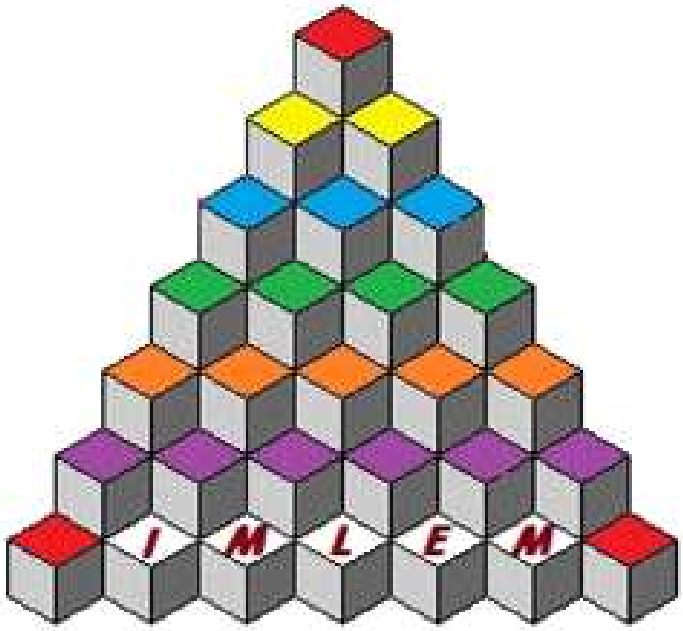
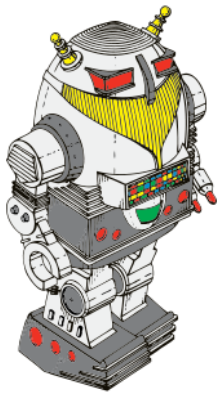
IMLEM Meet #5
March, 2014

Intermediate Mathematics League of Eastern Massachusetts



Category 1 *Calculator Meet*
Mystery
Meet #5 - March, 2014 *50th anniversary edition*



1) Evan has 27 mini-Minion toys. Connor gave $\frac{3}{5}$ of his own mini-Minion toys to Evan to bring Evan's total to 66. How many mini-Minions did Connor have in the beginning?

2) A certain scale only registers weights that are greater than 6 pounds. Bindy wants to know the weights of her puppy, bunny, and kitten.
Let P = the puppy's weight in pounds,
K = the kitten's weight in pounds, and
B = the bunny's weight in pounds.
If K + B = 7 and K + P = 8 and B + P = 9, then how many pounds does Bindy's kitten weigh?

3) Dow Nunda, a kangaroo, wants to reach the top of a hill. He starts at the base of the hill and is able to jump 12 feet per jump. However, it is winter, so the slope is slippery. He slips back 3 feet per jump after he lands. If the top of the hill is 400 feet from the base, then how many jumps must Dow make in order to reach the top of the hill?

ANSWERS

1) _____

2) _____ pounds

3) _____ jumps

Category 1 - Solutions
Mystery
Meet #5 - March, 2014

ANSWERS
1) 65
2) 3
3) 45

1) $\frac{3}{5}$ **of Connor's toys is 66 - 27, or 39. So,**
$\frac{5}{3}$ **x 39 = 65.**

2) The difference between the first two equations yields P - B = 1.
Adding this result to the third equation gives 2P = 10, and P = 5.
Since K + P = 8, then K = 3.

3) $\frac{400}{9}$ **gives the number of jumps required (just a bit over 44), given the number of net feet gained as 12 - 3, or 9. Dow's 44th jump brings him only 399 feet up the hill before he slid back three feet, so he needs to jump one more time in order to reach the summit.**

Category 2 *50th anniversary edition*

Geometry

Meet #5 - March, 2014 *Calculator Meet*

1) Melinda unwraps a gift contained in a cube-shaped box that has a volume of 729 cubic inches. How many square inches of wrapping paper are on the surface of the box (all sides)?

2) A cylindrical box of Earthquake Oats is 28 centimeters tall. Its circular top has a diameter of 12 centimeters. Wyatt puts 125 cubic centimeters of oats into individual tight-lock bags for his daily breakfasts. All oats must be in a bag, even if the last bag is not full. How many bags in all does Wyatt need in order to store a boxful of oats?

