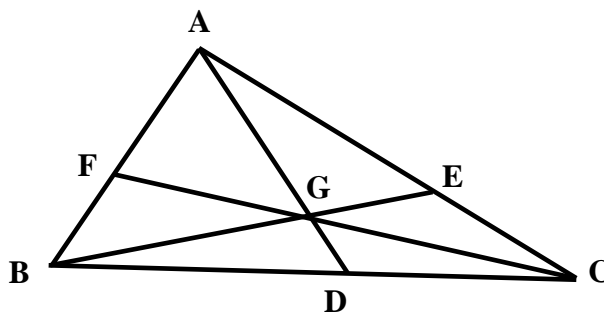
Answers

1) _____

2) _____

3) _____
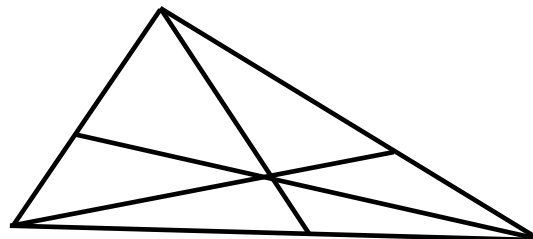
Solutions to Category 1
Mystery
Meet #2 - November, 2015

1) **Divide 115 by 7 to find the number of weeks ago that Bob's sister was born, then use the remaining numbers of days to count backwards from Thursday to the actual day of the week she was born. 115 / 7 = 16 with a remainder of 3. Three days before Thursday is Monday.**

2) **Let X = the other number. Then 5X = X + 5, 4X = 5, so X = 1.25.**

Answers
1) Monday
2) 1.25
3) 16

3) **Assigning letters to each vertex and then naming the triangles:**

```
 1  ABC
 2  ADB
 3  ADC
 4  BEA
 5  BEC
 6  CFA
 7  CFB
 8  CGA
 9  CGB
10  BGA
11  AGF
12  AGE
13  CEG
14  CGD
15  BGD
16  BGF
```

1) What is the maximum (most) number of non-overlapping squares of perimeter 8 centimeters that can be cut from a square that has an area of 144 square centimeters?

2) The area of rectangle MATH is 72 square millimeters. Points W and C are the midpoints, respectively, of sides AT and TH. How many square millimeters are in the area of triangle MWC ?
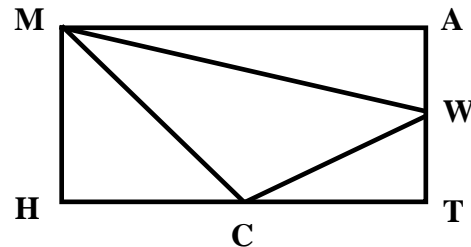
3) For Kamel to walk a mile in his local rectangular park, he must either walk the width 132 times or the perimeter 40 times. How many square feet are in the area of the park? (one mile = 5280 feet)

Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #2 - November, 2015

1) **A square of perimeter 8 has a side length of 8 / 4, or 2, and an area of 2x2, or 4. Divide the area of the large square by 4: 144 / 4 = 36 squares.**

2) **The area of triangle MWC = the area of rectangle MATH minus the sum of the areas of the lettered triangles X, Y and Z. Triangle A is 1/4 of the area of MATH. Triangle B is also 1/4 of the area of MATH. Triangle C is 1/8 of the area of MATH.**

Answers
1) 36
2) 27
3) 1040

MATH - (X + Y + Z)
= 72 - [(0.25)(72) + (0.25)(72) + (0.125)(72)]
= 72 - [18 + 18 + 9]
= 72 - [45]
= 27
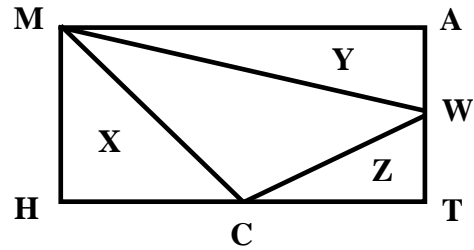
3) **Divide 5280 by 132 to get the width of the rectangle: 5280 / 132 = 40. Let X = the length of the rectangle. The perimeter is 40 + 40 + X + X, or 80 + 2X. The actual perimeter is 5280 divided by 40 (a different 40, which is the number of times Kamel had to traverse the perimeter to walk the mile).**

$$80 + 2X = 5280 / 40$$
$$80 + 2X = 132$$
$$2X = 52$$
$$X = 26$$

So, the length of the rectangle is 26 and the width is 40, so the area is length x width, or (26)(40) = 1040 square feet.

Check: 40 times the perimeter = 40 (132) = 5280
132 times the width = 132 (40) = 5280.

1) One prime factor of 272 is 2. What is the only other prime factor of 272 ?

2) How many of the following eleven numbers are factors of every positive whole number that is divisible by both 9 and 12 ?

