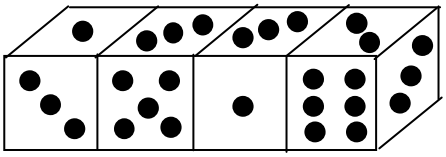
IMLEM Meet #5
March, 2015

Intermediate Mathematics League of Eastern Massachusetts

This is a calculator meet!

Category 1
Mystery
Meet #5 - March, 2015
Calculator meet

1) Four standard cubical dice, with faces numbered 1, 2, 3, 4, 5, and 6, are arranged as shown below with nine of the faces visible from this perspective. What is the total number of dots (called "pips") that are not visible in this view of the diagram, including the pips on the faces of the dice that touch one another?



2) Classrooms on one side of the corridor on the fourth floor at the Long Middle School are numbered with consecutive odd numbers from 401 through 499. Mrs. Abegg's room is 3/4 of the way from room 413 to room 469. What is Mrs. Abegg's room number?

3) Classrooms on other side of the corridor on the fourth floor at the Long Middle School (from problem #2) are numbered with consecutive even numbers from 400 through 498. If the doors of any two neighboring rooms are 38 feet apart, then how many feet apart are the doors of room number 408 and room number 482?

Answers

1) _____

2) _____

3) _____

March 26 is National Spinach Day.

Solutions to Category 1
Mystery
Meet #5 - March, 2015

Answers
1) 57
2) 455
3) 1406

1) The sum of the number of pips on any one die is 1 + 2 + 3 + 4 + 5 + 6 = 21. The sum on all four dice is 4 x 21, or 84. The sum of the pips shown is 1 + 3 + 3 + 5 + 3 + 1 + 2 + 6 + 3 = 27. The total number of pips on the faces hidden from view is 84 - 27, or 57.

2) Three-fourths of (469 - 413) is (3/4)(56) = 42. So, 42 is added onto 413 to make room #455.

3) Unlike in problem #2, it is important to note how many door distances of 38 feet there are from room #408 to room #482. That number is half of (482 - 408), or (1/2)(74) = 37. So, (38)(37) = 1406 feet.

Category 2
Geometry
Meet #5 - March, 2015
Calculator meet

1) The area of the top circular surface of a cylinder is 289π square feet. The height of the cylinder is 19 feet. How many cubic feet are in the volume of the cylinder? Express your answer in terms of π.

2) A cube and a rectangular solid have the same volume. The dimensions of the rectangular solid are 12 cm by 18 cm by 125 cm. How many square centimeters are in the area of one surface (face) of the cube?

3) A tennis ball has a diameter of 2.5 inches. Three tennis balls fit tightly inside a cylindrical can. What percent of the can is air that lies outside of the tennis balls? Use $\pi \approx 3.142$. Round your final answer to the nearest tenth of a percent.

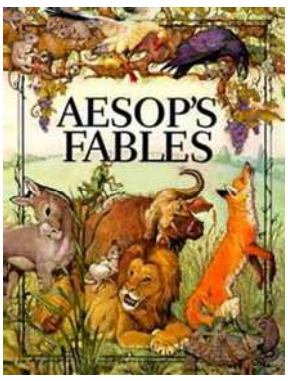
Answers

1) _____

2) _____

3) _____ **%**

On March 26, 1484, William Caxton printed his translation of Aesop's Fables.



Solutions to Category 2
Geometry
Meet #5 - March, 2015

Answers
1) 5491π
2) 900
3) 33.3

1) The solution is one simple step - that the volume of a cylinder is equal to the product of its height and the area of the top (or bottom) circular surface. (area of circular top)(height) = (289π)(19) = 5491π cubic feet.

2) The volume of the rectangular solid = (length)(width)(height) = (12)(18)(125) = 27,000 cc. The length of one side of the cube is equal to the cube root of 27,000, or 30 cm. The area of one surface of that cube is equal to the square of one side, or (30)(30) = 900 square cm.

3) The amount of air inside the can that lies outside the tennis balls is equal to the volume of the cylindrical, minus the volume of the tennis balls = $\pi r^2 h - 3\left(\frac{4}{3}\right)\pi r^3$. The radius of the top of the can is half the diameter of one of the tennis balls = (0.5)(2.5) = 1.25 inches. The height of the can is three times the diameter of one ball, = (3)(2.5) = 7.5 inches.