3) It takes 21 seconds to inflate a spherical balloon to a diameter of 8 inches. How long should it take to inflate a similar balloon to a diameter of 40 inches if it is inflated at the same rate as for the smaller balloon? If the answer is B minutes and C seconds, then give the value of B if C is less than 60.

ANSWERS

1) _____ sq. in.

2) _____ bags

3) _____ = B

Solutions to Category 2
Geometry
Meet #5 - March, 2014

Answers
1) 486
2) 26
3) 43

1) Take $\sqrt[3]{729}$ to find the length of one edge = 9 inches. Each face has an area = 9 x 9, or 81 square inches. Since there are six surfaces, the surface area is 6 x 81, or 486 square inches.

2) Since the formula for the volume of a cylinder depends on its radius, take half of the 12 cm diameter so that the radius is 6 cm long.
Volume of cylinder = $\pi r^2 h = \pi(6^2)(28) \approx$ 3166.72 cubic cm.
Divide the volume by 125 to find the number of bags needed:
$\frac{3166.72}{125} \approx$ **25.3 bags. So, 26 bags are required to hold all of the oats.**

3) The simplest approach requires this knowledge: The ratio of the volumes of two similar objects is equal to the cube of their linear ratio. (Also, while we are at it: The ratio of the surface areas of two similar objects is equal to the square of their linear ratio.)
Dividing the two diameters, 40 / 8, yields the fact that the larger balloon is five times the diameter of the smaller balloon, or 125 (five cubed) times its volume.
Multiply 21 by 125 to find the number of seconds required to inflate the larger balloon = 2625 seconds.
Divide 2625 by 60 to convert the time to minutes = 43.75 minutes = 43 minutes and 45 seconds. Since the question asks for the value of B, the number of minutes, then B = 43.
A more popular approach would be to calculate the two volumes and then divide to find how many times larger the big balloon is:
$$\frac{\frac{4}{3}\pi(20^3)}{\frac{4}{3}\pi(4^3)} = \frac{\frac{4}{3}\pi(8000)}{\frac{4}{3}\pi(64)} = \frac{8000}{64} =$$ **125.**
The remainder of the solution would be the same as for the former solution.

Category 3 *Calculator meet*
Number Theory
Meet #5 - March, 2014 *50th anniversary edition*

1) Seventeen students in homeroom 308 signed up for "Tiddlywinks for Beginners" and 15 signed up for "Advanced Thumb Wrestling." If there are 26 students in homeroom 308 and every student signed up for at least one activity, then how many students signed up for both activities?

2) Set A = { multiples of 6 between 20 and 70 }
Set B = { factors of 360 }
How many elements (numbers) are in $A \cap B$ (that is, the intersection of sets $A \cap B$) ?

3) Over the past few years, the following numbers of kids played on these Little League Baseball teams:

Number	Team
38	Red Sox
47	Yankees
33	Orioles
12	Red Sox and Yankees
9	Yankees and Orioles
5	Red Sox and Orioles
4	All three teams

If 416 kids played in all, then how many did not play on any of these three teams?

Answers

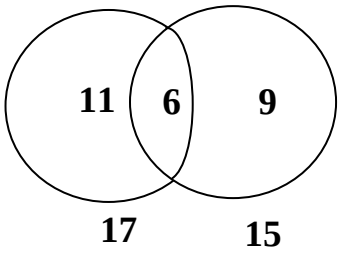
1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #5 - March, 2014

Answers	
1)	6
2)	4
3)	320

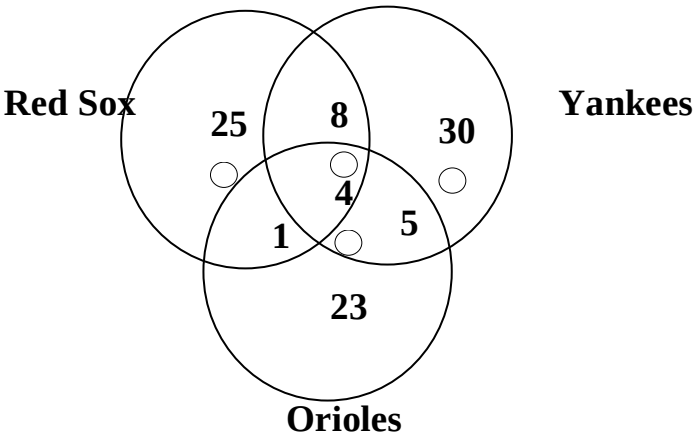
1) 

Let X = the number of students who signed up for both activities.

$$(17 + 15) - X = 26$$
$$32 - X = 26$$
$$X = 6$$

2) Set A = { 24, 30, 36, 42, 48, 56, 60, 66 }
Set B = (lots of elements, but only need to check whether 360 is divisible by the numbers in Set A }
360 is only divisible by 24, 30, 36, and 60. Thus, there are 4 members in the intersection of Sets A and B.