    3   4   6   8   18   24   27   36   45   72   108

3) To treat his cold, Wheezer takes some orange juice every 50 minutes, a cough drop every 80 minutes, and a teaspoon of honey every 96 minutes during his waking hours from 6:30 A.M. to 10:15 P.M. He took all three at 1:53 P.M. on Tuesday. On what day and at what time will he again take all three? You must include A.M. or P.M. in your answer.

Answers

1) _____

2) _____

3) _____ = day

    _____ = time

The United States Weather Bureau began its operation in November of 1870.

Solutions to Category 3
Number Theory
Meet #2 - November, 2015

1) The prime factorization of 272 is 2 x 2 x 2 x 2 x 17.
   So, the only other prime factor of 272 is 17.

2) Whole numbers that are divisible by both 9 and 12 are
   multiples of their LCM, namely, 36.
   The only numbers in the list that are factors of 36 are
   3, 4, 6, 18, and 36. So, there are five such numbers.

3) Find the LCM of 50, 80, and 96 by first factoring each
   number:

$$50 = 2 \text{ x } 5 \text{ x } 5$$
$$80 = 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 5$$
$$96 = 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3$$

The LCM is the product of the highest powers of any of the factors in any of
the three prime factorizations, = 2 x 2 x 2 x 2 x 2 x 3 x 5 x 5 = 2400.
Since Wheezer ends his waking hours at 10:15 P.M., 10:15 - 1:53 =
8 hours and 22 minutes, or 502 minutes. He still has 2400 - 502, or 1898
minutes, before the next confluence of his three doses. Dividing 1898 by 60
yields 31 hours and 38 minutes. From his waking time of 6:30 A.M. until his
bed time at 10:15 P.M. is 15 hours and 45 minutes. Two days of waking hours
would be 31 hours and 30 minutes, just 8 minutes shy of 31 hours and 18
minutes. So, he will take all three doses just 8 minutes into his fourth day -
Friday - at 6:38 A.M.

Check: Tuesday - 502 minutes
                Wednesday - 945 minutes
                 Thursday - 945 minutes
                   Friday - 8 minutes
                   Total - 502 + 945 + 945 + 8 = 2400 minutes.

Note: Both the day AND the time must be correct for a student to earn
             full credit.

1) I watched only **27** minutes of a three-hour baseball game. What percent of the game did I miss?

2) Abraham Lincoln was a tall man but he was **20%** taller at **228** centimeters when he wore his famous "stovepipe" hat. How many centimeters tall was his hat?

3) What is $\dfrac{5}{4}$ of **2.5%** of $1.\overline{7}$ of $0.2\overline{4}$ of **1440**? Express your answer as a common fraction.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - November, 2015

1) I missed 180 - 27, or 153 minutes of the game.
153 / 180 = 0.85 = 85%

2) Let X = Abraham Lincoln's height in centimeters.
$$X + (20\% \text{ of } X) = 228$$
$$X + 0.2X = 228$$
$$1.2X = 228$$
$$X = 228 / 1.2$$
$$X = 190$$
$$228 - 190 = 38$$
So, Lincoln is 190 cm tall while his hat is 38 cm tall.

3)
$$\frac{5}{4} \text{ of } 2.5\% \text{ of } 1.\overline{7} \text{ of } 0.2\overline{4} \text{ of } 1440$$

$$= \quad \frac{5}{4} \text{ of } \frac{25}{1000} \text{ of } \frac{16}{9} \text{ of } \frac{11}{45} \text{ of } 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

Factoring numerators and denominators and "cancelling" common
factors yields the fraction 176 / 9.

1) If $3N + 2Y = 25$, then what is the value of $12N + 8Y - 19$?

2) Romeo has two more 4-pound cantaloupes than 6-pound cantaloupes. The cantaloupes weigh 158 pounds in all. How many cantaloupes does Romeo have?

3) If $E \Psi H = \dfrac{E + H}{E - H}$ and $(12 \Psi 4) \Psi N = -0.5$ then what is the value

of N?

Answers
1) _____
2) _____
3) _____

The first cash register was patented on November 4, 1880.

Solutions to Category 5
Algebra
Meet #2 - November, 2015

1) $3N + 2Y = 25$, so $4(3N + 2Y) = 12N + 8Y = 100$,
   so $12N + 8Y - 19 = 100 - 19 = 81$.

2) Let $\quad$ C = the number of 6-pound cantaloupes
   $\quad\quad$ C + 2 = the number of 4-pound cantaloupes
   $\quad\quad\quad$ 6C = the weight, in pounds, of the 6-pounders
   $\quad\quad$ 4(C + 2) = the weight, in pounds, of the 4-pounders

$$6C + 4(C + 2) = 158$$
$$6C + 4C + 8 = 158$$
$$10C + 8 = 158$$
$$10C = 150$$
$$C = 15 \quad \text{So, there are } 15 \text{ 6-pounders.}$$
$$C + 2 = 17 \quad \text{and there are } 17 \text{ 4-pounders}$$

and there are $15 + 17$, or **32** cantaloupes in all.