Air space = (3.142)(1.25)(1.25)(7.5) - (3)(4/3)(3.142)(1.25)(1.25)(1.25)
= (36.820312) - 24-546875
= 12.27343

The percent of air inside the can that lies outside the three tennis balls is 12.27343 / 36.820312 = 0.3333 which, to the nearest tenth of a percent, is 33.3%.

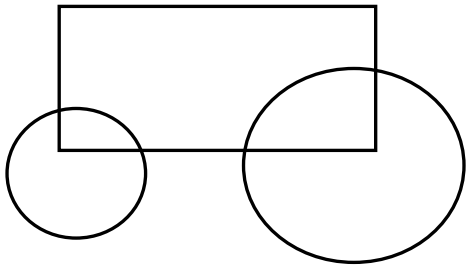
Category 3
Number Theory
Meet #5 - March, 2015
Calculator meet

1) Victoria has 83 coins in her pocket. Fifty-seven of the coins contain silver while 64 contain copper. The rest are silver-copper alloys, which are mixtures of the two metals. How many of the coins are a silver-copper alloy?

2) Set W = { multiples of 4 between 30 and 90 }
Set Y = { factors of 240 }

What is the sum of all the elements (members) in the set $W \cap Y$, that is, the intersection of sets W and Y ?

3) Santana arranged some of the toys that his school donated to Toys for Tots into roped areas as shown. All 824 trucks are in the rectangle. The 342 orange toys are in the smaller circle while the 487 red toys are in the larger circle. There are 72 red trucks and 119 orange trucks. If 2265 toys were donated in all, then how many of the toys were outside of the areas that Santana had roped off?



Answers

1) _____

2) _____

3) _____

Leonard Nimoy of Star Trek fame, born on March 26, 1931, lost his courageous battle with COPD and passed away peacefully on February 27, 2015.

Solutions to Category 3
Number Theory
Meet #5 - March, 2015

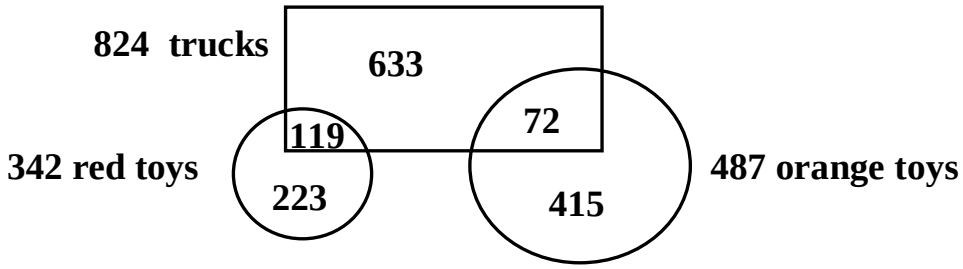
Answers
1) 38
2) 228
3) 803

1) Subtract the number of coins from the sum of silver + copper: (57 + 64) - 83 = 121 - 83 = 38.

2) Set W = { 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88 }
The elements in the intersection = {40, 48, 60, 80 }.
Their sum is 40 + 48 + 60 + 80 = 228.

3) The numbers in this diagram reflect how many toys have the assigned characteristics:

2265 = total number of toys



The number of toys that lie outside the roped area is

2265 - (633 + 72 + 119 + 415 + 223)

= 2265 - 1462

= 803

Category 4
Arithmetic
Meet #5 - March, 2015
Calculator meet

1) How many ways can five books be arranged on a book shelf that has space for exactly five books?

2) Candace has a pocketful of Skittles candies: 17 yellow, 28 red, 31 blue, and 8 green. If she reaches into her pocket and selects one at random, what is the probability that she will choose a red one? Express your answer as a common fraction (reduced to lowest terms).

3) If a fair coin is flipped six times, what is the probability that the result is four or more heads? Express your answer as a percent, rounded to the nearest whole percent.

. . . . . .

ANSWERS

1) _____

2) _____

3) _____

Sandra Day O'Connor, America's first female justice of the Supreme Court, was born on March 26, 1930.