3)

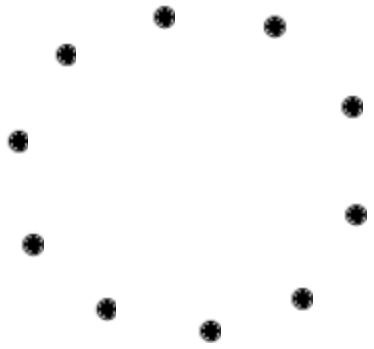


$$416 - (25 + 30 + 23 + 8 + 5 + 1 + 4)$$
$$= 416 - (96)$$
$$= 320$$

Category 4 *Calculator Meet*
Arithmetic

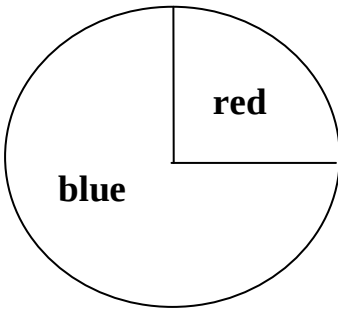
Meet #5 - March, 2014 *50th anniversary edition*

1) If three points are selected at random from ten points that are equally spaced in a circle, as shown, then how many triangles can be drawn?



2) How many different 9-letter "words," or arrangements of letters, can be made by using the letters in the name RAMANUJAN ?

3) A Bernoulli trial is a probability experiment that has only two possible outcomes. Consider a game where a player must spin a circular spinner that stops on either red or blue, as shown. The angle at the center of the spinner is a right angle.



Let n = the number of spins
x = the number of successes
p = the probability of success (desired outcome) for one trial
f = the probability of non-success (failure) for one trial.

The probability of achieving x successes for n spins is given by this formula:

$$({}_nC_x)(P^x)(f^{n-x})$$

If Liz spins the spinner 20 times, then what is the probability that it will stop on blue exactly 13 times? Express your answer as a percent, rounded to the nearest whole percent.

ANSWERS

1) _____

2) _____

3) _____ %

<u>footnote</u>: Jacob Bernoulli was a Swiss mathematician (1655 - 1705).

Solutions to Category 4
Arithmetic
Meet #5 - March, 2014

Answers
1) 120
2) 30,240
3) 11

1) $_{10}C_3 = \mathbf{120}$

2) $\dfrac{{}_9P_9}{(3!)(2!)} = \dfrac{(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(3)(2)(1)(2)(1)}$

$= (9)(8)(7)(5)(4)(3)$

$= \mathbf{30{,}240}$

Note: ${}_9P_9$ represents the number of arrangements if all nine letters were different. We divide that value by 3! to account for the repeated "A" and by 2! for the repeated "N."

3) $({}_{20}C_{13})(0.75^{13})(0.25^{7})$

$\approx (77{,}520)(0.023760)(0.000061035)$

≈ 0.1124

$\approx$ **11 %**

Category 5 *Calculator Meet*
Algebra

Meet #5 - March, 2014 *50th anniversary edition*

1) What are the two values of N that solve $N^2 - 4N = 32$ **?**

2) To fence a rectangular pasture, a farmer uses 110 meters of fencing to enclose a 750 square meter area. The length of the pasture is longer than the width. What is the length of the pasture ?

3) Hoses are attached to two outdoor faucets so that a backyard swimming pool can be filled. If both faucets are used, it takes two hours to fill the pool. If either faucet were used alone, then one faucet would take three hours less than the other to fill the pool. What is the least amount of time required to fill the pool if only one faucet is used?

ANSWERS

1) ____ and ____

2) _____ meters

3) _____ hours

Solutions to Category 5
Algebra
Meet #5 - March, 2014

Answers
1) - 4 and 8
2) 30
3) 3

1) $N^2 - 4N = 32$

$N^2 - 4N - 32 = 0$

$(N+4)(N-8) = 0$

Either $(N+4) = 0$ **or** $(N-8) = 0$

So, N = - 4 or N = 8

2) If the perimeter is 110 meters, then half the perimeter is 55 meters. So, the sum of the length and width is 55. Let X = one dimension and 55 - X = the other dimension.