3) $(12 \, \Psi \, 4) = \dfrac{12 + 4}{12 - 4} = \dfrac{16}{8} = 2.$

$(2 \, \Psi \, N) = \dfrac{2 + N}{2 - N} = -0.5.$

$-0.5 \, (2 - N) = 2 + N$

$-1 + 0.5N = 2 + N$

$-3 = 0.5N$

$N = -6$

Category 6
Team Round
Meet #2 - November, 2015

1) A square of area 256 square inches is cut into two rectangles whose areas differ by 32 square inches. How many inches are in the perimeter of the smaller rectangle?

2) What is the remainder when the product of all the prime numbers between 1 and 105 is divided by 105 ?

3) If the sum of the reciprocals of the first four positive integers that are squares of positive integers is $\frac{C}{D}$, then what is the sum C + D if $\frac{C}{D}$ is in lowest terms?

4) Five cans of cocoa can be traded for two bowls of beans. A bowl of beans can be traded for seven boxes of nuts. How many boxes of nuts is a can of cocoa worth? Express your answer as a common fraction.

5) A quadrilateral has the following vertices on a Cartesian (rectangular) coordinate plane. How many square units are in its area? Express your answer as a decimal. (- 7, - 3)   (-5, 4)   (4, 9)   (7, - 5)

6) Five distinct lines - none an edge of the paper - are drawn on a sheet of paper. Only two of the lines are parallel to each other. What is the greatest number of polygonal regions that can be created?

| ANSWERS |
| --- |
| 1) _____ |
| 2) _____ |
| 3) _____ |
| 4) _____ |
| 5) _____ |
| 6) _____ |

NOTE: *For the first time in over 20 years, since the inception of the Team Round,* **IMLEM** *is introducing a variety of Team Round formats. The standard one used since 1993 contained five questions, plus a sixth question that incorporated the answers to the first five. Today's round has six independent questions. A future round may have nine independent questions, or may be like the standard of the past 20 years. Enjoy this new* **IMLEM** *adventure!*

Solutions to Category 6
Team Round
Meet #2 - November, 2015

ANSWERS
1)   46
2)   0
3)   349
4) $\dfrac{14}{5}$
5)   121.5
6)   15

1) Let X = the smaller length into which one 16-inch side is divided, and $16 - X$ the the larger length.
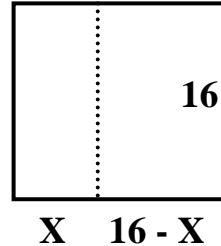
$$16(16 - X) - 16X = 32$$
$$256 - 16X - 16X = 32$$
$$256 - 32X = 32$$
$$224 = 32X$$
$$X = 7$$

The perimeter of the smaller is $7 + 7 + 16 + 16 = 46$.

2) The product of the prime numbers will include $3 \times 5 \times 7 = 105$. So, the product of the primes is divisible by 105 and the remainder is 0.

3) The first four positive integers that are squares of integers are $1, 4, 9,$ and 16. The product of their reciprocals is

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{144}{144} + \frac{36}{144} + \frac{16}{144} + \frac{9}{144} = \frac{205}{144} \qquad \text{Then } 205 + 144 = 349.$$

4) Let C = one can of cocoa
        B = one bowl of beans
        N = one can of nuts.
$5C = 2B$ and $1B = 7N$, so $2B = 14N$.
By the transitive property, $5C = 14N$, so $1C = \dfrac{14}{5}N$.

5) **The area of the quadrilateral is the area of the large outermost rectangle minus the sum of the four triangles and one small rectangle in the dotted regions outside the quadrilateral. Using the differences of coordinates to determine the lengths of bases and heights, we have the following (see next page):**
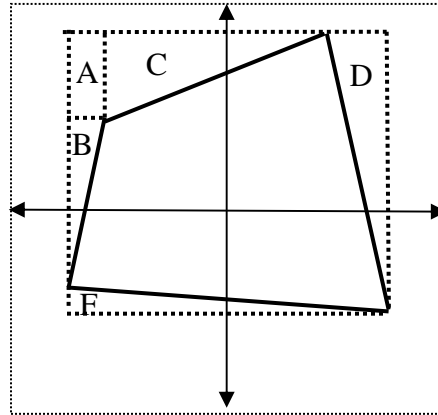
area of A (rectangle) = (2)(5) = 10
area of B (triangle) = (0.5)(2)(7) = 7
area of C (triangle) = (0.5)(5)(9) = 22.5
area of D (triangle) = (0.5)(3)(14) = 21
area of F (triangle) = (0.5)(2)(14) = 14
The total area of these five spaces is 10 + 7 + 22.5 + 21 + 14 = 74.5
The area of the outermost rectangle is (14)(14) = 196.

So, the area of the quadrilateral is 196 - 74.5, or 121.5.

6) **One possible way to display this phenomenon is as follows, with the numbers used to count the number of regions.**