Solutions to Category 4
Arithmetic
Meet #5 - March, 2015

Answers
1) 120
2) $\frac{1}{3}$
3) 34 (%)

1) 5 ! = 5x4x3x2x1 = 120

2) The total number of Skittles is 17 + 28 + 31 + 8 = 84.

$$\frac{red}{total} = \frac{28}{84} = \frac{14}{42} = \frac{7}{21} = \frac{1}{3}$$

3) Use the "choose" or "combination" function, since there is no importance as to the order in which the heads land.

number of (4 heads) = $_6C_4$ = $\frac{6!}{4!(6-4)!} = 15$

number of (5 heads) = $_6C_5$ = $\frac{6!}{5!(6-5)!} = 6$

number of (6 heads) = $_6C_6$ = $\frac{6!}{6!(6-6)!} = 1$

These numbers can be found in the 7th row of Pascal's Triangle:

1 6 15 20 15 6 1 where the sum is 64.

The total number of possible outcomes for tossing six coins is 2^6 **, or 64.**

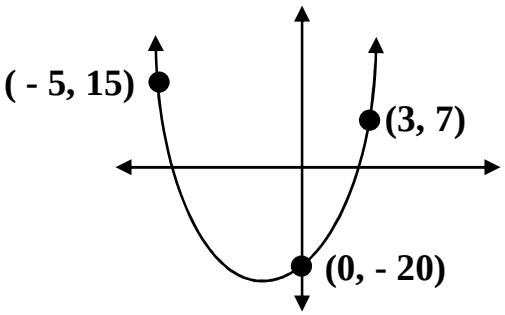
So, the probability of getting four or more heads = $\frac{15+6+1}{64} = \frac{22}{64} = 0.34375.$

Rounding to the nearest whole percent yields 34%.

Category 5
Algebra
Meet #5 - March, 2015
Calculator meet

1) If (N + 3)(N - 7) = 0, then what is the average of the two possible values of N that make this quadratic equation true?

2) The quadratic equation $Y = aX^2 + bX + c$, when graphed, is a parabola that passes through the points (3, 7) and (-5, 15) and has a Y-intercept of (0, -20), as shown. What is the value of a + b + c ?



3) A rocket is launched vertically from ground level at an initial velocity (starting speed) of 128 feet per second. For how many seconds is the rocket at least 112 feet above ground level? Use the quadratic equation where g = - 16 feet/second/second, the constant of gravity at the surface of the Earth, t is the time in seconds that the rocket is in the air, v is the initial velocity, h is the initial height of the rocket in feet, and y is the height in feet of the rocket at any time t seconds.

$$y = gt^2 + vt + h$$

. . . . . .

ANSWERS

1) _____

2) _____

3) _____

Jonas Salk announced his development of the polio vaccine on March 26, 1953 . . . "It is always with excitement that I wake up in the morning, wondering what my intuition will toss up to me, like gifts from the sea. I work with it and rely on it. It's my partner."

Solutions to Category 5
Algebra
Meet #5 - March, 2015

Answers	
1)	2
2)	- 15
3)	6

1) If (N + 3)(N - 7) = 0, then either N + 3 = 0 or N - 7 = 0, then N = - 3 or N = 7. The average of these solutions is (- 3 + 7) / 2, or 2.

2) One possible strategy:

1: substitute the X and Y coordinates of each of the known points into the general quadratic equation,
2: solve the resulting system to find the values of a, b, and c, and then
3: find the sum a + b + c.

for (3, 7): $7 = a(3^2) + b(3) + c$ **. . . or, simplified,** $7 = 9a + 3b + c$

for (- 5, 15): $15 = a((-5)^2) + b(-5) + c$ **. . . or, simplified,** $15 = 25a - 5b + c$

for (0, - 20): $-20 = a((0)^2) + b(0) + c$ **. . . or, simplified,** $-20 = c$.

Now substitute - 20 for c in the first two equations, yielding
7 = 9a + 3b - 20 and 15 = 25a - 5b - 20
Simplifying: 27 = 9a + 3b and 35 = 25a - 5b
Divide both sides of the first equation by 3 and both sides of the second equation by 5, yielding: 9 = 3a + b and 7 = 5a - b. Adding the two equations yields: 16 = 8a, so, a = 2 and then b = 3. So, a + b + c = - 15.

3) Substitute: G = - 16; V = 128; Y = 112; H = 0.