(length) (width) = area
(X) (55 - X) = 750
$55X - X^2 = 750$
$X^2 - 55X + 750 = 0$
(X - 30) (X - 25) = 0
Either X = 30 or X = 25.
The question calls for the larger dimension to be the length = 30.

3) (Rate) (Time) = Work.
Let X = the number of hours it would take for Hose A to fill the pool, if working alone. Let X + 3 = the number of hours it would take for Hose B to fill the pool, if working alone. So, the rate for Hose A is $\left(\frac{1}{X}\right)$ **of a job per hour and the rate for Hose B is** $\left(\frac{1}{X+3}\right)$ **of a job per hour.** $\left(\frac{1}{X}\right)(2)$ **represents the amount of work that Hose A does in 2 hours.** $\left(\frac{1}{X+3}\right)(2)$ **represents the amount of work that Hose B does in 2 hours.**

The amount of work of Hose A plus the amount of work of Hose B is equal to one job:

$$\left(\frac{1}{X}\right)(2) + \left(\frac{1}{X+3}\right)(2) = 1$$

$$\left(\frac{2}{X}\right) + \left(\frac{2}{X+3}\right) = 1$$

$$2X + 6 + 2X = X^2 + 3X$$

$$2(X + 3) + 2X = (X)(X + 3)$$

$$\text{So, } X = 3 \text{ or } X = -2$$

We will take the positive solution, 3, as it represents the number of hours it takes for Hose A to fill the pool while working alone. Since Hose B takes 3 hours longer, the answer to the question is 3 hours (the least amount of time it takes when only one faucet is used).

Category 6 *50th anniversary edition*

Team Round
Meet #5 - March, 2014

1) A cube's length, width, and height are each increased by 50%. By what percent is its volume increased? If the answer is N percent, then give the value of N.

2) In a new lottery game, Gigabux, a player must choose six different numbers from the set of integers from 1 through 70, inclusive. If every player selects a different set of numbers (order does not matter), then what is the greatest number of people who can play?

3) One natural number is two more than another. If the sum of their squares is 580, then what is the larger of the two numbers?

4) How many feet are in the perimeter of a rectangle whose diagonal is 75 feet if it is similar to a rectangle that is 36 by 48 feet?

5) Kaynine, a dog, is leashed to a 40-foot rope attached to the back corner of a square pen with an area of 289 square feet. The back of the pen is against a very long fence so that Kaynine cannot cross it. If the dog is outside the pen, on the same side of the fence as is the pen, then how many square feet of roaming space does Kaynine have? If the answer is $K\pi$, and K is a decimal, then what is the value of K ?

ANSWERS

1) _____ % = A

2) _____ = B

3) _____ = C

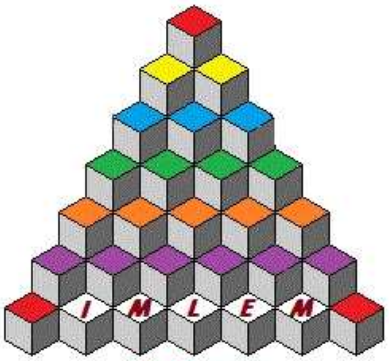
4) _____ = D

5) _____ = E

6) _____

6) Using the answers from questions #1-5 evaluate the following expression:

$$\sqrt{B + 900(A - 100) + 4(E - 60) - D + 10C - 30}$$



Solutions to Category 6
Team Round
Meet #5 - March, 2014

ANSWERS

1) 337.5 = A

2) 131,115,985 = B

3) 18 = C

4) 210 = D

5) 541.25 = E

6) 11,460

1) $(1.5\ L)(1.5\ W)(1.5\ H) = 3.375\ (LWH)$
$= 337.5\%\ (LWH)$
$= 337.5\%$ of original volume

2) ${}_{70}C_6 = 131,115,985$

3) Let X = the smaller natural number
X + 2 = the larger natural number

$$X^2 + (X+2)^2 = 580$$
$$X^2 + X^2 + 4X + 4 = 580$$
$$2X^2 + 4X + 4 = 580$$
$$X^2 + 2X + 2 = 290$$
$$X^2 + 2X - 288 = 0$$

(X - 16) (X + 18) = 0 So, X = 16 or - 18. Discard the solution of - 18 because it is not a natural number. If X = 16, then X + 2 = 18. Therefore, the larger of the two numbers is 18.