$112 = (-16)(T^2) + 128T + 0$	**Use the substitutions listed above.**
$0 = -16(T^2) + 128T - 112$	**Subtract 112 from both sides.**
$0 = T^2 - 8T + 7$	**Divide both sides by - 16.**
$0 = (T - 1)(T - 7)$	**Factor.**

T = 1 or T = 7. Therefore, the rocket was at or above 112 feet above the ground from 1 second until 7 seconds into the flight, so the rocket was in flight for the difference 7 - 1, or 6 seconds.

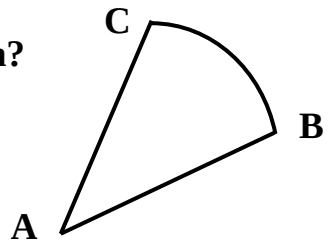
Category 6
Team Round
Meet #5 - March, 2015

1) What is the thousand's digit of the smallest 5-digit even number that can be written using the digits 1, 2, 5, 6, and 9 if each digit is used only once?

2) A cycle at a traffic light is 48 seconds. The light is red for 13 seconds, yellow for five seconds, and green for the rest of the time. If a car passes by this light, then what is the probability that it is not red? Express your answer as a decimal, rounded to the nearest hundredth.

3) When 2015 is divided by seven, the remainder is added to the units digit (one's place) of 7^{2015} What is the sum?

4) Sector ABC consists of radii AB and AC and arc BC, as shown. Arc BC measures 38 degrees. How many centimeters are in the perimeter of the sector? Use $\pi \approx 3.14$. AB = 7.62 cm. Round your answer to the nearest tenth of a centimeter.



5) A spherical ball of ice is tightly contained in a cubical box, such that it touches all six sides of the box. After the ice completely melts, how many inches deep is the water inside the box? The diameter of the ball is 24 inches. Use $\pi \approx 3.142$. Round your answer to the nearest tenth of an inch.

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

6) Using the answers to #1-5, evaluate the following expression:

$$\sqrt[N]{25A^C - 300B - 10(D+E) - 85}$$

Use $N = \sqrt{C}$.

Solutions to Category 6
Team Round
Meet #5 - February, 2015

ANSWERS	
1)	2 = A
2)	0.73 = B
3)	9 = C
4)	20.3 = D
5)	12.6 = E
6)	23

1) Use 1 as the ten-thousands place and use 6 as the larger even units digit. Then arrange the remaining digits to create 12,596.

2) If red = 13, then (not red) = 48 - 13, or 35. (not red)/total time = 35/48 = 0.729 . . . , or 0.73 rounded to the nearest hundredth.

3) 2015 divided by 7 yields 287 with a remainder of 6. The powers of 7 have units digits that occur in a cyclic pattern of four numbers: 7, 9, 3, 1, 7, 9, 3, 1, . . . so that the 2015th power of 7 is 3. Add: 6 + 3 = 9.

4) Arc BC is 38/360 of the circumference of a circle with as radius of 7.63, or (38/360)(2)(pi)(radius) = (38/360)(2)(3.14)(7.62) = 5.0512 . . The perimeter of the sector is the sum of the two radii and the arc = 7.62 + 7.62 + 5.0512 . ., or 20.2912 . . . which, when rounded to the nearest tenth, is 20.3 centimeters.

5) The volume of the ball is (4/3)(pi)(cube of the radius) = (4/3)(3.142)(24/2)(24/2)(24/2) = 7239.1676. The bottom surface of the cubical box has a length and width equal to the diameter of the ball, so its bottom area is (24)(24), or 576 square inches. The volume of the box is (area of bottom surface)(height) which equals the volume of the melted ice, so
7239.1676 = (area of bottom surface of cube)(height) = (576)(height).
So, height = 7239.1676 / 576, or 12.56799 . . . or 12.6 inches when rounded to the nearest tenth of an inch.

6) $\sqrt[N]{25A^C - 300B - 10(D+E) - 85} = \sqrt[3]{25\times 2^9 - 300(0.73) - 10(20.3 + 12.6) - 85}$

$= \sqrt[3]{25\times 512 - 219 - 10(32.9) - 85} = \sqrt[3]{12{,}800 - 219 - 329 - 85}$

$= \sqrt[3]{12{,}800 - 219 - 329 - 85} = \sqrt[3]{12{,}167} = 23.$ **(N = 3, or sqrt 9)**