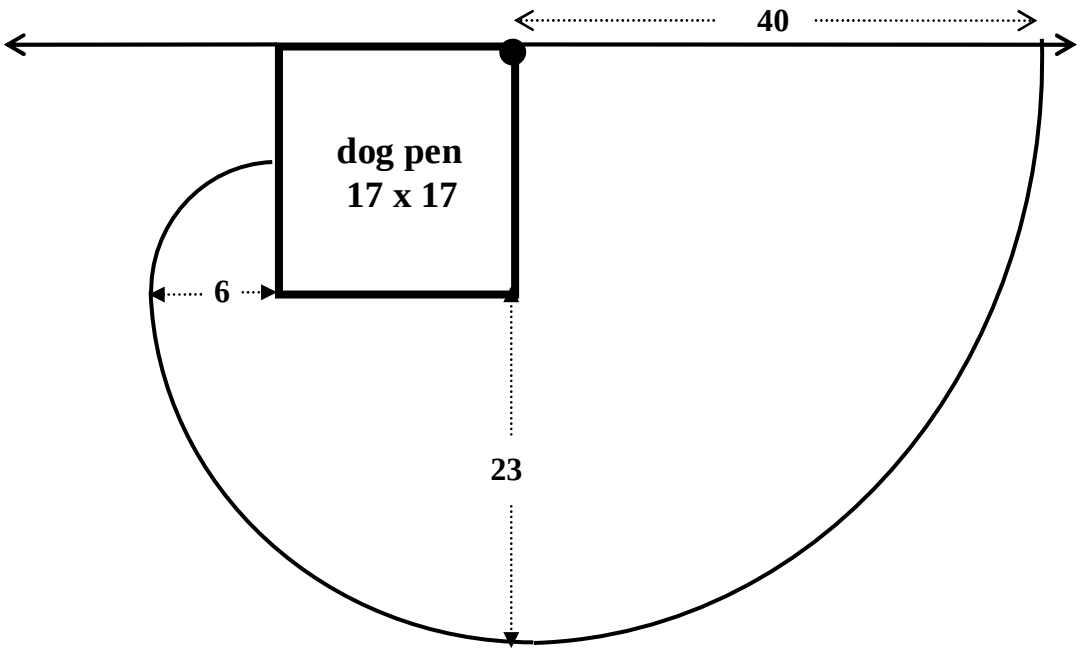
4) First, find the length of the diagonal of the smaller rectangle:
$36^2 + 48^2 = D^2$ ... $1296 + 2304 = D^2$ ... $3600 = D^2$ ... $60 = D$
Now divide 75 by 60 to find the scaling factor = 1.25.
So, the perimeter of the larger rectangle is $1.25\ [(2)(36) + (2)(48)]$
$= 1.25\ [72 + 96]$ which equals $1.25\ (168) = 210$ feet.

5) The pen whose area is 289 square feet has a length of 17 feet. The following diagram on the next page indicates the limit that the dog can roam is the rope is taut. Kaynine's roaming space is the sum of three sectors, each having a central angle of 90 degrees and equals:

$$\frac{1}{4}\pi r_1^2 + \frac{1}{4}\pi r_2^2 + \frac{1}{4}\pi r_3^2$$
$$= \frac{1}{4}\pi (40)^2 + \frac{1}{4}\pi (23)^2 + \frac{1}{4}\pi (6)^2$$
$$= \frac{1}{4}\pi (1600) + \frac{1}{4}\pi (529) + \frac{1}{4}\pi (36)$$

= $400\pi + 132.25\pi + 9\pi$

= 541.25π **So, K = 541.25.**



6) $\sqrt{B + 900(A - 100) + 4(E - 60) - D + 10C - 30}$

= $\sqrt{131{,}115{,}985 + 900(337.5 - 100) + 4(541.25 - 60) - 210 + 10(18) - 30}$

= $\sqrt{131{,}115{,}985 + 900(237.5) + 1925 - 210 + 180 - 30}$

= $\sqrt{131{,}115{,}985 + 213{,}750 + 1925 - 210 + 180 - 30}$

= $\sqrt{131{,}331{,}600}$

= **11,460**